Binary Consensus for Cooperative Spectrum Sensing in Cognitive Radio Networks

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***Abstract*—In this paper, we propose to use binary consensus algorithms for distributed cooperative spectrum sensing in cognitive radio networks. We propose to use two binary approaches, namely diversity and fusion binary consensus spectrum sensing. The performance of these algorithms is analyzed over fading channels. The probability of networked detection and false alarm are characterized for the diversity case. We then show that binary consensus cooperative spectrum sensing is superior to quantized average consensus in terms of agility, given the same number of transmitted bits.**

***Index Terms*—Binary Consensus, Cognitive Radio, Cooperative Spectrum Sensing.**

I. Introduction

Cognitive Radio (CR), introduced by Mitola [1], is expected to revolutionize the future of wireless technology. The need for such a technology comes from the fact that the demand for wireless services has increased rapidly in recent years. Furthermore, the utilization of the current allocated spectrum is inefficient based on the studies of Federal Communications Commission (FCC). As such, cognitive radio emerged as a possible solution by trying to use the spectrum holes and thus increasing the spectral efficiency.

In cognitive radios, unlicensed users constantly sense the unused parts of the spectrum and utilize them. Primary users (licensed users) have preassigned bandwidths in the spectrum. Therefore, CR users, known as secondary or unlicensed users, need to detect the presence/absence of the primary users and use the available spectrum to establish their own communications. To achieve this, the secondary users (nodes) should sense the signal power in the corresponding channels and make decisions on the presence of the primary users. This is called spectrum sensing, which is the first important step in cognitive radio networks. Spectrum sensing can be conducted locally or cooperatively. In local spectrum sensing, each secondary user makes a decision only based on its one-time sensing. In cooperative spectrum sensing, on the other hand, a group of secondary users decide collaboratively in order to improve the detection performance, in the presence of local sensing errors. For instance, poor link quality, due to multipath fading, deteriorates local detection. Furthermore, individual/local sensing is not capable of dealing with the hidden terminal problem. Therefore, cooperative spectrum sensing has been studied as an alternative approach [2]–[4].

There are two approaches for cooperative sensing: centralized [5] and distributed [6]. In centralized sensing, the measurements of the secondary users are collected by a fusion center (could be another secondary user), who makes the final decision and broadcasts it to the other secondary users [7]. Most of the approaches in cooperative spectrum sensing are centralized. Such approaches, however, are not as robust to node/link failures.

Distributed average consensus algorithms [8] have been a subject of several studies in recent years. Applications include distributed and parallel computing [9], wireless sensor networks [10], and cooperative control of multi-agent systems [11]. In such problems, the goal is to achieve average consensus, on local information, over a network of agents. In [6], average consensus approaches have been utilized for distributed cooperative spectrum sensing.

In [12], a framework for binary consensus, i.e., agreement over the occurrence of an event, is proposed. In this paper, we show how such binary consensus approaches can be utilized for fast and distributed cooperative spectrum sensing. We consider two binary consensus approaches in this paper: fusion and diversity. We show how our proposed framework results in a considerably higher agility, as compared to the average consensus spectrum sensing. The rapid convergence of spectrum sensing approaches is considerably important since the secondary users need to use the available spectrum as fast as possible.

II. Cooperative Spectrum Sensing Model

Consider a cooperative network of M secondary users trying to reach consensus on the existence of the primary users. Each secondary user has its own initial opinion, based on its local spectrum sensing. It will then exchange its information with other secondary users in order to improve its assessment of the existence of the primary users. The transmissions occur over fading channels and are furthermore corrupted by the receiver noise. As such, a communication link may not necessarily be established between a pair of secondary users due to poor link quality. Furthermore, the underlying topology of a group of secondary users that are cooperating for spectrum sensing can be time-varying. Therefore, we model the underlying network as an undirected random graph $\mathcal{G}(\mathcal{V}, \mathcal{E}(k))$, where $\mathcal{V} = \{1, \cdots, M\}$ represents the vertex set (the set of secondary users) and $\mathcal{E}(k)$ is the link set (the set of available communication links among the secondary users) at time k, in order to focus on the impact of network connectivity and rapidly-changing channels on cooperative spectrum sensing. In a random graph, the underlying topology changes from one time instant to the next. In each time step, the graph is not fully-connected and each link exists with the probability p. If a link exists, its quality is assumed perfect. Let CNR represent the ratio of channel power to receiver noise power. Then, there exists a link from node i to node j, at time k, if $\mathrm{CNR}_{i,j}(k) > \mathrm{CNR}_{\mathrm{TH}}$, i.e., the link quality is above a minimum acceptable threshold. We take $\mathrm{CNR}_{i,j}$s to be i.i.d. random variables with the same mean value as $\overline{\mathrm{CNR}} = \overline{\mathrm{CNR}}_{i,j}$. Thus, we assume that the secondary users operate over a small enough area such that the channel between each pair can be considered stationary. Let p represent the probability that a link exists, from node j to node i, at a given time. Assuming exponentially-distributed multipath fading, we have $p = \mathrm{prob}(\mathrm{CNR}_{i,j}(k) > \mathrm{CNR}_{\mathrm{TH}}) = e^{-\mathrm{CNR}_{\mathrm{TH}}/\overline{\mathrm{CNR}}}$.

