Abstract—In this paper, we consider a team of unmanned vehicles that are tasked with distributed transmit beamforming (virtual antenna array placement and design), in order to cooperatively transmit information to a remote station in realistic communication environments. We are interested in the energy-aware (both motion and communication energy) co-optimization of robotic paths and transmission powers for cooperative transmit beamforming under a reception quality requirement. We first consider the case where the channel is known. For this case, we propose an efficient approach for getting arbitrarily close to the optimum solution, which involves solving a series of multiple-choice knapsack problems. We then extend our analysis and methodology to the case where the channel is not known. The robots then probabilistically predict the channel at unvisited locations and integrate it with path planning and decision making for energy-aware distributed transmit beamforming. Finally, we extensively confirm our proposed approach with several simulation results with real channel parameters. Our results highlight the underlying trends of the optimum strategy and indicate a considerable energy saving.

I. INTRODUCTION

Networked robotic systems have been the focus of considerable research in recent years. Such systems are envisioned to carry out tasks such as search and rescue, surveillance, exploration, and sensing of the environment. Maintaining proper connectivity or transferring data to a remote station is a key enabling factor in many of these tasks, and the mobility of the unmanned nodes can play an important role in achieving proper connectivity by actively moving to places better for communication. Since unmanned vehicles typically have a limited energy budget, energy efficiency is of prime importance in these systems. Thus, energy-aware co-optimization of communication and motion strategies is needed to truly realize the full potentials of these systems, which is the main motivation for this paper.

Co-optimization of motion and communication strategies in robotic systems has recently attracted attention of both communication and robotics communities [2]–[10]. For instance, in [2], a node co-optimizes its motion speed and communication transmission rate, while a number of nodes utilize their mobility to form a communication relay network in [3]. In [6], robots act as collaborative relay beamformers, without considering motion energy costs. In [5], robots utilize mobility to maintain an optimal communication chain between a source and a destination node. In [11], the probability density function (pdf) of the distance traveled before the robot gets connected is derived.

On the communication side, distributed transmit beamforming is a cooperative communication strategy where a number of fixed transmitters cooperate to emulate a virtual centralized antenna array. For instance, consider the case where a node needs to transmit information to a remote station. If the corresponding link quality is not good, successful communication may not be possible. Instead, a number of transmitters can perform transmit beamforming, which means co-phasing and properly weighing their transmitted signals to communicate the same message while maintaining the same total communication power. In this manner, transmit beamforming creates an equivalent strong link to the receiving node. Transmit beamforming was originally proposed in the context of multiple co-located antennas for improving transmission quality of communication systems. More recently, it has been extensively studied in the context of fixed nodes that are spatially distributed over a given area [12], [13]. Then, the nodes align their transmission phases such that the wireless signals merge constructively at the remote station, thus providing dramatic gains in the signal to noise ratio (SNR). Using unmanned vehicles creates new possibilities for distributed transmit beamforming by enabling the transmitters to position themselves in better locations for beamforming, thus improving the overall performance significantly. However, several challenges for motion and communication co-planning need to be addressed before realizing this vision, which is the main motivation for this paper.

In this paper, we are interested in an energy-aware distributed transmit beamforming using unmanned vehicles. More specifically, we consider the problem where a team of unmanned vehicles are tasked with cooperatively transmitting information, via distributed transmit beamforming, to a remote station while minimizing the total energy consumption including both motion and communication energy costs. We are then interested in characterizing the optimal motion and communication strategies of the robots, including the optimization of the transmit power and robot paths. Fig. 1 shows an illustration of distributed robotic transmit beamforming.

As compared to the existing literature on distributed beamforming, most work are not concerned with unmanned vehicles and the resulting challenges in terms of path planning and motion energy. In [6], where robots act as collaborative relay beamformers, motion energy-related issues are not considered, resulting in a different problem formulation. Moreover, there is no channel learning and prediction. Finally, the motion of the

This work is supported in part by NSF NetS award 1321171 and NSF RI award 1619376. A small part of this work has appeared in [1]. As compared to the conference version, which only considered motion energy consumption, this paper considers both communication and motion energy consumption, resulting in a different formulation and a more extensive analysis/optimization.

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In this paper, we adopt a model where the motion energy consumption is proportional to the distance traveled, similar to the one adopted in [16], [17]. Thus, motion energy \( \epsilon = \kappa_M d \), where \( d \) is the distance traveled by the robot and \( \kappa_M \) is a constant that depends on the environment (e.g., friction coefficient, terrain) and the mass of the vehicle. This model is a good match for wheeled robots (see [16] for discussion).

### A. Motion Energy Model

In this section we first introduce our energy consumption models for both motion and communication. We then review distributed transmit beamforming and the corresponding power gains that it provides. Finally, we briefly summarize probabilistic modeling and prediction of wireless channels.

#### A. Motion Energy Model

In this paper, we adopt a model where the motion energy consumption is proportional to the distance traveled, similar to the one adopted in [16], [17]. Thus, motion energy \( \epsilon = \kappa_M d \), where \( d \) is the distance traveled by the robot and \( \kappa_M \) is a constant that depends on the environment (e.g., friction coefficient, terrain) and the mass of the vehicle. This model is a good match for wheeled robots (see [16] for discussion).

#### B. Communication Energy Model

We consider a generic model of communication rate of the form \( R = \eta_1 B \log_2 \left( 1 + \eta_2 \frac{P_B}{N_0} \right) \), where \( \eta_1, \eta_2 \leq 1 \) are constants, \( B \) is the available bandwidth, \( P_B \) is the received power and \( N_0 \) is the noise power.  \(^1\)For capacity approaching codes (such as turbo codes and LDPC), the constants for a binary symmetric channel correspond to \( \eta_1 = 1 - \epsilon \) and \( \eta_2 = 1 \) where \( \epsilon \) is the multiplicative gap to capacity [18]. For an uncoded MQAM modulation scheme with a target bit error rate of BER\(_{th}\), we obtain \( \eta_1 = 1 \) and \( \eta_2 = 1.5 / \ln(5\text{BER}_{th}) \) [19]. The communication energy incurred in transmitting \( l \) bits of data can then be expressed as

\[
\text{Communication Energy} = \frac{l}{\eta_1 B \log_2 \left( 1 + \eta_2 \frac{P_B}{N_0} \right)} P_0, \quad (1)
\]

where \( P_0 \) is the transmit power.

#### C. Distributed Transmit Beamforming

Distributed transmit beamforming is a form of cooperative communication where several nodes that are distributed in a given space emulate a centralized antenna array [12]. The nodes simultaneously transmit the same message with phases such that the signals combine constructively at the remote station. Channel state information (CSI), i.e., information about the channel, is required at the transmitters for the implementation of distributed transmit beamforming.

Consider \( N \) robots in an environment. Let \( h_i = \alpha_i e^{j\theta_i} \) denote the complex baseband channel from robot \( i \) to the remote station with \( \alpha_i \) and \( \theta_i \) denoting the channel amplitude and phase respectively. Ideally, node \( i \) transmits \( w_i s(t) \) where \( w_i = \rho_i e^{-j\phi_i} \) is the complex beamforming weight and \( s(t) \) is the complex baseband signal to be transmitted. As can be seen, setting \( \angle w_i = -\angle h_i = -\theta_i \) is the crucial step in obtaining a constructive interference and thus beamforming gains. The received signal is then \( (\sum_{i=1}^{N} h_i w_i) s(t) = \sum_{i=1}^{N} (\alpha_i \rho_i) s(t) \) resulting in the received SNR of \( \frac{P_B (\sum_{i=1}^{N} \alpha_i \rho_i)^2}{\eta_1 B \log_2 \left( 1 + \eta_2 \frac{P_B}{N_0} \right)} P_0 \). Constraining \( \rho_i \leq 1 \) imposes a maximum power of \( P_0 \) on each node. We stress here the difference from the traditional centralized transmit beamforming where a total transmit power is enforced, i.e. \( \sum_{i=1}^{N} \rho_i^2 \leq 1 \). However, in distributed beamforming, the

\(^1\)Note that the communication rate is adaptive as it is a function of the received power.
nodes are separated and have their own power supply. We thus impose individual power constraints instead. Note that the position of node \( i \) affects \( \alpha_i \), the corresponding channel amplitude, and therefore the overall received SNR. Thus, by properly designing robotic paths and transmit power (\( \rho_i \)), using unmanned vehicles can significantly improve distributed transmit beamforming, as we shall see in this paper.

