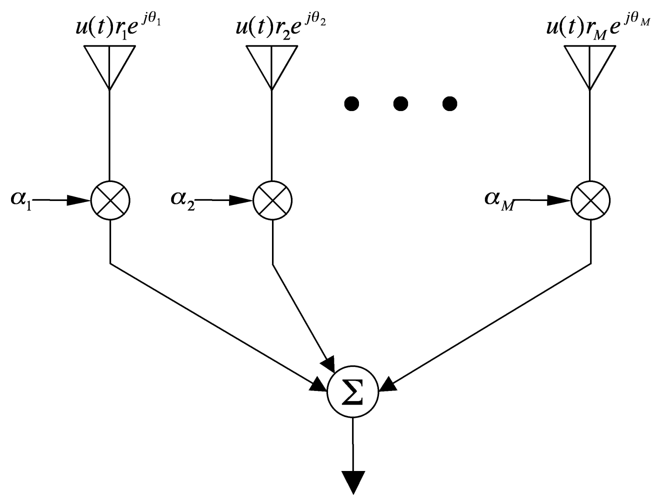
Wireless Communications
Lecture 7

[Diversity]

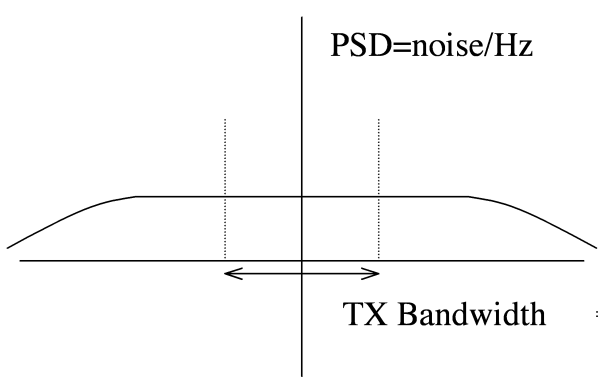
- Goal : Improve signal quality by providing alternatives for mitigating fading.
- Different types:
  - Space $\begin{cases} \text{RX} \\ \text{TX} \end{cases}$
  - Time
  - Frequency
  - Polarization
- This RX space diversity : putting multiple antennas at uncorrelated distance in the receiver
  - Microscopic diversity: mitigate small-scale fading.
  - Macroscopic diversity: mitigate large-scale fading.

[Microscopic diversity]



Instantaneous $\gamma_i = \text{SNR}_i = \frac{|ch|^2 P_T}{N_R}$,

$r(t)$	:	$\lvert ch(t)\rvert$
$u(t)$	:	TX baseband linear combiner
N_R	:	receiver noise power
P_T	:	TX power $\quad P_T = \mathbf{E}[u^2(t)]$



$\Rightarrow N_R = N_B \times \text{BW}, \; N_B = \text{Noise/Hz}$

Received signal at the output of the combiner is

$$\underbrace{\sum_{i=1}^{M} \alpha_i r_i e^{j\theta_i} u(t)}_{\text{signal}} + \sum_{i=1}^{M} \alpha_i n_i, \quad n_i : \text{RX noise at the } i\text{th branch}$$

Then the received SNR is

$$\gamma_{\Sigma} = \text{Total instantaneous SNR} = \frac{P_T |\sum_{i=1}^{M} \alpha_i e^{j\theta_i} r_i|^2}{N_R \sum_{i=1}^{M} |\alpha_i|^2}$$

1) Selection Combining:

γ_{Σ}*: maximum* SNR_i *is chosen.*

The cumulative distribution function (cdf) of γ_{Σ} is

$$P_{\gamma_{\Sigma}}(\gamma_T) = prob\{\gamma_{\Sigma} \le \gamma_T\} = prob\{\gamma_1, \gamma_2, \cdots, \gamma_M \le \gamma_T\}$$

If uncorrelated on different braches

$$\begin{aligned} P_{\gamma_{\Sigma}}(\gamma_T) &= \prod_{i=1}^{M} prob\{\gamma_i \le \gamma_T\}, \quad \gamma_i \sim \text{exponential pdf: } p_{\gamma_i}(\gamma_i) = \lambda_i e^{-\lambda_i \gamma_i} \quad \lambda_i = \frac{1}{\text{SNR}_i} \\ &= \prod_{i=1}^{M} (1 - e^{-\lambda_i \gamma_T}) \end{aligned}$$

If all the same λ, $\quad P_{\gamma_{\Sigma}}(\gamma_T) = (1 - e^{-\lambda \gamma_T})^M$.