As mentioned in Section I, all the secondary users utilize energy detectors for local sensing. An energy detector [13] consists of a square-law function, followed by an integrator. Let B and T denote the bandwidth of the bandpass filter and the integration duration of a local energy detector respectively. Moreover, we assume that all the secondary users utilize energy detectors with the same parameters. Let $r_i(t)$ represent the received signal of the ith secondary user, in sensing of the primary user. We have the following two hypotheses

$$r_i(t) = \begin{cases} n_i(t) & \mathcal{H}_0 \\ h_i(t)s(t) + n_i(t) & \mathcal{H}_1, \end{cases}$$

where $s(t)$ is the unknown signal of the primary user, $n_i(t)$ is the zero-mean additive white Gaussian receiver noise of the ith CR user, and $h_i(t)$ is the channel gain from the primary user to the ith user, which has a Rayleigh distribution. Let γ_i denote the Signal-to-Noise Ratio (SNR) from the primary user to the ith secondary user. Furthermore, $x_i(0)$ represents the output of the energy detector of the ith node at time $t = 0$.[1] $x_i(0)$ has the following distribution [13]

$$x_i(0)|\gamma_i \sim \begin{cases} \chi^2_{2TB} & \mathcal{H}_0 \\ \chi^2_{2TB}(2\gamma_i) & \mathcal{H}_1, \end{cases}$$

where χ^2_{2TB} and $\chi^2_{2TB}(2\gamma_i)$ are the central and non-central chi-square densities respectively, with $2TB$ degrees of freedom and non-centrality parameter $2\gamma_i$. The CR users then communicate among themselves in order to improve their assessments. We define the performance metric of our cooperative network as follows:

Definition 1. For a cooperative spectrum sensing algorithm, we define the probability of networked detection and false alarm, in the kth time step, as follows:

$$P_d(k) = \mathrm{prob}(\text{all the nodes vote for } 1|\mathcal{H}_1)$$
$$P_f(k) = \mathrm{prob}(\text{at least one node votes for } 1|\mathcal{H}_0).$$

[1]It should be noted that there is only a one-time local sensing and the time progression of the next section is due to communication and consensus iterations among the secondary users.

By *vote* we refer to the binary decision of a secondary user, i.e., voting 1 means that a secondary user decides that a primary user exists while voting 0 denotes otherwise. In the following section, we propose a framework for binary consensus-based cooperative spectrum sensing over fading channels. Moreover, we characterize the probabilities of networked detection and false alarm.

III. Distributed Consensus Algorithms for Cooperative Spectrum Sensing

Consider a cognitive radio network with M secondary users. In our proposed binary consensus cooperative spectrum sensing framework, each secondary user makes a binary decision (vote) on the existence of primary users, based on its local sensing. The secondary users then exchange binary votes over fading channels and update their votes based on the communicated information. This process will go on for a while. The goal of binary consensus is to achieve the majority of the initial votes. In our case, however, we are interested in cooperative spectrum sensing which may not correspond to the majority of the initial votes. For instance, due to low reception qualities from the primary user, the majority of the initial votes may not correctly reflect the existence of the primary users. Let $b_i(0) \in \{0, 1\}$ represent the binary decision or vote of the ith secondary user at time step $k = 0$. $b_i(k) = 1$ indicates that the ith secondary user decides that a primary user exists while $b_i(k) = 0$ indicates otherwise. Define $\pi_{11} \triangleq \mathrm{prob}(b_i(0) = 1|\mathcal{H}_1)$ as the probability that the ith secondary user votes for 1, given $\mathcal{H}_1$ hypothesis. Under the Rayleigh fading assumption, γ_is are exponentially distributed. Furthermore, we assume that all the secondary users experience the same average SNR, in the reception from the primary user, denoted by $\overline{\gamma}$. Thus, as shown in [13], we can write