D. Overview of Channel Modeling and Prediction

1) Probabilistic Channel Modeling [19]: A communication channel is well modeled as a multi-scale random process with three major dynamics: path loss, shadowing and multipath fading [19]. Let \( \Gamma(q_1) = |h(q_1)|^2 \) represent the received channel power from a transmitter at location \( q_1 \in W (W \subseteq \mathbb{R}^2 \) is the workspace) to the remote station located at \( q_b \). The received channel power in dB, \( \Gamma_{db}(q_1) = 10 \log_{10} \Gamma(q_1) \), can be expressed as \( \Gamma_{db}(q_1) = \Gamma_{PLdb}(q_1) + \Gamma_{SHdb}(q_1) + \Gamma_{MPdb}(q_1) \) where \( \Gamma_{PLdb}(q_1) = K_{dB} - 10 n_{PL} \log_{10} ||q_1 - q_b|| \) is the distance-dependent path loss with \( n_{PL} \) representing the path loss exponent, and \( \Gamma_{SHdb} \) and \( \Gamma_{MPdb} \) are random variables denoting the impact of shadowing and multipath respectively. \( \Gamma_{SHdb}(q_1) \) is best modeled as a Gaussian random variable with an exponential spatial correlation, i.e.,

\[
\mathbb{E} \{ \Gamma_{SHdb}(q_1) \Gamma_{SHdb}(q_2) \} = \nu_{SH} e^{-\frac{||q_1 - q_2||}{\beta_{SH}}} \text{ where } \nu_{SH} \text{ is the shadowing power and } \beta_{SH} \text{ is the decorrelation distance.}
\]

2) Realistic Channel Prediction [10], [20]: Let \( q = [q_1 \cdots q_m]^T \) denote the vector of channel power measurements (in dB) from the same environment, and \( q = [q_1, q_2] \) denote the vector of the corresponding positions.

Lemma 1 (See [20] for proof): A Gaussian random vector, \( \Gamma_{dB}(p) = [\Gamma_{dB}(p_1) \cdots \Gamma_{dB}(p_k)]^T \sim N \left( \Gamma_{dB}(p), C_{dB}(p) \right) \) can best characterize the vector of channel power (in the dB domain) when transmitting from unvisited locations \( p = [p_1 \cdots p_k]^T \), with the mean and covariance matrix given by

\[
\Gamma_{dB}(p) = \mathbb{E} \{ \Gamma_{dB}(p) \} = G_{p} \theta + \Psi_{p,q} \Phi_{q}^{-1} (\hat{\gamma} - G_{q} \theta) \text{ and } C_{dB}(p) = \mathbb{E} \left\{ \Gamma_{dB}(p) - \Gamma_{dB}(p) \right\} (\Gamma_{dB}(p) - \Gamma_{dB}(p))^T | G_{q,db}, \hat{\gamma}, \beta_{SH}, \nu_{SH}, \nu_{MP} = \Phi_{p} - \Psi_{p,q} \Phi_{q}^{-1} \Phi_{q,p} \text{ respectively, where } G_{p} = [1_k - D_p], G_{q} = [1_m - D_q], 1_m (1_k) \text{ represents the } m \text{-dimensional (} k \text{-dimensional) vector of all ones, } D_q = [10 \log_{10}(||q_1 - q_b||) \cdots 10 \log_{10}(||q_m - q_b||)]^T, D_p = [10 \log_{10}(||p_1 - q_b||) \cdots 10 \log_{10}(||p_k - q_b||)]^T \text{ and } \nu_{SH} \text{ is the position of the remote station. Furthermore, } \Phi_{g}, \Phi_{p} \text{ and } \Psi_{p,q} \text{ denote matrices with entries } \Phi_{p,j_1,j_2} = \nu_{SH} e^{-||p_{j_1} - q_{j_2}||/\beta_{SH}} + \nu_{MP} \delta_{j_1,j_2}, \text{ where } \Phi_{p,j_1,j_2} = \nu_{SH} e^{-||p_{j_1} - q_{j_2}||/\beta_{SH}} + \nu_{MP}\delta_{j_1,j_2} \text{ and } \Psi_{p,q} = \left[ \nu_{SH} e^{-||p_{j_1} - q_{j_2}||/\beta_{SH}} \right]_{j_1,j_2} \text{ and } \delta_{i,j} = \left\{ \begin{array}{ll} 1, & \text{if } i = j \ , \\ 0, & \text{else.} \end{array} \right. \text{ Moreover, } \theta, \beta_{SH}, \nu_{SH} \text{ and } \nu_{MP} \text{ denote the path loss parameters, the decorrelation distance of shadowing, the power of shadowing (in dB) and the power of multipath (in dB) respectively. The } ^\dagger \text{ symbol denotes the estimate of the corresponding parameter.}
\]

The underlying parameters can be estimated based on the a priori measurements as well. See [20] for more details on the estimation of the underlying parameters and the performance of this framework with real data and in different environments.

III. MOTION ENERGY-AWARE COOPERATIVE ROBOTIC BEAMFORMING

Consider the case where the robots are distributed over the space such that the required cooperative received SNR is not satisfied. The robots thus need to move to new positions that satisfy the cooperative connectivity requirement while minimizing the motion energy consumption. We start by looking at the case of perfect channel knowledge (i.e., the robots know the uplink channel quality for transmission from any unvisited location), and show how this problem can be optimally solved by posing it as a multiple-choice knapsack problem. We then extend our analysis to the stochastic case where the nodes predict the channel based on a small number of a priori channel samples, as discussed in Section II-D2.

In this section we focus on motion energy minimization, assuming a non-adaptive communication transmit power case.

A. Perfect Channel Knowledge

The perfect channel knowledge assumption would be a good approximation for environments where path loss is dominant and channel has a low variance around path loss. In our case, this serves as a starting point for our analysis, which will then be extended to the general case of an unknown channel. Consider \( N \) robots in a workspace \( W \subseteq \mathbb{R}^2 \). Let \( d_i(x_i) = ||x_i - x_0|| \) be the distance traveled by robot \( i \) with \( x_i \) and \( x_0 \) denoting the initial and final position respectively. Let \( h(x_i) = \alpha(x_i) e^{j\theta(x_i)} \) be the uplink channel from position \( x_i \) to the remote station with \( \alpha(x_i) \) and \( \theta(x_i) \) denoting the channel amplitude and phase respectively.

Since communication cost is not penalized in this setup, the optimal thing for the nodes would be to maximize the SNR at the remote station, subject to the individual power constraints. We then set \( \rho_i = 1 \), which corresponds to each node transmitting at the maximum allowed power, and the complex beamforming weight as \( w_i = e^{-j\theta(x_i)} \), for the \( i \)th node. The received signal power, after beamforming, is then given by

\[
P_R = P_0 \left( \sum_{i=1}^{N} \alpha(x_i) \right)^2.
\]

