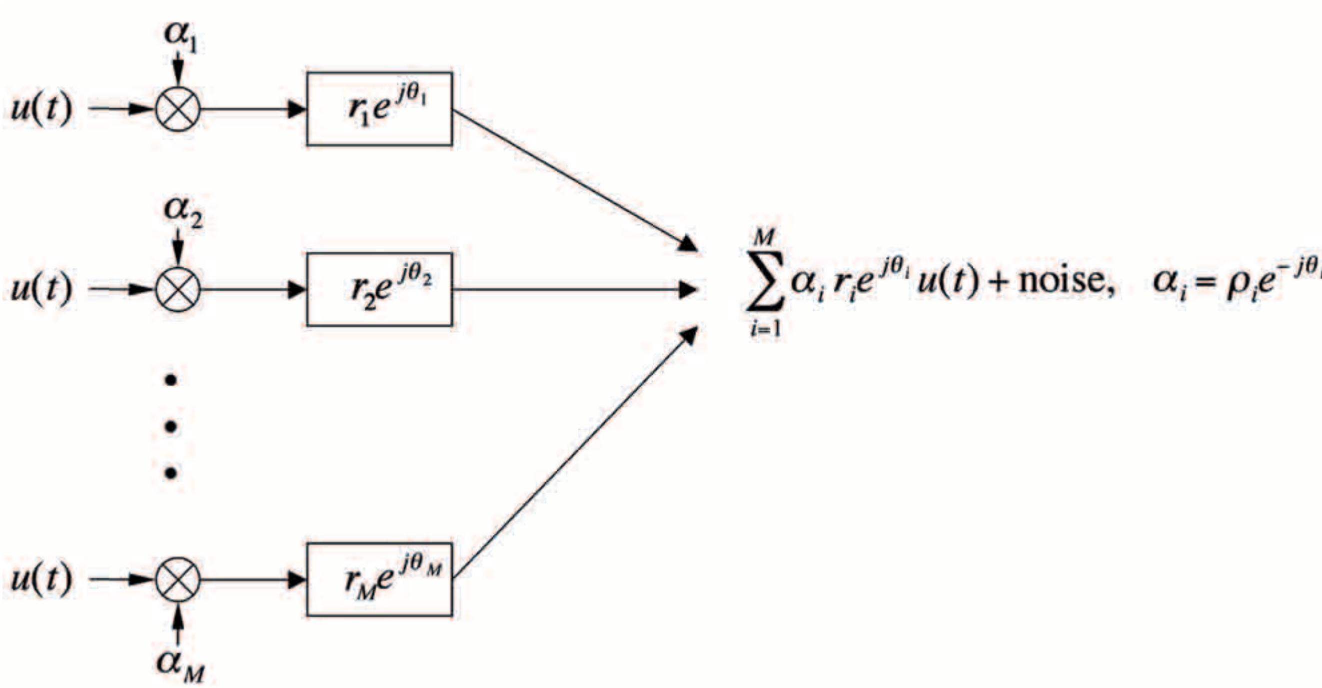
Wireless Communications
Lecture 8

[TX diversity] Channel is known at the TX.



Total TX power $= \sum |\alpha_i|^2 P_T, \quad \sum_{i=1}^{M} \rho_i^2 = 1$. The received signal is

$$r(t) = \text{overall received signal} = \sum r_i \rho_i u(t) + \text{noise}$$

The SNR is

$$\gamma_\Sigma = \frac{\left[\sum r_i \rho_i\right]^2 P_T}{N_R}$$

Find max $\left[\sum r_i\rho_i\right]^2$ such that $\sum \rho_i^2 = 1$ with Lagrange integral