$$\begin{aligned}\pi_{11} &= \int_0^\infty \mathrm{prob}(b_i(0) = 1|\mathcal{H}_1, \gamma_i)\frac{1}{\overline{\gamma}}e^{-\frac{\gamma_i}{\overline{\gamma}}}d\gamma_i \\ &= e^{-\frac{\eta}{2}}\sum_{n=0}^{TB-2}\frac{1}{n!}\Big(\frac{\eta}{2}\Big)^n + \Big(\frac{\overline{\gamma}+1}{\overline{\gamma}}\Big)^{TB-1} \\ &\times \left(e^{-\frac{\eta}{2(\overline{\gamma}+1)}} - e^{-\frac{\eta}{2}}\sum_{n=0}^{TB-2}\frac{\eta}{n!}\frac{\overline{\gamma}}{2(\overline{\gamma}+1)}\right), \end{aligned} \tag{1}$$

where η is the local decision threshold of the secondary users optimized from the Maximum Likelihood (ML) test. Similarly we have

$$\begin{aligned}\pi_{10} &\triangleq \mathrm{prob}(b_i(0) = 1|\mathcal{H}_0) \\ &= \mathrm{prob}(x_i(0) > \eta|\mathcal{H}_0) = 1 - \frac{\Gamma_l(TB, \frac{\eta}{2})}{\Gamma(TB)}, \end{aligned} \tag{2}$$

where $\Gamma(\cdot)$ and $\Gamma_l(\cdot,\cdot)$ are Gamma and lower incomplete Gamma functions respectively. We next consider two binary consensus approaches for spectrum sensing [12]: *fusion* and *diversity*. In the fusion approach, each secondary user fuses the received votes of the secondary users that it can communicate with, namely neighbors, and updates its state based on the majority of the received votes. It will then send its updated vote to all its neighbors in the next time step. This process will go on until the given time for cooperative sensing is reached. This strategy is suitable, in particular, when the graph connectivity is low as it creates virtual links between nodes. In the case of diversity, on the other hand, each node uses its transmissions to repeat its initial vote, without fusing its received information. It then fuses its received votes only at the end of the given time. This strategy is more robust to link errors. We next apply these approaches to cooperative spectrum sensing.

A. Diversity-Based Binary Consensus for Spectrum Sensing

In this part, we apply diversity binary consensus to cooperative spectrum sensing. In this strategy, each secondary user utilizes its transmissions to repeat its initial vote. Consider the case where the CR network is given $K+1$ time steps (K transmissions) to reach an agreement. Each node can use all its transmissions to repeat its initial vote and only fuses the received information at the end. This strategy can, in particular, be useful in reducing the impact of link failures. Let $\mathbf{b}(k) = [b_1(k), \cdots, b_M(k)]^T$ represent the vector of the votes of all the secondary users at time step k, where ^{T} denotes matrix/vector transpose. Then, the dynamics of the network evolve as follows, given K transmissions,

$$\begin{aligned} &\mathbf{b}(k) = \mathbf{b}(k-1), \quad k \in \{1, \cdots, K-1\} \\ &\mathbf{b}(K) = \mathrm{Dec}\left(\frac{1}{M}\Big(\mathbf{b}(0) + \frac{1}{Kp}\sum_{t=0}^{K-1}\mathbf{A}(t)\mathbf{b}(t)\Big)\right), \end{aligned} \tag{3}$$

where $\mathbf{A}(k) = [a_{ij}(k)]_{1\le i,j\le M}$ is an $M \times M$ adjacency matrix of the CR network, at time step k, with $a_{ii}(k) = 0$. The off-diagonal elements of the adjacency matrix are Bernoulli random variables with $\mathrm{prob}(a_{ij}(k) = 1) = p$ for $i \neq j$, and $\mathrm{Dec}(z) = \begin{cases} 1 & z \ge 0.5 \\ 0 & z < 0.5 \end{cases}$. Note that if $\mathrm{Dec}(\cdot)$ is applied to a vector, it functions entry-wise. In [12], it was shown that diversity binary consensus algorithms achieve asymptotic majority consensus.[2]

Let $S(0) = \mathbf{1}^T\mathbf{b}(0)$, the sum of the initial votes of M secondary users, represent the state of the network, where $\mathbf{1}$ is an $M \times 1$ all-one vector. In the following lemma and the corollary that follows, we characterize the probability of networked detection and false alarm of diversity-based cooperative spectrum sensing.

Lemma 1. *Assume a cognitive radio network, with M secondary users communicating over a rapidly-changing network topology, where p denotes the probability of the existence of a link at any time step. For a sufficiently-large odd M, the probability of networked detection at time step K is approximated by*

$$\begin{aligned} P_d(K) \approx \sum_{i=0}^{M}\binom{M}{i}&\left[(1-\pi_{11})Q\left(\frac{(\frac{M}{2}-i)\sqrt{K}}{\sqrt{\frac{1-p}{p}i}}\right)\right]^{M-i} \\ &\times\left[\pi_{11}Q\left(\frac{(\frac{M}{2}-i)\sqrt{K}}{\sqrt{\frac{1-p}{p}|i-1|}}\right)\right]^{i}, \end{aligned} \tag{4}$$

where $Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^{+\infty} e^{-t^2/2}dt$ and π_{11} is defined in Eq. (1).