A Quality of Service (QoS) requirement, such as a target bit error rate, would result in a minimum required received power at the remote station, which we denote as \( P_{R,th} \). We then need \( P_R = P_0 \left( \sum_{i=1}^{N} \alpha(x_i) \right)^2 \geq P_{R,th} \) or equivalently \( \sum_{i=1}^{N} \alpha(x_i) \geq \sqrt{\frac{P_{R,th}}{P_0}} \), which results in the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \kappa M \sum_{i=1}^{N} d_i(x_i) \\
\text{subject to} & \quad \sum_{i=1}^{N} \alpha(x_i) \geq \alpha_{R,th} \quad \text{(2)}
\end{align*}
\]

where \( \alpha_{R,th} = \sqrt{\frac{P_{R,th}}{P_0}} \), \( N(x_i) \subseteq W \) is the neighborhood around \( x_i \) that the \( i \)th node is constrained to stay in, and \( x = [x_1 \cdots x_N]^T \) are the final positions of the robots.
Optimal Solution: We next show how to repose the optimization problem of (2) as a multiple-choice knapsack problem, which is a well studied problem in the computer science literature and can be solved optimally [21]. We first discretize our workspace $W$ into $M$ cells with centers $r_j \in W$, for $j \in \{1, \ldots, M\}$. The motion cost is then given by $J_{\text{MEMP}}(z_{ij}) = \kappa_M \sum_{j=1}^{N} \sum_{i \in N_j} d_i(r_j) z_{ij}$ and the optimization problem of (2) can be reformulated as

$$\begin{align*}
\text{minimize} & \quad J_{\text{MEMP}}(z_{ij}) \\
\text{subject to} & \quad \sum_{i} \sum_{j \in N_i} \alpha(r_j) z_{ij} \geq \alpha_{R,\text{th}} \quad \forall j \in N_i \\
& \quad \sum_{j \in N_i} z_{ij} = 1, \quad z_{ij} \in \{0, 1\}, \quad \forall j \in N_i, \quad \forall i,
\end{align*}$$

where $d_i(r_j)$ is the distance to cell $j$ for robot $i$, $\alpha(r_j)$ is the channel amplitude from cell $j$ to the remote station and $N_i \subseteq \{1, \ldots, M\}$ is the set of cells present in $N(x_i)$. A value of $z_{ij} = 1$ implies that robot $i$ chooses to move to cell $j$. We refer to this problem as the Motion Energy Minimization Problem (MEMP), with the optimal value of $J_{\text{MEMP}}^\ast$.

Lemma 2: MEMP of (3) can be posed as a multiple-choice knapsack problem (MCKP).

Proof: Define $\{\pi_{ij}\}$ and $\{\omega_{ij}\}$ as $\pi_{ij} = \max_{k \in N_i} d_i(r_k) - d_i(r_j)$ and $\omega_{ij} = \max_{k \in N_i} \alpha(r_k) - \alpha(r_j)$. We have

$$\begin{align*}
\sum_{i=1}^{N} \sum_{j \in N_i} \pi_{ij} z_{ij} = \sum_{i=1}^{N} \max_{k \in N_i} d_i(r_k) \sum_{j \in N_i} z_{ij} - \sum_{i=1}^{N} \max_{j \in N_i} d_i(r_j) z_{ij} = \sum_{i=1}^{N} \max_{j \in N_i} d_i(r_j) \sum_{i \in N_j} z_{ij} - \sum_{i=1}^{N} \max_{j \in N_i} d_i(r_j) z_{ij},
\end{align*}$$

where the second equality follows for any feasible solution since $\sum_{j=1}^{N} z_{ij} = 1$. Similarly, $\sum_{i=1}^{N} \max_{j \in N_i} \omega_{ij} z_{ij} = N \max_{i} \alpha(r_i) - \sum_{j \in N_i} \alpha(r_j) z_{ij}$. MEMP of (3) can then be posed as

$$\begin{align*}
\text{maximize} & \quad \sum_{i} \sum_{j \in N_i} \pi_{ij} z_{ij} \\
\text{subject to} & \quad \sum_{j \in N_i} \omega_{ij} z_{ij} \leq c \quad \forall j \in N_i, \quad \forall i,
\end{align*}$$

where $c = N \max_{i} \alpha(r_i) - \alpha_{R,\text{th}}$. Equation (4) is the standard form of the multiple-choice knapsack problem (MCKP). Although MCKP is NP-hard, the true solution can be efficiently found for several cases that arise in practice [21]. In this paper, we thus utilize the minimal algorithm developed by Pisinger [21] to optimally solve the resulting MCKP.

Remark 1: Let $J_{\text{OPT}}^\ast_{\text{MCKP}}$ denote the optimal value of the objective function of (4). The optimal values of the two formulations (3) and (4) are then related as follows:

$$J_{\text{MEMP}}^\ast = \kappa_M \left( \sum_{i} \max_{k \in N_i} d_i(r_k) - J_{\text{OPT}}^\ast_{\text{MCKP}} \right).$$

B. Probabilistic Channel Prediction

In realistic scenarios, the uplink channel values in transmission from unvisited locations may not be known to the robots a priori. We next consider this realistic case. The robots then utilize the stochastic prediction approach of Section II-D2 to predict the channel strength when transmitting from an unvisited location, using a small number of a priori channel samples in the same environment. Optimization of path planning for cooperative beamforming can then be posed as follows in this case,

$$\begin{align*}
\text{minimize} & \quad \kappa_M \sum_{i} d_i(x_i) \\
\text{subject to} & \quad \Pr\left( \sum_{i} \alpha(x_i) < \alpha_{R,\text{th}} \right) < \Pr_{\text{out}} \quad \forall x_i \in N\left(x_i^0\right), \quad i = 1, \ldots, N,
\end{align*}$$

where $\Pr(.)$ denotes the probability of the argument, $\alpha(x_i) = \sqrt{T(x_i)}$ is the random variable that represents the received channel amplitude when the $i$th node transmits from $x_i$, and $\Pr_{\text{out}}$ is the maximum tolerable outage probability.

As discussed in Section II, the predicted channel power (in dB) when transmitting from unvisited locations $x = [x_1 \ldots x_N]^T \in W^N$ can be represented as a Gaussian random vector $\Gamma_{\text{db}}(x) \sim N\left(\bar{\Gamma}_{\text{db}}(x), C_{\text{db}}(x)\right)$, where $\bar{\Gamma}_{\text{db}}(x)$ and $C_{\text{db}}(x)$ are the estimated mean and covariance matrix respectively. The channel amplitude at locations $x_i, \alpha(x_i) = [\alpha(x_1) \ldots \alpha(x_N)]^T = \left[\sqrt{T(x_1)} \ldots \sqrt{T(x_N)}\right]^T$ is a lognormal random vector, i.e., $20 \log_{10} \alpha(x_1) \ldots 20 \log_{10} \alpha(x_N)^T \sim N\left(\bar{\Gamma}_{\text{db}}(x), C_{\text{db}}(x)\right).$

$$\sum_{i=1}^{N} \alpha(x_i) = \sum_{i=1}^{N} [\alpha(x_1) \ldots \alpha(x_N)]^T$$

is then the sum of lognormal random variables.

As established in the literature, the lognormal distribution is a good approximation for the distribution of the sum of lognormal random variables [22]. Let $\alpha_{\text{sum}}$ with distribution $20 \log_{10} \alpha_{\text{sum}} \sim N\left(\mu_{\text{sum}}, \sigma_{\text{sum}}^2\right)$ denote the lognormal random variable approximating $\sum_{i=1}^{N} \alpha(x_i).$ $\mu_{\text{sum}}$ and $\sigma_{\text{sum}}$ can be found, based on $\bar{\Gamma}_{\text{db}}(x)$ and $C_{\text{db}}(x)$, by using the extended Fenton-Wilkinson (F-W) method [22]. The details are given in Appendix A. The outage probability inequality in (5) can then be expressed as $\mu_{\text{sum}} + \sigma_{\text{sum}} Q^{-1}\left(1 - \Pr_{\text{out}}\right) \geq 20 \log_{10} \alpha_{R,\text{th}}$, where $Q(.)$ denotes the $Q$ function. Equation (5) can then be posed as

$$\begin{align*}
\text{minimize} & \quad \kappa_M \sum_{i} d_i(x_i) \\
\text{subject to} & \quad \mu_{\text{sum}} + \sigma_{\text{sum}} Q^{-1}\left(1 - \Pr_{\text{out}}\right) \geq 20 \log_{10} \alpha_{R,\text{th}} \\
& \quad x_i \in N\left(x_i^0\right), \quad i = 1, \ldots, N.
\end{align*}$$

We refer to this as the Motion Energy Stochastic Setting (MESS) minimization problem. The optimization problem (6) can then be solved by using existing optimization toolboxes. We next propose an alternative approach for the case of stochastic channel knowledge, based on our proposed MEMP approach of (3) of Section III-A.

