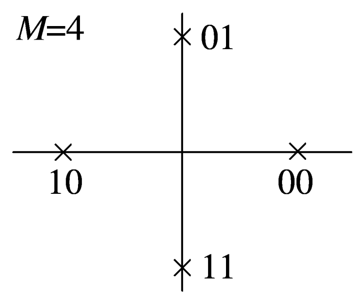
Wireless Communications
Lecture 9

See Cioffi's class (379) of Stanford University for more details.

[MPSK] The modulated signal of MPSK is defined as

$$u(t) = \sum d_{(n)} g(t - nT_s),$$
$$\text{where} \quad a_m = \left\{ e^{j2\pi \frac{(m-1)}{M}} \right\}, \quad 1 \le m \le M, \quad M = 2^k$$



Given $d_{(n)}$ is sent, the energy over one period is

$$\mathbf{E}\{|u(t)|^2\} = E_g, \quad \text{independent of } M$$

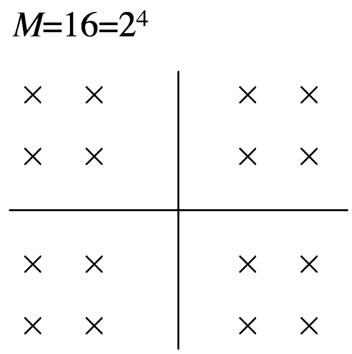
Note that the passband energy is approximately 1/2 of baseband energy.

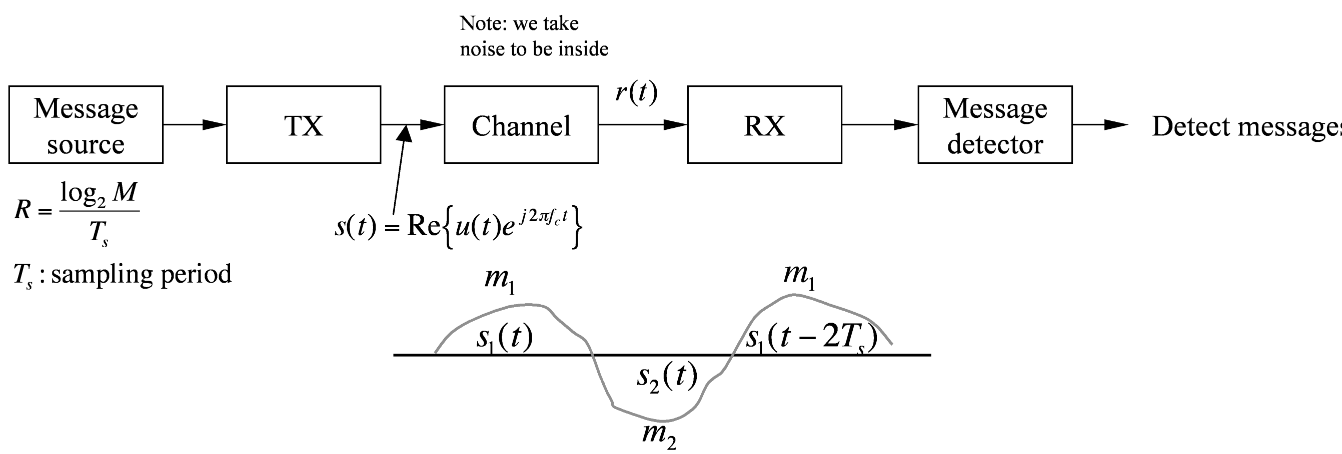
[QAM] The encoded signal of QAM is

$$d_n = a_n + jb_n = A_n e^{j\theta_n}$$

The transmitted energy is

$$E_{s_n} \simeq \frac{1}{2} E_g A_n^2$$





[Example] Send $+A$ for 1 and $-A$ for 0

$$\Rightarrow s_i(t) = \begin{cases} \pm A g(t) \cos \omega_c t & 1 \le i \le 2 \\ 0 & \text{else} \end{cases}$$

where $g(t)$: Pulse shaper

[Vector Representation] To have a general model for all modulations: *Take a signal defined on* $[0, T_s]$ *and write it as a function of its basis functions.*

$$s_i(t) = \sum_{j=1}^{N} s_{ij} \varphi_j(t)$$

The inner product of $\varphi_i(t)$ is

$$<\varphi_j(t), \varphi_k(t)> = \int_0^{T_s} \varphi_j(t)\varphi_k(t)dt = \begin{cases} 1 & k = j \\ 0 & \text{else} \end{cases}$$

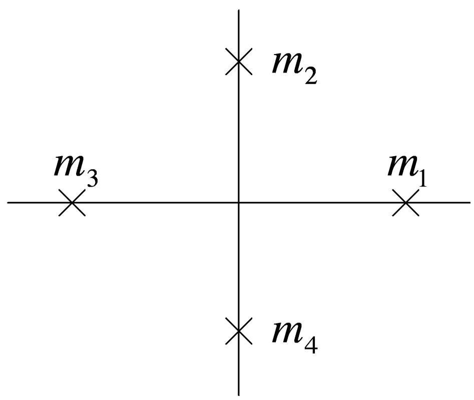
The vector representation of transmitted signal $s_i(t)$ becomes

$$\vec{s_i} = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}$$

The geometric visualization

$$< s_1(t), s_2(t) > = \underbrace{< \vec{s_1}, \vec{s_2} >}_{\text{Eclidean Inner product}} = \vec{s_1}^T \vec{s_2}$$

