[Review of previous lecture]

- Small-scale fading

\[ s(t) = \text{Re} \left\{ u(t) e^{j2\pi f_c t} \right\} \]

Send a cosine, e.g. \( u(t) = 1 \)

\[
\tau(t) = \text{Re} \left\{ \sum_{n=1}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} e^{j2\pi f_c t} \right\} \\
= \sum_{n=1}^{N(t)} \alpha_n(t) \cos(\phi_n(t)) \cos(\omega_c t) - \sum_{n=1}^{N(t)} \alpha_n(t) \sin(\phi_n(t)) \sin(\omega_c t)
\]

\[ ch(t) = ch_I(t) + jch_Q(t) \] Complex Gaussian random variable

The autocorrelation and cross-correlation of \( ch_I \) and \( ch_Q \) are

\[
A_{ch_I}(\tau) = A_{ch_Q}(\tau) = \frac{1}{2} \sum_{n=1}^{N(t)} \alpha_n^2(t) \cos(2\pi \Delta f_n \tau) \\
= \frac{1}{2} \sum_{n=1}^{N(t)} \alpha_n^2(t) J_0(2\pi f_D \tau)
\]

\[
A_{I,Q}(\tau) = \frac{1}{2} \sum_{n=1}^{N(t)} \alpha_n^2(t) \sin(2\pi \Delta f_n \tau) \\
= 0
\]

- Power Spectrum

\[
\mathcal{F}\{A_I(\tau)\} = S_I(f) = \frac{\Omega_p}{2\pi f_D \sqrt{1 - \left( \frac{f}{f_D} \right)^2}}, \quad |f| < f_D
\]

What is \( \int S_I(f) df \)?
To get uncorrelated, \( f_D \tau = 0.4\lambda \).
- Amplitude of channel, $|ch(t)|$ is Rayleigh distributed. $|ch(t)|^2$ is exponential distributed.

- Jakes Spectrum

$$A_{|ch|}(\tau) = \frac{\pi}{8} P J_0^2(2\pi f_D \tau) + \frac{\pi}{2} P$$

$$\mathbb{E}[|ch|^2] = 2P = \Omega_p, \quad \mathbb{E}^2[|ch|] = \frac{\pi}{2} P. \quad (See \ Jakes \ for \ proof)$$
[Deriving PSD in another way]

Let’s assume

- large number of arriving paths.
- can account for antenna gain and non-uniform distribution of received paths.
- the paths are planar paths.

Let $p(\alpha)$ be probability distribution of path power/angle.

$$\int p(\alpha) d\alpha = 1$$

Total power $= P \int p(\alpha) G(\alpha) d\alpha$

where $P$ is total power for isotropic antenna $G = 1$. In frequency domain:

$$S(f)|df| = P[p(\alpha)G(\alpha)|d\alpha| + P(p(\alpha)G(-\alpha)|d\alpha| \quad 0 < \alpha < \pi$$

where $S(f)$ denotes power spectrum.

$$f = f_c + \frac{v_c}{f_D} \cos \alpha$$

$$df = -f_D \sin \alpha d\alpha$$

$$|df| = f_D |\sin \alpha| |d\alpha|$$

Is it valid to do the change of variable and $|df|$ small enough?

$$S(f) = \frac{P[p(\alpha)G(\alpha) + p(-\alpha)G(-\alpha)]}{f_D \sin \alpha} \quad \alpha = \cos^{-1} \left( \frac{f - f_c}{f_D} \right), \quad 0 < \alpha < \pi$$

$$= \frac{P[p(\alpha)G(\alpha) + p(-\alpha)G(-\alpha)]}{f_D \sqrt{1 - \left( \frac{f - f_c}{f_D} \right)^2}}, \quad 0 \leq \alpha \leq \pi \Rightarrow \left| \frac{f - f_c}{f_D} \right| < 1$$

Therefore, to translate it to all $f$, $1/2$ of $S(f)$ will be in $f > 0$ and $1/2$ in $f < 0$.

[Example] For $p(\alpha) = 1/2\pi, G(\alpha) = 1$

$$S(f) = \frac{2P}{2\pi f_D \sqrt{1 - \left( \frac{f - f_c}{f_D} \right)^2}}$$
Log-normal Shadowing] \( |ch| \) is Rayleigh but \( |\bar{ch}| \) is log-normal.

- We have \( u = 10 \log |ch|^2 \),

\[
P_U(u) = \frac{1}{\sqrt{2\pi}\sigma_u} e^{-\frac{(u-m_u)^2}{2\sigma_u^2}}, \quad m_u = 10 \log \alpha - n10 \log d \text{ (Path Loss)}
\]

\[
E[|ch|^2] = E[10^{u/10}]
\]

\[
10^{m_u} = e^{\beta u} \\
\alpha u \ln 10 = \beta u \\
\beta = \alpha \ln 10
\]

\[
E[e^{\frac{u}{10}}] = \ldots \\
= \exp \left\{ \frac{m_u}{g} + \frac{\sigma_u^2}{2g^2} \right\}, \quad g = \frac{10}{\ln 10}
\]

- Distribution of \( \psi = |\bar{ch}|^2 \),

\[
P(\psi) = \frac{\partial}{\partial \psi} P_U(u = f(\psi)), \quad u = 10 \log \psi
\]

\[
\psi = 10^{u/10} = e^{\ln 10^{u/10}}
\]

\[
\frac{\partial \psi}{\sigma_\psi} = \frac{\ln 10}{10^{-\psi}}
\]

\[
P(\psi) = \frac{10}{\ln 10^{-\psi}} \frac{1}{\sqrt{2\pi}\sigma_u} e^{-\frac{(10 \log \psi - m_u)^2}{2\sigma_u^2}}
\]