1) Approximation using analysis of MEMP of (3): In Section III-A, we showed how the motion energy-aware optimization problem can be solved for the case of perfect channel knowledge. That analysis and the corresponding solution can be used to find an approximate solution for the stochastic case, as we show next. As introduced earlier, the channel power
in dB, $20 \log_{10} \alpha(r_{j})$, has the distribution $20 \log_{10} \alpha(r_{j}) \sim \mathcal{N}(\mu(r_{j}), \sigma^{2}(r_{j}))$, where $r_{j}$ is the $j^{th}$ cell, as defined in Section III-A, and $\mu(r_{j}) = \Gamma_{dB}(r_{j})$ and $\sigma^{2}(r_{j}) = C_{dB}(r_{j})$ are obtained by evaluating Lemma 1 at $p = r_{j}$ (scalar). Consider $\hat{\alpha}(r_{j})$ such that $20 \log_{10} \hat{\alpha}(r_{j}) = \mu(r_{j}) - \zeta \sigma(r_{j})$ for some constant $\zeta > 0$. $\hat{\alpha}(r_{j})$ provides a conservative estimate of the channel amplitude. We then approximate $\alpha(r_{j})$ by $\hat{\alpha}(r_{j})$ in (3), which results in the following optimization problem:

$$\minimize \quad J_{\text{MEMP}}(\{z_{ij}\})$$

subject to

$$\sum_{i} \sum_{j \in N_{i}} \hat{\alpha}(r_{j}) z_{ij} \geq \alpha_{R,\text{th}}$$

(7)

Equation (7) can then be efficiently solved using the proposed approach of Section III-A for MEMP of (3). We next relate the optimization problem of (7) to the original optimization problem of (5) by finding a bound on the probability that the obtained solution satisfies the inequality constraint of (3). We first need the following lemma.

**Lemma 3:** $20 \log_{10} \alpha(r_{j})$ is positively correlated as a result of the exponential correlation, where $j$ is the cell chosen by robot $i$. We then have $\Pr(20 \log_{10} \alpha(r_{j}) \geq \xi_{i}, \forall i) \geq \prod_{i=1}^{N_{i}} \Pr(20 \log_{10} \alpha(r_{j}) \geq \xi_{i})$, for any $\xi_{i} \in \mathbb{R}$.

**Proof:** See [23].

**Lemma 4:** Let $\{z_{ij}\}$ be the solution of (7), and let $j_{i}$ be such that $z_{ij_{i}} = 1$. Then the probability that this solution results in an outage in (3) is bounded as follows:

$$\Pr\left(\sum_{i=1}^{N} \alpha(r_{j_{i}}) < \alpha_{R,\text{th}}\right) \leq 1 - [Q(-\zeta)]^{N}.$$  

**Proof:** The probability of successful transmission is $\Pr\left(\sum_{i=1}^{N} \alpha(r_{j_{i}}) \geq \alpha_{R,\text{th}}\right) \geq \Pr\left(\sum_{i=1}^{N} \alpha(r_{j_{i}}) \geq \sum_{i=1}^{N} \hat{\alpha}(r_{j_{i}})\right)$ since $\sum_{i=1}^{N} \hat{\alpha}(r_{j_{i}}) \geq \alpha_{R,\text{th}}$ for a feasible solution of (7). Further,

$$\Pr\left(\sum_{i=1}^{N} \alpha(r_{j_{i}}) \geq \sum_{i=1}^{N} \hat{\alpha}(r_{j_{i}})\right) \geq \Pr\left(20 \log_{10} \alpha(r_{j_{i}}) \geq 20 \log_{10} \hat{\alpha}(r_{j_{i}}), \forall i\right) \geq \prod_{i=1}^{N_{i}} \Pr\left(20 \log_{10} \alpha(r_{j_{i}}) \geq 20 \log_{10} \hat{\alpha}(r_{j_{i}}), \forall i\right) \geq [Q(-\zeta)]^{N}$$

where the second inequality follows from Lemma 3.

**IV. ENERGY-AWARE COOPERATIVE ROBOTIC BEAMFORMING**

In this section, we extend our results of Section III to include the communication energy cost as well, i.e., we are interested in finding the most energy efficient way (considering both motion and communication) for the robots to cooperatively transmit the data to a remote station. The robots need to determine new locations for transmission as well as the transmission powers such that they minimize the total energy consumption while satisfying the cooperative connectivity requirement. As in Section III, we start with the scenario of perfect channel knowledge, for which we obtain an $\epsilon$-suboptimal solution by showing that solving our problem is equivalent to solving a series of multiple-choice knapsack problems. We then extend our analysis to the stochastic setting with probabilistic channel prediction and incorporate channel uncertainty into our formulation.

**A. Perfect Channel Knowledge**

In this case, the robots perform distributed transmit beamforming with complex weights $w_{i} = r_{i} e^{j \theta_{i}(x_{i})}$, if node $i$ moves to $x_{i}$, where $0 \leq r_{i} \leq 1$ and $\theta_{i}(x_{i})$ is as described in Section II-C. The received power is then given as $P_{R} = P_{0} \left(\sum_{j=1}^{N} \alpha(x_{i}) \rho_{j}\right)^{2}$. As can be seen from Section II-B, imposing a minimum transmission rate requirement results in a minimum required received power which we denote by $P_{R,\text{th}}$ for a given $\eta_{1}, \eta_{2} \leq 1$. For instance, in the case of uncoded MQAM, a bit error rate requirement of BER$_{th}$, results in $\eta_{1} = 1$ and $\eta_{2} = 1.5/\ln(5)$BER$_{th}$, and a minimum spectral efficiency requirement would translate to a minimum required received power $P_{R,\text{th}}$. Imposing this results in $P_{R} = P_{0} \left(\sum_{j=1}^{N} \alpha(x_{i}) \rho_{j}\right)^{2} \geq P_{R,\text{th}}$ or equivalently $\sum_{j=1}^{N} \alpha(x_{i}) \rho_{j} \geq \sqrt{P_{R,\text{th}}/P_{0}} = \alpha_{R,\text{th}}$, with $\rho_{i}^{2}P_{0}$ denoting the transmit power of robot $i$.

The total energy cost is then given as $J_{\text{TE}}(x_{i}) = \sum_{i} \frac{1}{n_{i}} B \log_{2} \left(1 + \frac{n_{i}P_{i} \sum_{j=1}^{N} \alpha(x_{i}) \rho_{j}}{n_{i}P_{i} \sum_{j=1}^{N} \alpha(x_{i}) \rho_{j}^{2}}\right)^{2}$, and the resulting optimization problem can be expressed as

$$\minimize \quad J_{\text{TE}}(x_{i})$$

subject to

$$\sum_{i=1}^{N} \alpha(x_{i}) \rho_{i} \geq \alpha_{R,\text{th}}$$

(8)

We refer to this as the Total Energy Minimization Problem (TEMP), with the optimal value of $J_{\text{TE}}^{\text{opt}}$. We begin by characterizing properties of the optimal communication strategy as well as of the optimal solution $(\{z_{ij}\}, \rho)$ of TEMP of (9). This is one of the key intermediate steps that allows us to pose our problem as a series of multiple choice knapsack problems.

In the following lemma, we show that the inequality for the cooperative connectivity requirement in TEMP of (9) is satisfied with equality in the optimal solution.
Lemma 5: Let $(\{z_{ij}^{\text{OPT}}\}, \rho^{\text{OPT}})$ be an optimal solution of TEMP of (9). Let $j_i^{\text{OPT}}$ be such that $z_{ij}^{\text{OPT}} = 1$. Then the solution satisfies $\sum_{i=1}^{N} \alpha(r_{ij}^{\text{OPT}}) \rho^{\text{OPT}} = \alpha_{R,h}$. 

Proof: See Appendix B for the proof.

Next, consider the case where the positions of the robots are fixed and the only objective is to minimize the total transmit power (not energy) while satisfying the communication requirement. Lemma 6 characterizes the optimal solution of this case, as follows.