Proof: Let $\mathbf{Y}(K) = \mathbf{b}(0) + \frac{1}{Kp}\sum_{t=0}^{K-1}\mathbf{A}(t)\mathbf{b}(t)$ and $y_i(K)$ represent the ith entry of $\mathbf{Y}(K)$. In [12], it was shown that, for a sufficiently-large M, we can evoke the Central Limit Theorem (CLT) to approximate the distribution of $y_i(K)$ with a Gaussian density, with mean $\mu_{y_i(K)} = S(0)$ and variance $\sigma^2_{y_i(K)} = |S(0) - b_i(0)|\frac{1-p}{pK}$. Therefore, we have

$$\begin{aligned} &\mathrm{prob}\big(y_1(K) > \frac{M}{2}, \cdots, y_M(K) > \frac{M}{2}|S(0)\big) \\ &\approx \prod_{i=1}^{M} Q\left(\frac{\frac{M}{2}-S(0)}{\sqrt{|S(0)-b_i(0)|\frac{1-p}{p}}}\sqrt{K}\right) \\ &= Q^{M-S(0)}\left(\frac{(\frac{M}{2}-S(0))\sqrt{K}}{\sqrt{S(0)\frac{1-p}{p}}}\right)Q^{S(0)}\left(\frac{(\frac{M}{2}-S(0))\sqrt{K}}{\sqrt{|S(0)-1|\frac{1-p}{p}}}\right). \end{aligned} \tag{5}$$

The last equality is written by noting that $b_i(0)$s are either 0 or 1. We can then derive the probability of networked detection of the secondary users as follows,

$$\begin{aligned} P_d(K) = \sum_{i=0}^{M} &\mathrm{prob}\big(b_1(K)=1, \cdots, b_M(K)=1|S(0)=i\big) \\ &\times \mathrm{prob}(S(0)=i|\mathcal{H}_1), \end{aligned} \tag{6}$$

where $\mathrm{prob}(S(0)=i|\mathcal{H}_1) = \binom{M}{i}\pi_{11}^i(1-\pi_{11})^{M-i}$. Substituting Eq. (5) in Eq. (6) results in the Lemma. ∎

We then have the following for the asymptotic value of $P_d(K)$

$$\lim_{K\to\infty} P_d(K) = \sum_{i=\lceil \frac{M}{2}\rceil}^{M}\binom{M}{i}(1-\pi_{11})^{M-i}\pi_{11}^i. \tag{7}$$

Therefore, the asymptotic behavior of diversity-based binary consensus spectrum sensing, over random graphs, is independent of the network connectivity and only depends on the number of secondary users and $\overline{\gamma}$ (through π_{11}).

Corollary 1. *For the cognitive radio network of Lemma 1, the probability of false alarm is*

$$\begin{aligned} P_f(K) = 1 - \sum_{i=0}^{M}\binom{M}{i}&\left[(1-\pi_{10})\left(1-Q\left(\frac{(\frac{M}{2}-i)\sqrt{K}}{\sqrt{\frac{1-p}{p}i}}\right)\right)\right]^{M-i} \\ &\times\left[\pi_{10}\left(1-Q\left(\frac{(\frac{M}{2}-i)\sqrt{K}}{\sqrt{\frac{1-p}{p}|i-1|}}\right)\right)\right]^{i}, \end{aligned}$$

[2] Note that this, however, does not mean that the asymptotic probability of networked detection is 1 for spectrum sensing.

where π_{10} is defined in Eq. (2).

Proof: Proof is similar to that of Lemma 1. ■

As expected, we can see from Corollary 1 and Eq. (2), that the probability of false alarm is independent of $\overline{\gamma}$ (it is only a function of noise parameters). Similarly, as we discussed for $P_d(K)$, the asymptotic value of $P_f(K)$ is independent of the network connectivity (p). Fig. 1 shows the probability of networked detection, for a network of $M = 51$ secondary users. The theoretical approximation, for the probability of networked detection, is compared to the true value obtained from simulation, for $\overline{\text{CNR}} = 0$ dB. It can be seen that Eq. (4) matches considerably well with the true probabilities.