$$\left[\sum r_i\rho_i\right]^2 - \lambda\left(\sum \rho_i^2 - 1\right)$$

$$\begin{aligned}
&\Rightarrow \quad \partial/\partial\rho_j = 2\left[\sum r_i\rho_i\right] r_j - 2\lambda\rho_j = 0\\
&\Rightarrow \quad \rho_j = \frac{r_j\sum r_i\rho_i}{\lambda}\\
&\Rightarrow \quad \sum \rho_i^2 = 1 = \frac{1}{\lambda^2}\sum r_i^2\left[\sum r_j\rho_j\right]^2 = 1\\
&\Rightarrow \quad \left[\sum r_j\rho_j\right]^2\sum r_i^2 = \lambda^2\\
&\Rightarrow \quad \lambda = \sum r_j\rho_j\sqrt{\sum r_i^2}\\
&\Rightarrow \quad \rho_j = \frac{r_j\sum r_i\rho_i}{\sum r_i\rho_i\sqrt{\sum r_i^2}} = \frac{r_j}{\sqrt{\sum r_i^2}}
\end{aligned}$$

SNR is similar to RX diversity

$$\gamma_\Sigma = \frac{P_T}{N_R}\frac{\left[\sum r_j^2\right]^2}{\sum r_j^2} = \frac{P_T}{N_R}\sum r_j^2 = \sum_i \mathrm{SNR}_i$$

[Macroscopic Diversity]

- Microscopic diversity does not help with shadowing.
- $\alpha_i = |ch_i| \Rightarrow P_r = \overline{|ch_i|^2}P_T$= received power. $P_{r_{\mathrm{dB}}}$ is in normal distribution.
- Receive from two branches

$$\begin{aligned}
\text{outage Probability} &= prob\{P_{r_{1,\mathrm{dB}}}, P_{r_{2,\mathrm{dB}}} < P_{\min}\}\\
&= prob\{P_{r_{1,\mathrm{dB}}} < P_{\min}\} \times prob\{P_{r_{2,\mathrm{dB}}} < P_{\min}\}\\
&= \left[1 - Q\left(\frac{P_{\min} - \overline{P_{r_{1,\mathrm{dB}}}}}{\sigma_{1,\mathrm{dB}}}\right)\right]\left[1 - Q\left(\frac{P_{\min} - \overline{P_{r_{2,\mathrm{dB}}}}}{\sigma_{2,\mathrm{dB}}}\right)\right]
\end{aligned}$$

It is like selection diversity.

[Handoff]

[Digital Communication with AWGN channel]

- The transmitted signal is

$$\begin{aligned} s(t) &= \text{Re}\left\{u(t)e^{j2\pi f_c t}\right\} \\ &\quad u(t) = A(t)e^{j\phi(t)} \\ &\quad u_I(t) = A(t)\cos\phi(t) \\ &\quad u_Q(t) = A(t)\sin\phi(t) \end{aligned}$$

- Signal energy is defined as

$$E_u = \int_{T_1}^{T_2} |u(t)|^2 dt,$$

$$\begin{aligned} \text{norm signal} &= \|u(t)\| = \sqrt{\int_{T_1}^{T_2} |u(t)|^2 dt} \\ \text{Euclidean distance} &= \|x_1(t) - x_2(t)\| = \sqrt{\int_{T_1}^{T_2} |x_1(t) - x_2(t))|^2 dt} \end{aligned}$$

[Linear Modulation]

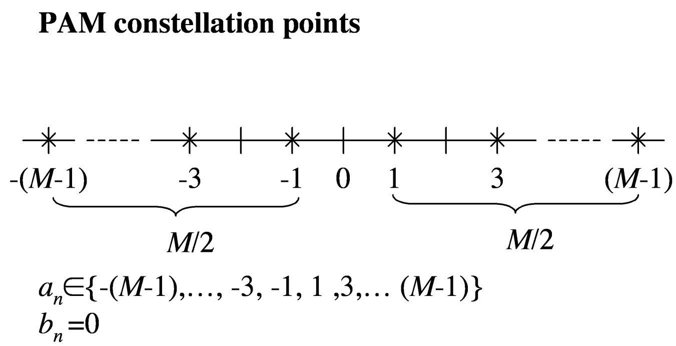
The modulated signal, $u(t)$ is

$$u(t) = \sum d_n g(t - nT_s), \quad g(t) : \text{Pulse shaper}$$

When the sampling period is longer than the period of carrier frequency such as $T_s \gg 1/f_c$, $d_n = a_n + jb_n$. If M possibility where $M = 2^k \Rightarrow \log M/T_s = k$ bits/T_s = bit rate. The modulated signal becomes

$$u(t) = \sum a_n g(t - nT_s) + j \sum b_n g(t - nT_s)$$

[PAM]