The probability density function (pdf) of γ_{Σ} is

$$p_{\gamma_{\Sigma}}(\gamma_T) = \prod_{i=1}^{M} (1 - e^{-\lambda_i \gamma_T}) \sum_{i=1}^{M} \frac{\lambda_i e^{-\lambda_i \gamma_T}}{1 - e^{-\lambda_i \gamma_T}}$$

If $\lambda_i = \lambda_j, \quad \forall i, j$

$$\begin{aligned} p_{\gamma_{\Sigma}}(\gamma_T) &= M(1 - e^{-\lambda \gamma_T})^{M-1} \lambda e^{-\lambda \gamma_T} \quad \text{with} \\ \bar{\gamma}_{\Sigma} &= M\lambda \int_0^{\infty} \gamma_T (1 - e^{-\lambda \gamma_T})^{M-1} e^{-\lambda \gamma_T} d\gamma \\ &= \frac{1}{\lambda} \sum_{i=1}^{M} \frac{1}{i} \end{aligned}$$

Note: As M↑, impact of diversity ↓.

2) Equal Gain Combining:

Co-phase the signals: $\alpha_i = e^{-j\theta_i} \Rightarrow$

$$\gamma_\Sigma = \frac{P_T|\sum r_i|^2}{N_R \sum_M} = \frac{P_T}{MN_R}\left(\sum r_i\right)^2$$

The outage probability is

$$P_{\gamma_\Sigma}(\gamma_T) = prob\left\{\frac{P_T}{MN_R}\left(\sum r_i\right)^2 \le \gamma_T\right\}$$

No close form expression exists. For i.i.d branches

$$\bar{\gamma}_\Sigma = \frac{\sum \gamma_i}{M}$$

So, the average does not change but the probability of outage ↓.

3) Maximal Ratio Combining:

Find optimum α_i

$$\gamma_\Sigma = \frac{P_T}{N_R}\frac{|\sum \alpha_i e^{j\theta_i} r_i|^2}{\sum |\alpha_i|^2}$$

$\alpha_i = \beta_i e^{-j\theta_i} \Rightarrow$ cophased.

$$\begin{aligned}
\gamma_\Sigma &= \frac{P_T}{N_R}\frac{(\sum \beta_i r_i)^2}{\sum \beta_i^2} \\
&\Rightarrow \frac{\partial \gamma_\Sigma}{\partial \beta_j} = 0 \\
&\Rightarrow \left(\sum \beta_i r_i\right)(2r_j)\sum \beta_i^2 = 2\beta_j\left(\sum \beta_i r_i\right)^2 \\
&\Rightarrow r_j \sum \beta_i^2 = \beta_j \sum \beta_i r_i \\
&\Rightarrow \beta_j = r_j
\end{aligned}$$

Weight is proportional to link quality.

$$\begin{aligned}
\gamma_\Sigma &= \frac{P_T}{N_R}\frac{\left(\sum r_i^2\right)^2}{\sum r_i^2} = \frac{P_T}{N_R}\sum r_i^2 \\
&= \sum \gamma_i, \quad \text{sum of SNR of all the paths}
\end{aligned}$$

If i.i.d, γ_Σ is χ^2(chi distribution) with expected value $\bar{\gamma}_\Sigma = M\bar{\gamma}$ and variance $2M\bar{\gamma}$. The probability density function (pdf) is

$$p_{\gamma_\Sigma}(\gamma) = \frac{\gamma^{M-1} e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}^M (M-1)!}, \quad \gamma \geq 0$$

The outage probability for a given threshold γ_T is

$$\begin{aligned} P_{\text{out}} &= prob\{\gamma_\Sigma < \gamma_T\} = \frac{1}{\bar{\gamma}^M (M-1)!} \int_0^{\gamma_T} \gamma^{M-1} e^{-\gamma/\bar{\gamma}} d\gamma \\ &= \ldots \text{integrating by part} \\ &= 1 - e^{-\gamma_T/\bar{\gamma}} \sum_{k=1}^{M} \frac{(\gamma_T/\bar{\gamma})^{k-1}}{(k-1)!} \end{aligned}$$