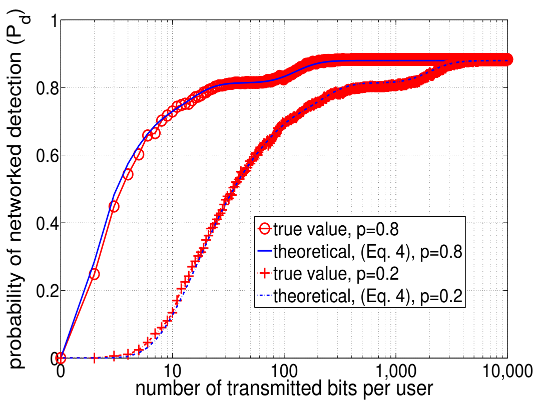

Fig. 1. Theoretical v.s. simulated probability of networked detection for diversity-based binary consensus cooperative spectrum sensing, with $M = 51$ and $\overline{\gamma} = 2$ dB.

B. Fusion-Based Binary Consensus for Spectrum Sensing

In this part, we apply the fusion binary consensus approach [12] to cooperative spectrum sensing. In this case, each secondary user updates its binary decision, at every step, based on the received votes from its neighbors. In the next time step, it then transmits its updated vote to its neighbors. The dynamics of the fusion strategy can be expressed as

$$\mathbf{b}(k) = \text{Dec}\left(\frac{1}{M}\Big(\mathbf{I} + \mathbf{A}(k)\Big)\mathbf{b}(k-1)\right), \quad k \in \mathbb{Z}^+.$$

In [12], the properties of fusion binary consensus were discussed. It was shown that the fusion case does not necessarily reach the majority of the initial votes, if less than $M - 1$ nodes vote the same initially, while diversity binary consensus achieves the majority of the initial votes almost surely.[3] The characterization of the probability of networked detection for fusion binary spectrum sensing, however, is challenging and an open problem.

The advantage of the fusion strategy, on the other hand, is that local information propagates faster through the network than the diversity approach. Consequently, the agility of fusion binary consensus is higher. Therefore, the fusion strategy is suitable, in particular, when the graph connectivity is low as it creates virtual links between nodes. Thus, there are some tradeoffs between diversity and fusion approaches, in terms of agility and asymptotic behavior, as we discussed in [12], as well as in Section IV.

[3]Note that the majority of the initial votes may not still correspond to an accurate networked detection if $\overline{\gamma}$ is too small.

C. Average Consensus for Spectrum Sensing

In [6], authors proposed applying average consensus techniques for cooperative spectrum sensing. In this part, we mathematically characterize a lower bound for the probability of networked detection for this approach. We then compare its agility with our binary consensus-based algorithms, in terms of probability of networked detection. The standard average consensus dynamics, over random graphs, evolve as follows [8]

$$\mathbf{X}(k+1) = \mathbf{P}(k+1)\mathbf{X}(k), \tag{8}$$

where $\mathbf{X}(k) = [x_1(k), \cdots, x_M(k)]^T$ is an $M \times 1$ state vector at time step k, and $\mathbf{P}(k)$ is a doubly stochastic matrix, i.e., $\sum_{j=1}^M P_{ij} = \sum_{i=1}^M P_{ij} = 1$ for $i, j \in \{1, \cdots, M\}$, corresponding to the underlying graph of the CR users at time k. Note that $\mathbf{X}(0) = [x_1(0), \cdots, x_M(0)]^T$ contains the local measurements and $\mathbf{X}(k)$, for $k \in \mathbb{Z}^+$, is the vector of updated sensing of the nodes after k steps of fusion. We then have $\mathbf{P}(k) = \mathbf{I} - \epsilon(\mathbf{D}(k) - \mathbf{A}(k))$, where $\mathbf{A}(k)$ is the adjacency matrix of the CR network and $\mathbf{D}(k)$ is a diagonal matrix whose diagonal entries are the degrees of the nodes of the graph, i.e., $\mathbf{D}(k) = \text{diag}(d_1(k), \cdots, d_M(k))$, with $d_i(k) = \sum_{j=1}^M a_{ij}(k)$. Let Δ denote the maximum degree of the network, then $\epsilon \in (0, 1/\Delta)$ [8]. Since we assume that each node can potentially be connected to any other node, then $\Delta = M - 1$. Using Perron's theorem [14], we have $1 = |\lambda_1(\mathbf{P}(k))| > |\lambda_2(\mathbf{P}(k))| \geq \cdots \geq |\lambda_M(\mathbf{P}(k))|$, where $\lambda_1(\cdot), \lambda_2(\cdot), \cdots, \lambda_M(\cdot)$ represent the eigenvalues of the stochastic matrix $\mathbf{P}(k)$. Note that the algebraic multiplicity of λ_1 is one.