Lemma 6: Consider the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \rho \\
\text{subject to} & \quad \sum_{i} \rho_{i}^{2} \\
& \quad \sum_{i} \alpha_{i} \rho_{i} \geq \alpha_{R,h} (10) \\
& \quad 0 \leq \rho_{i} \leq 1, \quad i = 1, \ldots, N.
\end{align*}
\]

The optimal solution for (10) is $\rho_{i} = \min\{\alpha_{i}, 1\}$ where $\lambda > 0$ is such that $\sum_{i=1}^{N} \min\{\alpha_{i}, 1\} \alpha_{i} = \alpha_{R,h}$. 

Proof: See Appendix C for the proof.

We use Lemma 6 as building blocks, we next characterize the optimal communication strategy of TEMP of (9) given the final optimal positions of the robots.

Lemma 7: Let $(\{z_{ij}^{\text{OPT}}\}, \rho^{\text{OPT}})$ be an optimal solution of TEMP of (9). Let $j_i^{\text{OPT}}$ be such that $z_{ij}^{\text{OPT}} = 1$. Then $\rho^{\text{OPT}} = \min\{\alpha(r_{ij}^{\text{OPT}}), 1\}$ where $\lambda > 0$ is such that $\sum_{i=1}^{N} \min\{\alpha(r_{ij}^{\text{OPT}}), 1\} \alpha(r_{ij}^{\text{OPT}}) = \alpha_{R,h}$. 

Proof: We prove this by contradiction. Assume that $\rho_{i}^{\text{OPT}} \neq \rho_{i}^{*}$ where $\rho_{i}^{*} = \min\{\alpha(r_{ij}^{\text{OPT}}), 1\}$, for $\lambda > 0$, such that $\sum_{i=1}^{N} \min\{\alpha(r_{ij}^{\text{OPT}}), 1\} \alpha(r_{ij}^{\text{OPT}}) = \alpha_{R,h}$. Then, $(\{z_{ij}^{\text{OPT}}\}, \rho^{\text{OPT}})$ is a feasible solution of (9) since $\sum_{i=1}^{N} \alpha(r_{ij}^{\text{OPT}}) \rho_{i}^{\text{OPT}} = \alpha_{R,h}$. The cost of the optimal solution then becomes $J_{\text{TEMP}}(\{z_{ij}^{\text{OPT}}\}, \rho^{\text{OPT}}) = \kappa_{M} \sum_{i=1}^{N} d_{i}(r_{ij}^{\text{OPT}}) + \kappa_{C} \sum_{i=1}^{N} \rho_{i}^{\text{OPT}}^{2} \geq \kappa_{M} \sum_{i=1}^{N} d_{i}(r_{ij}^{\text{OPT}}) + \kappa_{C} \sum_{i=1}^{N} \rho_{i}^{*2} = J_{\text{TEMP}}(\{z_{ij}^{\text{OPT}}\}, \rho^{*})$, following from Lemma 5 and Lemma 6, where $\kappa_{C} = \frac{\mu_{C} L_{B} \log_{2}(1 + \frac{\rho_{i}^{*}}{\sigma_{n}^{2}})}{\rho_{i}^{*}}$. Thus, we have a contradiction, as we found a feasible solution with a lower cost.

Lemma 8: Let $(\{z_{ij}^{\text{OPT}}\}, \lambda^{\text{OPT}})$ be an optimal solution of (11). Let $j_i^{\text{OPT}}$ be such that $z_{ij}^{\text{OPT}} = 1$ and let $\rho_{i}^{*} = \min\{\lambda^{\text{OPT}}(r_{ij}^{\text{OPT}}), 1\}$. Then $(\{z_{ij}^{\text{OPT}}\}, \rho^{\text{OPT}})$ is an optimal solution of TEMP of (9).

Proof: Without loss of generality, let $\lambda^{*} = \min\{\lambda^{*}(r_{ij}^{\text{OPT}}), 1\}$ for some $i$. We first show that the connectivity requirement inequality in (11) is satisfied with equality for the optimal solution, i.e., $\sum_{i=1}^{N} \min\{\lambda^{*}(r_{ij}^{\text{OPT}}), 1\} \alpha(r_{ij}^{\text{OPT}}) = \alpha_{R,h}$. 

Consider a robot such that $\sum_{i=1}^{N} \min\{\lambda^{*}(r_{ij}^{\text{OPT}}), 1\} \alpha(r_{ij}^{\text{OPT}}) = \alpha_{R,h}$. Clearly we have $\lambda < \lambda^{*}$, which implies $\min\{\lambda(r_{ij}^{\text{OPT}}), 1\} \geq \min\{\lambda(r_{ij}^{\text{OPT}}), 1\}$ for some $i$. Hence $J_{\lambda}(\{z_{ij}^{*}\}, \lambda) < J_{\lambda}(\{z_{ij}^{\text{OPT}}\}, \lambda^{*})$, resulting in a contradiction. Thus $\sum_{i=1}^{N} \min\{\lambda^{*}(r_{ij}^{\text{OPT}}), 1\} \alpha(r_{ij}^{\text{OPT}}) = \alpha_{R,h}$.

Next, we show via contradiction that $(\{z_{ij}^{\text{OPT}}\}, \rho^{\text{OPT}})$, obtained from an optimal solution of (11), is an optimal solution of TEMP of (9). Assume $(\{z_{ij}^{\text{OPT}}\}, \rho^{\text{OPT}})$ is not an optimal solution of (9) and let $(\{z_{ij}^{\text{OPT}}\), \rho^{\text{OPT}})$ be an optimal solution instead. From Lemma 7, we have that $\rho^{\text{OPT}} = \min\{\lambda^{\text{OPT}}(r_{ij}^{\text{OPT}}), 1\}$, where $\lambda^{\text{OPT}} > 0$ is such that $\sum_{i=1}^{N} \min\{\lambda^{\text{OPT}}(r_{ij}^{\text{OPT}}), 1\} \alpha(r_{ij}^{\text{OPT}}) = \alpha_{R,h}$.

Then, we have $J_{\text{TEMP}}(\{z_{ij}^{\text{OPT}}\}, \rho^{\text{OPT}}) = \kappa_{M} \sum_{i=1}^{N} d_{i}(r_{ij}^{\text{OPT}}) + \kappa_{C} \sum_{i=1}^{N} \min\{\lambda^{\text{OPT}}(r_{ij}^{\text{OPT}}), 1\}^{2} \geq \kappa_{M} \sum_{i=1}^{N} d_{i}(r_{ij}^{\text{OPT}}) + \kappa_{C} \sum_{i=1}^{N} \rho_{i}^{*2} = J_{\text{TEMP}}(\{z_{ij}^{\text{OPT}}\}, \rho^{*})$, which implies that $(\{z_{ij}^{\text{OPT}}\), \rho^{\text{OPT}})$ is an optimal solution of (11) with a lower cost than $(\{z_{ij}^{\text{OPT}}\}, \lambda^{*})$, resulting in a contradiction.