The transmitted signal over one symbol time is given by

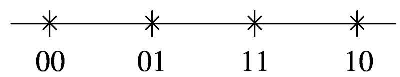
$$s_m(t) = \mathrm{Re}\{a_m g(t) e^{j2\pi f_c t}\} = a_m g(t)\cos(2\pi f_c t)$$

The energy is

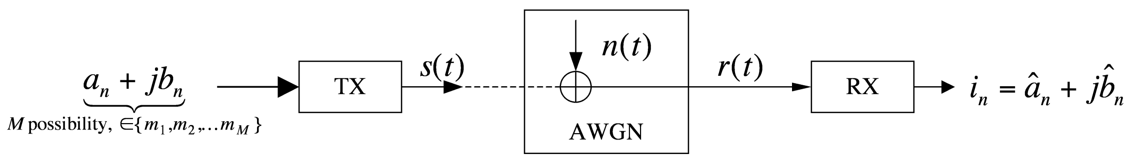
$$\begin{aligned} E_{s_m}^{T_s} &= \int_0^{T_s} s_m^2(t)dt \\ &= \int_0^{T_s} a_m^2 g^2(t)\cos^2(2\pi f_c t)dt \\ & \quad a_m \text{ and } E_g \text{ for } [0, T_s] \text{ are constants} \\ &= \frac{1}{2} A_m^2 E_g \end{aligned}$$

Gray coding to minimize the error if the adjacent is closer. For example $M = 4 = 2^k, \quad k = 2$,

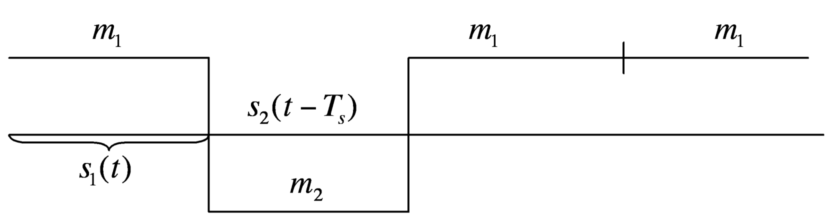
M=4, after encoded (Gray code)



$$P_e = \sum_i prob\{\hat{a}_n + j\hat{b}_n \neq m_i | m_i \text{ is sent}\} prob\{m_i \text{ is sent}\}$$



Goal of the RX to minimize P_e.

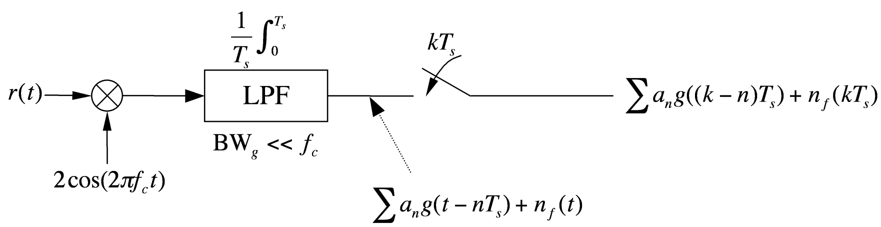


$s_1(t), s_2(t), \cdots, s_M(t)$ in one period

[Receiver for PAM]

$$r(t) = \sum a_n g(t - nT_s)\cos(2\pi f_c t) + n(t)$$

$$\text{where} \quad g(zT_s) = \begin{cases} 0 & z \neq 0 \\ 1 & z = 0 \end{cases}$$