As the average consensus dynamics evolve, all the secondary users compare their current state to a predefined threshold and make a binary decision on the existence of the primary users. Let ρ denote the predefined threshold. This parameter can be optimally designed, using an ML detection rule for the asymptotic average consensus desired value, which is $\bar{x}(0) = \frac{1}{M}\sum_{j=1}^M x_j(0)$. Define $\mathcal{F} \triangleq \Big\{\mathbf{X}(0) | \bar{x}(0) \geq \rho\Big\}$. Let $f_\zeta(\cdot)$ represent the probability density function (PDF) of a general random variable ζ. Furthermore, let $P_d(k) = \text{prob}(x_1(k) > \rho, \cdots, x_M(k) > \rho | \mathcal{H}_1)$ denote the probability of networked detection for average consensus spectrum sensing, at time k. We have

$$P_d(k) = \int_{\mathbf{Z} \in \mathcal{F}} \text{prob}(\mathbf{X}(k) \geq \rho\mathbf{1} | \mathcal{H}_1, \mathbf{X}(0) = \mathbf{Z}) f_{\mathbf{X}(0)}(\mathbf{Z}) d\mathbf{Z}$$
$$+ \int_{\mathbf{Z} \in \mathcal{F}^C} \text{prob}(\mathbf{X}(k) \geq \rho\mathbf{1} | \mathcal{H}_1, \mathbf{X}(0) = \mathbf{Z}) f_{\mathbf{X}(0)}(\mathbf{Z}) d\mathbf{Z}. \tag{9}$$

Under the assumption of independent initial assessments, i.e.,

$x_i(0)$s, we have $f_{\mathbf{X}(0)}(\mathbf{Z}) = \Pi_{i=1}^M f_{x_i(0)}(z_i)$.[4] The second term on the right hand side of Eq. (9) is equal to zero. However, finding a closed from expression for $P_d(k)$ is still challenging. Thus, we derive a lower bound in order to analyze the performance of average consensus spectrum sensing. Let $\mathbf{\Delta}(k) = \mathbf{X}(k) - \bar{x}(0)\mathbf{1}$ represent the error vector. We then have

$$\begin{aligned}&\text{prob}(\mathbf{X}(k) \geq \rho\mathbf{1}|\mathcal{H}_1, \mathbf{X}(0) \in \mathcal{F})\\ &\quad \geq \text{prob}\big(\|\mathbf{\Delta}(k)\|^2 \leq (\bar{x}(0) - \rho)^2|\mathcal{H}_1, \mathbf{X}(0)\big)\\ &\quad \geq 1 - \frac{E\{\|\mathbf{\Delta}(k)\|^2\}}{\big(\bar{x}(0) - \rho\big)^2},\end{aligned} \tag{10}$$

where $E\{\cdot\}$ denotes the expectation operator, which is taken over the graph randomness. The last line in Eq. (10) is derived using Chebyshev's inequality under the assumption of large enough k.

Next we derive an upper bound for $E\{\|\mathbf{\Delta}(k)\|^2\}$. We have $\mathbf{\Delta}(k) = \mathbf{P}(k)\mathbf{\Delta}(k-1)$ [15]. By conditioning on the previous step and using Rayleigh-Ritz theorem [14], we have

$$\begin{aligned}&E\big\{\|\mathbf{\Delta}(k)\|^2|\mathbf{\Delta}(k-1)\big\}\\ &\quad = \mathbf{\Delta}^T(k-1)E\big\{\mathbf{P}^T(k)\mathbf{P}(k)\big\}\mathbf{\Delta}(k-1)\\ &\quad \leq \lambda_2\big(E\{\mathbf{P}^T(k)\mathbf{P}(k)\}\big)\|\mathbf{\Delta}(k-1)\|^2.\end{aligned}$$

Then, through induction, we can write

$$E\big\{\|\mathbf{\Delta}(k)\|^2\big\} \leq \lambda_2^k\big(E\{\mathbf{P}^2(k)\}\big)\|\mathbf{\Delta}(0)\|^2. \tag{11}$$

Next we characterize $\lambda_2(\cdot)$. For simplicity we drop the time index k. We have $\mathbf{P} = \mathbf{I} - \epsilon(\mathbf{D} - \mathbf{A})$. Furthermore, the off-diagonal entries of the adjacency matrix are Bernoulli distributed random variables with the probability $\text{prob}(a_{ij} = 1) = p$ for $i \neq j$. The diagonal entries of matrix $\mathbf{D}$ are binomial random variables $d_i \sim \mathcal{B}(M-1, p)$ for $i \in \{1, \cdots, M\}$. Next we characterize the second largest eigenvalue of $E\{\mathbf{P}^2(k)\}$. We have $E\{\mathbf{A}^2\} = ((M-1)p - (M-2)p^2)\mathbf{I} + (M-2)p^2\mathbf{1}\mathbf{1}^T$, and $E\{\mathbf{D}^2\} = \big((M-1)(M-2)p^2 + (M-1)p\big)\mathbf{I}$.