$\epsilon$-Suboptimal Solution: In this subsection we pose a series of multiple-choice knapsack problems and relate their solution to TEMP of (9) to obtain an $\epsilon$-suboptimal solution. In this context, $\epsilon$ is a positive variable that determines how close to the optimal solution we can get. Basically, for each fixed value of $\lambda$, we have a multiple-choice knapsack problem, as can be seen from (11), which we can solve optimally. We then discretize $\lambda$ uniformly with $\epsilon$ determining the corresponding resolution. Let $\lambda_{k} = \frac{\epsilon}{\alpha_{\text{max}}}$ for $k \in \{T_{1} - 1, \ldots, T_{2} - 1\}$ and $\lambda_{T_{2}} = \frac{1}{\alpha_{\min}}$, where $\epsilon_{1} = \frac{2N(\alpha_{\text{max}}/\alpha_{\text{max}}^{0})^{2}}{\alpha_{\min}}$, $\alpha_{\text{max}} = \max_{j=1,\ldots,M}\alpha(r_{j})$ and $\alpha_{\text{min}}^{0} = \min\{\alpha(r_{j}) : i = 1, \ldots, N\}$ denotes the minimum channel amplitude among the initial positions of the robots. Furthermore, $T_{1} = \left\lfloor \frac{\alpha_{\text{max}}}{\alpha_{\text{min}}^{0}} \right\rfloor$, $T_{2} = \frac{\alpha_{\text{max}}^{0}}{\alpha_{\text{min}}^{0}}$, and $2N\left(\frac{\alpha_{\text{max}}}{\alpha_{\text{max}}^{0}}\right)^{2}$ determine the range of $\lambda$, as explained next. Since $\sum_{i=1}^{N} \min\{\lambda(r_{ij}), 1\} \alpha(r_{ij}) \leq \sum_{i=1}^{N} \sum_{j=1}^{M} (\lambda_{T_{1}} - 1) \alpha(r_{ij}) = \left\lfloor \frac{\alpha_{\text{max}}}{\alpha_{\text{min}}^{0}} \right\rfloor - 1 \epsilon_{1} N \alpha_{\text{max}} < \alpha_{R,h}$, $\lambda < \lambda_{T_{1}} - 1$ could not be a feasible solution of (11). Moreover, an optimal solution $(\{z_{ij}^{\text{OPT}}\}, \lambda^{*})$ would not involve a robot incurring motion energy to get to a location with a worse channel amplitude, resulting in $\alpha(r_{j}) \geq \alpha_{\text{min}}^{0}$ for all $i$. Since $\lambda_{T_{2}} \alpha_{\text{min}}^{0} = 1$, we have that $\lambda^{*} > \lambda_{T_{2}}$ is an optimal solution, then $\lambda_{T_{2}}$ is also an optimal solution. Thus, we need to only consider $\lambda \in (\lambda_{T_{2}}, -1)$ in (11), which results in the
following optimization problem for each $k \in \{T_1, \cdots, T_2\}$:

$$
\begin{align*}
\text{minimize } & \quad J_{k}(\{z_{ij}\}) \\
\text{subject to } & \quad \sum_{i \in N} \left[ \min\{\lambda_k \alpha(r_j), 1\} \right] \alpha(r_j) z_{ij} \geq \alpha R_{th} \\
& \quad \sum_{j \in N} z_{ij} = 1, \quad z_{ij} \in \{0,1\}, \quad \forall \ j \in N_i, \quad \forall \ i,
\end{align*}
$$

(12)

where $J_{k}(\{z_{ij}\}) = \sum_{i \in N} \left(\min\{\lambda_k \alpha(r_j), 1\}\right)^2 z_{ij}$, with the optimum value of $J_{k}^{\text{OPT}}$. This optimization problem can be solved similar to (3) by posing it as a knapsack problem through a change of variable as shown in Section III.

Let $J_{k,\text{min}} = \min_{k \in \{T_1, \cdots, T_2\}} J_{k}^{\text{OPT}}$. In order to find $J_{k,\text{min}}$, we need to solve $T_2 - T_1 + 1 \leq \left[ \frac{2N \alpha_{\text{max}}}{\alpha_{\text{min}}}, \frac{2N \alpha_{\text{max}}}{\alpha_{\text{min}}}, 1 \right]$ multiple choice knapsack problems. As can be seen, the number of knapsack problems to be solved grows linearly with $N$ and $\frac{1}{\gamma}$. In the following theorem, we show how we can get arbitrarily close to the optimal solution by solving this set of knapsack problems.

**Theorem 1:** Let $m = \arg \min_{k \in \{T_1, \cdots, T_2\}} J_{k}^{\text{OPT}}$. Let $\{z_{ij}^{\ast}\}$ be a solution of (12) when $k = m$, and $z_{ij}^{\ast}$ be such that $z_{ij}^{\ast} = 1$. Consider a $\lambda^{\ast}$ such that $\lambda^{\ast} \leq \lambda_{m}$ and

$$
\sum_{i=1}^{N} \left[ \min\{\lambda^{\ast} \alpha(r_j), 1\} \right] \alpha(r_j) \geq \alpha R_{th}.
$$

Further, set $\rho^{\ast} = \min\{\lambda^{\ast} \alpha(r_j), 1\}$. Then, $\{z_{ij}^{\ast}, \rho^{\ast}\}$ is a feasible solution of TEMP of (9) and

$$
J_{\text{TEMP}}(\{z_{ij}^{\ast}, \rho^{\ast}\}) \leq J_{\text{OPT}}^{\ast} + \kappa C.
$$

**Proof:** It is straightforward to see that $\{z_{ij}^{\ast}, \rho^{\ast}\}$ is a feasible solution of TEMP of (9). Moreover, $J_{\text{OPT}}^{\ast}(\{z_{ij}^{\ast}, \lambda^{\ast}\}) = J_{\lambda^{\ast}}(\{z_{ij}^{\ast}\}, \lambda^{\ast}) \leq J_{\lambda_{m}}(\{z_{ij}^{\ast}\}, \lambda_{m}) = J_{\lambda_{m}}^{\ast} = J_{\text{OPT}}^{\ast} \text{ since } J_{\lambda}(\cdot) \text{ is a non-decreasing function of } \lambda.$

Let $\{\{z_{ij}^{\ast}, \lambda^{\ast}\}, \text{OPT}\}$ be an optimal solution of (11). From Lemma 8 we have that $J_{\text{OPT}}^{\ast}(\{z_{ij}^{\ast}, \lambda^{\ast}\}) = J_{\lambda^{\ast}}(\{z_{ij}^{\ast}\}, \lambda^{\ast})$. Let $\lambda^{\text{OPT}}$ be $J_{\text{OPT}}^{\ast}(\{z_{ij}^{\ast}\}, \rho^{\ast}) \leq J_{\lambda_{m}}^{\ast} \leq J_{\lambda}(\{z_{ij}^{\ast}\}, \lambda_{m}) = J_{\lambda_{m}}$, as we established earlier. Thus, there exists a $k \in \{T_1, \cdots, T_2\}$ such that $\lambda^{\text{OPT}} = \lambda_{k-1}, \lambda_{k}$. We then have

$$
\begin{align*}
J_{\text{TEMP}}^{\ast} & \leq J_{\text{OPT}}^{\ast} + \kappa C \sum_{i=1}^{N} \lambda_{k}^{2} \left(\alpha \left(\lambda r_{j}^{\text{OPT}}\right)\right)^{2} \leq J_{\text{OPT}}^{\ast} + \kappa C \sum_{i=1}^{N} \lambda_{k}^{2} \left(\alpha \left(\lambda r_{j}^{\text{OPT}}\right)\right)^{2} \leq J_{\text{OPT}}^{\ast} + \kappa C \\
& \leq J_{\text{OPT}}^{\ast} + \kappa C.
\end{align*}
$$

**Remark 2:** $\kappa C$ is the communication energy cost of a single robot when it transmits at maximum power and the robots satisfy the cooperative connectivity requirement with equality.

**Remark 3:** Solving TEMP of (9) through a brute-force search of space is infeasible even for moderately small values of the number of robots ($N$). For instance, if $M$ is the number of points in the discretized workspace and if we represent each $\rho_{i}$ by $k$ bits, then the computational complexity of an exhaustive search is $M^{N}2^{kN}$. On the other hand, with our proposed $\epsilon$-suboptimal solution, the number of multiple-choice knapsack problems to solve grows linearly with $N$ and $\frac{1}{\gamma}$. We note that while we can solve (8) with an existing solver, there is no guarantee that the solver will find the global optimum since the objective function is non-convex. Theorem 1 then allows us to get arbitrarily close to the optimal solution with a low computational complexity.