The output of receiver is

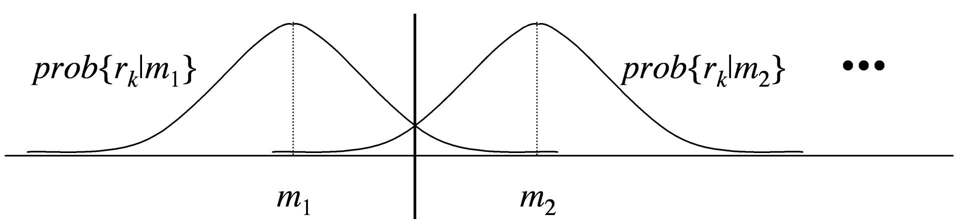
$$r(k) = a_k + n_f(k)$$

[Decision Regions] MAP (Maximum a posteriori): Choose m_i if max $prob\{m_i \text{ is sent}|r(k)\}$

$$prob\{a_k = m_i|r(k)\} \quad = \quad \frac{prob\{r_k|a_k = m_i\}prob\{a_k = m_i\}}{prob\{r_k\}}$$

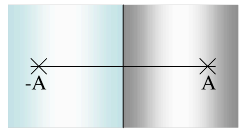
If $prob\{a_k = m_i\}$ equal,
$\Rightarrow$ m_i that maximizes $prob\{m_i|r_k\}$ will maximize $prob\{r_k|m_i\}$.

ML(Maximum Likelihood)
$\Rightarrow$ choose m_i if $prob\{r_k|m_i\}$ is maximized.



Decision Regions: Points closest to each message: (*Voronoi Cell*)

BPSK



$$P_e = Q(A/\sigma), \quad A^2 : \text{Baseband signal power such that } \mathbf{E}\{a_k^2\} = A^2$$

Pulse shaper $g(t)$ has the constraint $\int_0^T g^2(t)dt = 1$. We define SNR as A^2/σ^2. Then, P_e becomes.

$$P_e = Q(\sqrt{\text{SNR}}) \quad \text{Fading SNR} = |ch|^2 \text{SNR}_{\text{AWGN}}$$

$$\begin{aligned}
\bar{P}_e &= \int_0^\infty Q(\sqrt{\text{SNR}})\lambda e^{-\lambda \text{SNR}} d\text{SNR} \\
&\quad \text{define SNR} = x \\
&= -e^{-\lambda x} Q(\sqrt{x})\Big|_0^\infty - \int_0^\infty e^{-\lambda x} \frac{1}{\sqrt{2\pi}} e^{-x/2} \frac{1}{2} x^{-1/2} dx \\
&= \frac{1}{2} - \frac{1}{2}\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\lambda x} e^{-x/2} x^{-1/2} dx \\
&= \frac{1}{2} - \frac{1}{2\sqrt{2\pi}} \int_0^\infty \frac{e^{-x(\lambda+1/2)}}{\sqrt{x}} dx \\
&\quad \sqrt{x} = u \rightarrow \frac{dx}{2u} = du \\
&= \frac{1}{2} - \frac{1}{2\sqrt{2\pi}} \int_0^\infty 2u \frac{e^{-u^2(\lambda+1/2)}}{u} du \\
&= \frac{1}{2} - \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-u^2(\lambda+1/2)} du \\
&\quad \frac{1}{2\sigma^2} = \lambda + \frac{1}{2} \rightarrow \sigma = \frac{1}{\sqrt{2(\lambda+1/2)}} \\
&= \frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{2\lambda+1}} \\
&\quad \lambda = \frac{1}{\text{SNR}} = \frac{1}{\bar{\gamma}} \\
&= \frac{1}{2}\left[1 - \frac{\sqrt{\bar{\gamma}}}{\sqrt{2+\bar{\gamma}}}\right] \\
&= \frac{1}{2}\left[1 - \frac{\sqrt{0.5\bar{\gamma}}}{\sqrt{1+0.5\bar{\gamma}}}\right]
\end{aligned}$$