[Example] Linear modulation 4PSK,



$N = 2, M = 4, 1 \leq i \leq 4$

$$\begin{aligned} s_i(t) &= a_i g(t)\cos(2\pi f_c t) - b_i g(t)\sin(2\pi f_c t), \quad 0 \leq t \leq T_s \\ & \quad a_i + jb_i \in \{m_1, m_2, m_3, m_4\} \text{(Constellation points)} \\ \varphi_1(t) &= g(t)\cos(2\pi f_c t)\sqrt{\frac{2}{T_s}} \\ \varphi_2(t) &= g(t)\sin(2\pi f_c t)\sqrt{\frac{2}{T_s}} \end{aligned}$$

Approximation :

$$\int_0^{T_s} \varphi_1^2(t)dt = \int_0^{T_s} \varphi_2^2(t)dt = 1, \quad \int_0^{T_s} g^2(t)dt = 1, \quad \text{since } f_c T_s \gg 1$$

$s_i(t)_{\substack{1\le i\le 4\\ 0\le t\le T_s}}$ corresponds to $m_1, \ldots, m_4$.

$$s_1(t) \leftrightarrow m_1 \Rightarrow \vec{s}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

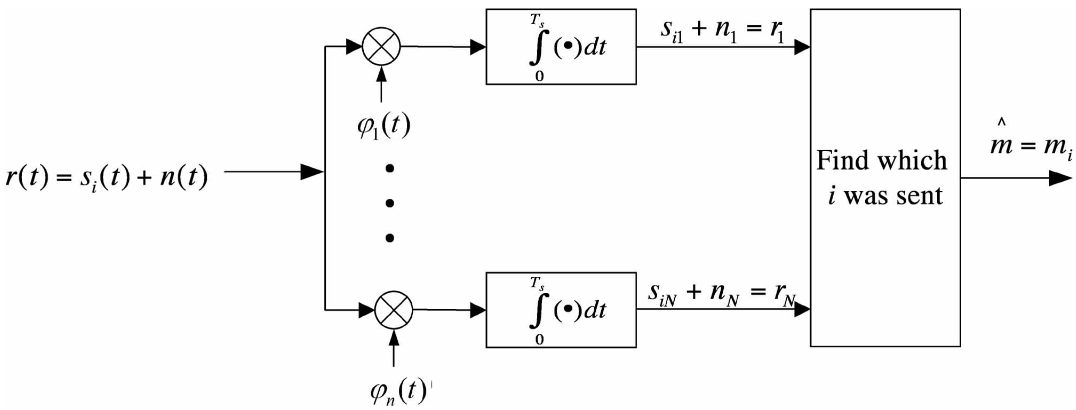
Distance between constellation points =

$$\begin{aligned} \|\vec{s}_i - \vec{s}_j\| &= \sqrt{\sum_{k=1}^{N}(s_{ik} - s_{jk})^2} \\ &= \sqrt{\int_0^{T_s}(s_i(t) - s_j(t))^2 dt} \end{aligned}$$

[Example] ISDN digital phone standard for public phone.

$$\begin{aligned} M &= 4, \quad \varphi_1(t) = \sqrt{\frac{1}{T_s}}\text{sinc}(t/T_s), \quad T_s = 1/80\text{KHz} \\ \text{data rate} &= 2 \text{ bits}/T_s = 160\text{kbps} \end{aligned}$$

For AWGN channel,



Receiver structure matched filter

[Example] The modulated signal of PAM is $a_n g(t) \cos(2\pi f_c t)$

$$\varphi_1(t) = g(t) \cos(2\pi f_c t) \longrightarrow \text{LPF}$$

[Detection]

$$P_e = \sum_i prob\{\hat{m} \neq m_i\} prob\{m_i \text{ was sent}\}$$

- MAP: Maximum a Posteriori Detector

  Choose m_i if $prob\{m_i | \overbrace{r_1, r_2, \ldots, r_N}^{\vec{r}}\}$ is maximized

  $$prob\{m_i | r_1, r_2, \ldots, r_N\} = \frac{prob\{\vec{r} | m_i\} prob\{m_i\}}{prob\{\vec{r}\}}, \quad \text{(Bayes' rule)}$$

  MAP Decision:

  $$prob\{\vec{r} | m_i\} prob\{m_i\} > prob\{\vec{r} | m_k\} prob\{m_k\}, \quad \forall k \neq i$$

- ML detector : Max $prob\{\vec{r}|m_i\}$ if $prob\{m_i\}$ equal $\Rightarrow$ MAP becomes ML

$$\begin{aligned} p(\vec{r}|m_i) &= prob\{r_1, r_2, \ldots, r_N|m_i\} \\ &= \prod_j p(r_j|m_i) \\ &= \frac{e^{-\frac{1}{2\sigma_n^2}\sum_{j=1}^{N}(r_j - s_{ij})^2}}{(2\pi\sigma_n^2)^{N/2}} \end{aligned}$$

It is maximized when the ith one is chosen if $\|\vec{r} - \vec{s}_i\|^2 \leq \|\vec{r} - \vec{s}_k\|^2, \quad \forall k.$ (Voronoi cells)