### B. Probabilistic Channel Prediction

As discussed earlier, in realistic scenarios, the unmanned vehicles do not know the uplink channel when transmitting from unvisited locations. As such, they will probabilistically predict the channel based on a small number of a priori measurements in the same environment, as summarized in Section II-D2. The energy-aware (both motion and communication) cooperative beamforming problem (8), can then be extended to the following in this stochastic setting:

$$
\begin{align*}
\text{minimize } & \quad J_{\text{TE,ST}}(x, \rho) \\
\text{subject to } & \quad \operatorname{Pr}(\sum_{i \in N} \alpha(x_{r}) \rho_{i} < \alpha R_{th}) < \operatorname{Pr}_{\text{out}} \\
& \quad 0 \leq \rho_{i} \leq 1, \quad x_{r} \in \mathcal{N}(x_{r}), \quad i = 1, \cdots, N,
\end{align*}
$$

(14)

where $J_{\text{TE,ST}}(x, \rho) = \sum_{i=1}^{N} d_{i}(x_{r}) + \kappa M \sum_{i=1}^{N} \frac{1}{P_{0}} \sum_{j} \frac{\alpha_{ij}(x_{r})}{\rho_{j}}$, with $x = \{x_{1} \cdots x_{N}\}^T$ and $\rho = \{\rho_{1} \cdots \rho_{N}\}^T$ as optimization variables and $E[\cdot]$ representing the average of the argument. In this case, the average is taken over $\alpha(x_{r}), \forall i$. The vector $\alpha(x_{r}) = \alpha(x_{N})^T$ is a lognormal random vector with distribution $[20 \log_{10} \alpha(x_{1}) \cdots 20 \log_{10} \alpha(x_{N})] \sim \mathcal{N}(\mu_{\rho}, \Gamma_{\rho}(x_{r})) \sim \mathcal{N}(\mu_{\rho}, \Gamma_{\rho}(x_{r}))$ where $\Gamma_{\rho}(x_{r})$ and $\Gamma_{\rho}(x_{r})$ are the estimated mean and covariance matrix of the predicted channel power respectively, and $[\mu_{\rho}]_{i} = \log_{10} \alpha_{ij}$. Let $\alpha_{\sum,\rho}$ with distribution $20 \log_{10} \alpha_{\sum,\rho} \sim \mathcal{N}(\mu_{\sum,\rho}, \sigma_{\sum,\rho})$ denote the lognormal random variable approximating $\sum_{i=1}^{N} \alpha(x_{r}) \rho_{i}$. $\mu_{\sum,\rho}$ and $\sigma_{\sum,\rho}$ can be found, based on $\mu_{\rho}$, $\Gamma_{\rho}(x_{r})$, and $\Gamma_{\rho}(x_{r})$, by using the extended Fenton-Wilkinson method [22]. Similar to Section III, the objective then becomes $J_{\text{TESS}}(x, \rho) = \mu_{\sum,\rho}$, and our optimization problem can be rewritten as

$$
\begin{align*}
\text{minimize } & \quad J_{\text{TESS}}(x, \rho) \\
\text{subject to } & \quad \mu_{\sum,\rho} + \sigma_{\sum,\rho} Q^{-1} \left(1 - \operatorname{Pr}_{\text{out}}\right) > 20 \log_{10} \alpha R_{th} \\
& \quad 0 \leq \rho_{i} \leq 1, \quad i = 1, \cdots, N, \quad x_{r} \in \mathcal{N}(x_{r}), \quad i = 1, \cdots, N,
\end{align*}
$$

(15)

which can then be solved by using existing optimization toolboxes. We refer to this as the Total Energy Stochastic Setting (TESS) minimization problem.
amplitude with a high probability. We then approximate \( \alpha(r_j) \) by \( \hat{\alpha}(r_j) \) in (9), which results in the following optimization:

\[
\begin{align*}
\text{minimize} & \quad J_{\text{TESS}}(z_{ij}, \rho) \\
\text{subject to} & \quad \sum_i \sum_{j \in N_i} \hat{\alpha}(r_j) \rho_i z_{ij} \geq \alpha_{R,\text{th}} \\
& \quad 0 \leq \rho_i \leq 1, \ i = 1, \cdots, N \\
& \quad \sum_{j \in N_i} z_{ij} = 1, \ z_{ij} \in \{0, 1\}, \ \forall j \in N_i, \ \forall i.
\end{align*}
\]

where \( J_{\text{TESS}}(z_{ij}, \rho) = \kappa_M \sum_i \sum_{j \in N_i} d_i(r_j) z_{ij} + \eta_1 B \log_2 \left( 1 + \eta_2 P_0 \sum_{j \in N_i} \hat{\alpha}(r_j) \rho_i z_{ij} \right) \). Equation (16) can then be efficiently solved using the proposed approach of Section IV-A for TEMP. We next relate the optimization problem of (16) to the original optimization problem of (14) by finding a bound on the probability that the obtained solution satisfies the inequality constraint of (9).

**Lemma 9:** Let \( (z_{ij}, \rho) \) be the solution obtained when solving (16) using the proposed approach of Section IV-A, and let \( j_i \) be such that \( z_{ij_i} = 1 \). The probability that this solution results in an outage in (9) is bounded as follows:

\[
\Pr \left( \sum_i \alpha(r_j) \rho_i < \alpha_{R,\text{th}} \right) < 1 - \left[ \frac{\kappa}{Q(\kappa)} \right]^N.
\]

The proof is similar to the proof of Lemma 4.

**V. SIMULATION RESULTS**

Consider a scenario where 6 robots are located in a 50 m \( \times \) 50 m workspace with the remote station at the origin and initial positions as shown in Fig. 5. The channel is generated using the probabilistic channel model described in Section II-D1, with the following parameters that were obtained from real channel measurements in downtown San Francisco [15]: \( n_{PL} = 4.4, \nu_{SH} = 6.76 \) and \( \beta_{SH} = 22.6 \) m. Moreover, the multipath fading is taken to be uncorrelated Rician fading with the parameter \( K_{\text{Ric}} = 3.9 \). We consider a bandwidth of \( B = 10 \) MHz and the received noise power is taken to be a realistic value of \(-100 \) dBmW [24]. We consider uncoded MQAM modulation with a BER tolerance of \( 10^{-5} \) and a minimum spectral efficiency requirement (transmission rate divided by bandwidth) of** 4. This corresponds to \( \eta_2 = 0.1515 \) and a minimum received SNR requirement of \( 20 \) dB which, for the given noise power, corresponds to a received power requirement of \( P_{R,\text{th},dBm} = -80 \) dBmW. We take the maximum transmission power of a node to be \( P_{0,dBm} = 27 \) dBmW [25], which results in \( \alpha_{R,\text{th},dB} = -53.5 \) dB. The amount of data to be transferred is 800 bits/Hz. The robots are situated far enough from the remote station that they do not satisfy the received power requirement at their initial positions (see Fig. 5). The neighborhood \( N_i \), within which the final position of robot \( i \) is constrained to lie in, is taken to be the entire workspace. The optimization problems of MEMP, MESS, TEMP and TESS, can be solved centrally by either one of the robots or by the remote station.

**A. Perfect Channel Knowledge**

We first analyze the trends of the motion energy-aware (MEMP) and the total energy-aware (TEMP) approaches as the communication load \( (l/B) \) varies in Figures 3, 4 and 5. TEMP of (9) is solved via the set of multiple-choice knapsack problems of Theorem 1 with \( \epsilon = 0.05 \). As shown in Theorem 1, the optimal value lies at most \( 0.05\kappa_C \) below the value obtained by solving the family of knapsack problems. This confidence bound is also shown in Fig. 3. It can be seen that the confidence bound is very close to the solution obtained by using Theorem 1 for solving TEMP, which confirms that Theorem 1 can get arbitrarily close to the optimal solution with a considerably low computational complexity. Each data point on the plots is obtained by averaging over 100 channels generated for the given set of channel parameters.

![Fig. 3: Total energy (sum of motion and communication) consumption of MEMP and TEMP for different communication loads for the case of perfect channel knowledge. TEMP provides a considerable energy saving, as expected. MEMP refers to the case where only motion energy is minimized while communication energy is also adapted and co-optimized in TEMP.](image)

Fig. 3 shows the average total energy consumption of MEMP and TEMP. The figure also shows the corresponding error bars, representing the standard deviation of the total energy consumption for each data point. As can be seen, TEMP provides a significant energy saving as the communication load \( l/B \) increases. In TEMP, with increasing \( l/B \), the time for transmission increases as well, as a direct consequence of Lemma 5. Transmission power is thus penalized more and the robots travel larger distances to get to the locations with a better channel quality, allowing them to utilize a lower transmit power for communication. More specifically, Fig. 4 shows the total distance traveled as \( l/B \) varies. We can see that TEMP travels larger distances as \( l/B \) increases. Fig. 4 also shows how the total communication transmission power of TEMP decreases with increasing \( l/B \). This is due to the fact that in TEMP, by incurring more motion energy, the nodes can find spots with a better channel quality, resulting in a lower communication energy and a lower total energy consumption. Fig. 5 shows the behavior of the solution of MEMP and TEMP for communication loads of \( l/B = 100 \) bits/Hz and \( l/B = 1500 \) bits/Hz. The background color encodes the channel power to the remote station. The lighter (darker) areas correspond to regions with better (worse) channel quality. As expected, the TEMP solution moves larger distances in the high communication load case to get to locations with a better channel quality.