Moreover, $[\mathbf{DA}]_{ij} = \sum_{l=1}^M a_{il}a_{ij} = \sum_{l=1, l\neq i,j}^M a_{il}a_{ij} + a_{ij}^2$. Therefore, $E\big\{\mathbf{DA}\big\} = ((M-2)p^2 + p)(\mathbf{1}\mathbf{1}^T - \mathbf{I})$. We then have

$$\begin{aligned}E\{\mathbf{P}^2(k)\} &= \mathbf{I} - 2\epsilon(E\{\mathbf{D}\} - E\{\mathbf{A}\})\\ &\quad + \epsilon^2(E\{\mathbf{A}^2\} + E\{\mathbf{D}^2\} - 2E\{\mathbf{DA}\})\\ &= \big(M(M-2)\epsilon^2p^2 + 2M\epsilon^2p - 2M\epsilon p + 1\big)\mathbf{I}\\ &\quad + \big(2\epsilon p - (M-2)\epsilon^2p^2 - 2\epsilon^2p\big)\mathbf{1}\mathbf{1}^T.\end{aligned}$$

$E\{\mathbf{P}^2(k)\} = \alpha\mathbf{I} + \beta\mathbf{1}\mathbf{1}^T$, for any α, $\beta \in \mathbb{R}$, is a special form of a circulant matrix. It is straightforward to see that the eigenvalues of a circulant matrix are $\lambda_1 = \alpha + M\beta$ and $\lambda_j = \alpha$ for $j \in \{2, \cdots, M\}$. Therefore,

$$\lambda_2\big(E\{\mathbf{P}^2(k)\}\big) = M(M-2)\epsilon^2p^2 + 2M\epsilon^2p - 2M\epsilon p + 1.$$

[4] In [13], the authors show that the probability density function of $x_i(0)$ can be represented as the convolution of a χ^2 PDF, with $2TB - 2$ degrees of freedom, and an exponential PDF, with parameter $2(\overline{\gamma} + 1)$.

Combining Eq. (9), (10) and (11) yields

$$\begin{aligned}P_d(k) &= \int_{\mathbf{Z}\in\mathcal{F}} \text{prob}(\mathbf{X}(k) \geq \rho\mathbf{1}|\mathcal{H}_1, \mathbf{X}(0) = \mathbf{Z})f_{\mathbf{X}(0)}(\mathbf{Z})d\mathbf{Z}\\ &\geq \text{prob}(\mathcal{F}) - \lambda_2^k\big(E\{\mathbf{P}^2(k)\}\big)\\ &\qquad \times \int_{\mathbf{Z}\in\mathcal{F}} \frac{\|\mathbf{Z} - \frac{1}{M}\mathbf{1}^T\mathbf{Z}\mathbf{1}\|^2}{\big(\frac{1}{M}\mathbf{1}^T\mathbf{Z} - \rho\big)^2} f_{\mathbf{X}(0)}(\mathbf{Z})d\mathbf{Z}.\end{aligned} \tag{12}$$

Since $\lambda_2(\cdot) < 1$, the lower bound tends to prob$(\mathcal{F})$ for large enough k. Moreover, from Eq. (9), it can be easily confirmed that $\lim_{k\to\infty} P_d(k) = \text{prob}(\mathcal{F})$, which is the exact asymptotic value. Also we can see that the asymptotic value of $P_d(k)$ is only a function of M and $\overline{\gamma}$ and does not depend on the network connectivity.

IV. Simulation Results and Discussion

In this section, we compare the performance of our binary consensus-based cooperative spectrum sensing with that of average consensus. As mentioned earlier, in average consensus spectrum sensing, local measurements, $x_i(0)$s, are exchanged among the secondary users. In a realistic scenario, measurements need to be quantized before transmission. Therefore, in this section, we simulate quantized average consensus and compare its detection performance with the binary consensus schemes for cooperative spectrum sensing. Let $q(\cdot)$ denote an R-bit midtread quantizer. This quantizer is a mapping $q : \mathbb{R} \to \mathbb{Z}$, converting $z \in \mathbb{R}$ to its nearest integer value $n \in \mathbb{Z}$

$$q(z) = \begin{cases} 2^R - 1, & z \geq 2^R - 1.5 \\ n, & n - 0.5 \leq z < n + 0.5 \\ 0, & z \leq 0.5. \end{cases}$$

It is straightforward to rewrite Eq. (8) for the quantized case as $\mathbf{X}(k+1) = \mathbf{X}(k) + (\mathbf{P}(k+1) - \mathbf{I})q(\mathbf{X}(k))$, where $q(\cdot)$ is applied entry-wise to its vector argument. Next, we compare the performance of binary consensus schemes with quantized average consensus. In order to have a fair comparison, we quantize the transmitted data of the average consensus case to R bits per transmission and evaluate the performance of these algorithms in terms of the number of transmitted bits per CR user. We further assume that all the secondary users have the same $\overline{\gamma}$ and investigate two cases of low and high network connectivity ($p = 0.2$ and $p = 0.8$). Moreover, we set $R = 5$ bits to achieve the best agility for average consensus algorithm (optimal R was found through simulation). We take $\epsilon = \frac{1}{M-1}$ and $TB = 5$.

Fig. 2 and 3 compare the performance of the three consensus-based spectrum sensing approaches, for the two cases of low and high network connectivity respectively. It can be seen that, although the SNR to the primary user is low ($\overline{\gamma} = 2$ dB), fusion-based binary consensus outperforms both diversity-based binary consensus and quantized average consensus spectrum sensing, in terms of agility. Moreover, it can be seen that the agility of diversity-based binary consensus spectrum sensing improves tremendously by increasing network connectivity. Network connectivity, however, does not impact the performance of fusion-based strategy significantly. The reason is that when the graph connectivity is low, fusion-based strategy creates virtual links between the secondary users. In Eq. (7) and (12), we show that the asymptotic behavior of diversity-based binary consensus and average consensus spectrum sensing is independent of the network connectivity and depends only on M and $\overline{\gamma}$. Fig. 2 and 3 also confirm this.

Fig. 4 shows the performance of these approaches for a higher level of SNR ($\overline{\gamma} = 6$ dB) and the case of low connectivity. Comparing Fig. 2 and 4, it can be seen that increasing SNR, which corresponds to more correct initial votes, improves the performance of all the three approaches, with a more considerable impact on the agility of the binary-based consensus approaches.

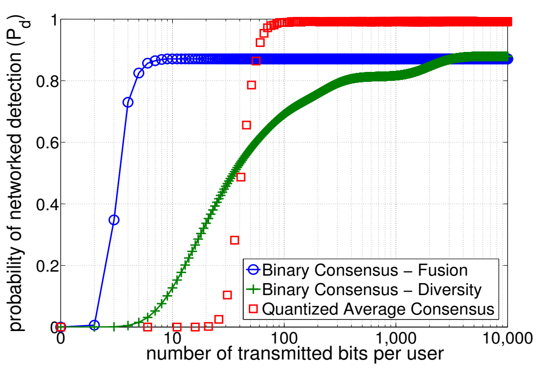

Fig. 2. Comparison of the probability of networked detection for binary consensus and quantized average consensus schemes, with $M = 51$, $p = 0.2$, $\overline{\gamma} = 2$ dB and $R = 5$ bits.

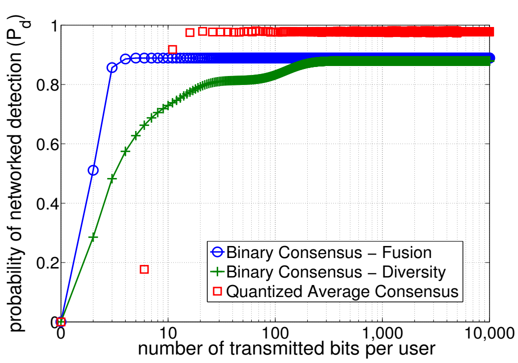

Fig. 3. Comparison of the probability of networked detection for binary consensus and quantized average consensus schemes, with $M = 51$, $p = 0.8$, $\overline{\gamma} = 2$ dB and $R = 5$ bits.

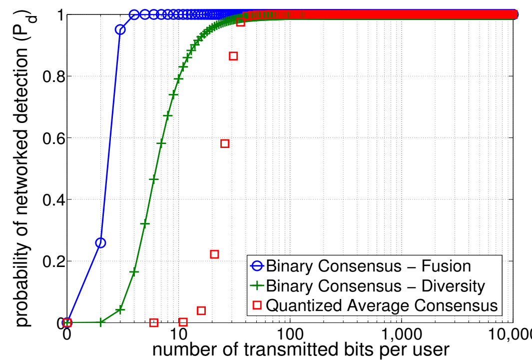

Fig. 4. Comparison of the probability of networked detection for binary consensus and quantized average consensus schemes, with $M = 51$, $p = 0.2$, $\overline{\gamma} = 6$ dB and $R = 5$ bits.

V. Conclusion

In this paper, we proposed to use binary consensus algorithms for distributed cooperative spectrum sensing in cognitive radio networks. We discussed two approaches, namely diversity and fusion binary consensus. The performance of diversity-based spectrum sensing is studied in terms of probability of networked detection and false alarm. Binary consensus-based spectrum sensing is then compared to quantized average consensus approach. We furthermore derived a lower bound for the case of average consensus. We showed that binary consensus cooperative spectrum sensing is superior to quantized average consensus, in terms of agility, given the same number of transmitted bits.

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