**B. Probabilistic Channel Prediction**

We next consider the case where the channel is not known in the transmission from unvisited locations. We consider the workspace of Fig. 8. The robots are assumed to have 5% a priori channel measurements in this workspace. The robots then utilize the channel prediction framework of Section
II-D2 for probabilistically predicting the channel at unvisited locations.\(^3\) Channel and system parameters are as summarized earlier in this section, with \(P_{\text{out}} = 0.2\). Fig. 6 shows the average total energy consumption as a function of \(l/B\) for both MESS and TESS. The figure also shows the corresponding error bars which represents the standard deviation of the total energy consumption for each data point. MESS refers to the case where only motion energy is minimized, for the case of probabilistic channel prediction, while communication energy is also adapted and co-optimized in TESS. Curves marked by TESS and MESS denote the total energy consumption when the nodes move to the final locations and experience the true channel values. The label ‘TESS predicted’ in Fig. 6, on the other hand, is obtained from (15) using predicted channel values and the lognormal approximation. In other words, the predicted curve is what the nodes predict to consume while the TESS curve is the true consumption. As expected, we see a significant performance improvement when using the total energy-aware approach (TESS), as compared to the motion energy-aware approach (MESS), especially as the communication load increases. Fig. 7 shows the total distance traveled by the robots for both MESS and TESS. Similar to the behavior of the perfect channel knowledge case, an increase in the communication load results in a larger penalization of the transmit power, and as a result TESS travels larger distances to get to locations with a better channel quality. Then TESS can use lower transmission powers, as can be seen in Fig. 7. Fig. 8 shows the behavior of MESS and TESS for communication loads of \(l/B = 100\) bits/Hz and \(l/B = 1500\) bits/Hz. The background color encodes the estimated channel power to the remote station. The lighter (darker) areas correspond to a better (worse) channel quality.

\(^3\)MESS and TESS are solved using MATLAB’s fmincon solver. fmincon requires the objective and the constraints to be twice differentiable and is thus unable to handle uncorrelated multipath. We then assume an exponentially correlated multipath in the channel predictor with a very small decorrelation distance of \(\lambda_d = 0.033\) m, which has a negligible impact on the prediction performance. It should be noted that this is only for prediction purposes and that the real channel has an uncorrelated multipath.

![Fig. 4: Total (left) distance traveled and (right) transmission power utilized by MEMP and TEMP as a function of communication load and for the case of perfect channel knowledge.](image)

![Fig. 5: Solution of MEMP and TEMP for (left) low \((l/B = 100\) bits/Hz\) and (right) high \((l/B = 1500\) bits/Hz\) communication loads for the case of perfect channel knowledge. The background represents the uplink channel power with lighter (darker) regions corresponding to a better (worse) channel quality. Readers are referred to the color pdf for a better viewing.](image)

![Fig. 6: Energy consumption of MESS and TESS for different communication loads for the case of probabilistic channel prediction. TESS provides a considerable energy saving, as expected. MESS refers to the case where only motion energy is minimized for the case of probabilistic channel prediction while communication energy is also adapted and co-optimized in TESS.](image)

![Fig. 7: Total (left) distance traveled and (right) transmission power utilized by MESS and TESS for different communication loads for the case of probabilistic channel prediction.](image)

![Fig. 8: Solution of MESS and TESS for (left) low \((l/B = 100\) bits/Hz\) and (right) high \((l/B = 1500\) bits/Hz\) communication loads for the case of probabilistic channel prediction. The background represents the estimated channel power with lighter (darker) regions corresponding to a better (worse) channel quality. Readers are referred to the color pdf for a better viewing.](image)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>TESS of (15) with target (P_{\text{out}} = 0.2)</th>
<th>MESS of (6) with target (P_{\text{out}} = 0.2)</th>
<th>Approx. of (16) with (\zeta = 0.1)</th>
<th>Approx. of (7) with (\zeta = 0.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob. of outage</td>
<td>0.108</td>
<td>0.178</td>
<td>0.028</td>
<td>0.020</td>
</tr>
</tbody>
</table>

TABLE I: Probability of outage for TESS and MESS as well as for the approximations of (16) and (7) of Sections IV-B1 and III-B1. We can see that the target \(P_{\text{out}}\) is satisfied for TESS and MESS.

Remark 4 (Computational complexity): In our simulations, the implementation is done in MATLAB except for the MCKP solver, which is in C, and is adapted from David Pisinger’s implementation [21]. The simulations were run on a 3.4 GHz
The first and second moments of $\sum_ia_i$ are given by

$$u_1 = E\left[\sum_ia_i\right] = \sum_ie^{\mu_i + \Sigma_i/2}$$

and $u_2 = E\left[\sum_i\alpha_i^2\right] = \sum_ie^{2\mu_i + 2\Sigma_i} + 2\Sigma_i\sum_{j=1}^{\infty}e^{\mu_j + \mu_j/2}(\Sigma_i + \Sigma_j + 2\Sigma_{ij}),$$

where $\mu_i$ is the $i$th entry of $\mu$ and $\Sigma_{ij}$ is the $ij$th entry of $\Sigma$. In the extended Fenton-Wilkinson method [22], the first and second moments of $\sum_ia_i$ and $\sum_\alpha_i$ are equated to obtain $\mu_{sum} = \frac{1}{2}(\Sigma_i / u_1)$ and $\sigma_{sum}^2 = \frac{1}{2}(\Sigma_i / u_1)$. For both perfect channel knowledge cases (MEMP and TEMP), for both the total energy-aware and motion energy-aware cases. We can see that this approach can be used in the extended Fenton-Wilkinson method [22].

B. Proof of Lemma 5

We first prove the following lemma, which we shall use in proving Lemma 5.

Lemma 10: Let $f : \mathbb{R}^n_+ \rightarrow \mathbb{R}_+$ with $f(\rho) = \ln(1 + \xi(\sum_{i=1}^n\alpha_i\rho_i)^2)$, where $\xi, \alpha > 0$. Given $\rho$, let $I = \{i : \frac{\partial}{\partial \rho_i} > 0\}$, and let $v \in \mathbb{R}_+$ be such that its $i$th element is $v(i) = \begin{cases} -\alpha_i, & i \in I \\ 0, & \text{else} \end{cases}$. Then $f(\rho)$ is strictly decreasing in direction $v$, i.e., $(\nabla f)^Tv < 0$.

Proof: Let $y = \xi(\sum_{i=1}^n\alpha_i\rho_i)^2$. Also, $\alpha_k \left(\sum_{i=1}^n\rho_i^2\right) = \rho_k \left(\sum_{i=1}^n\alpha_i\rho_i\right) + \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. Inserting this in (17) and rearranging in $(\nabla f(\rho))^Tv = \frac{2}{\ln(1+y)}\left(\sum_{i=1}^n\alpha_i\rho_i\right)^2(1+y)\ln(1+y)\ln(1+y) - \frac{y}{(1+y)\ln(1+y)}\left(\sum_{i=1}^n\alpha_i\rho_i\right)^2(1+y)\ln(1+y)$. We have $\frac{dy}{dy}(1+y)\ln(1+y) - y = 0$ for $y > 0$. Also, $(1+y)\ln(1+y) - y = 0$ for $y > 0$. Thus $(1+y)\ln(1+y) - y > 0$, which results in $(\nabla f(\rho))^Tv < 0$. We denote $\frac{\partial}{\partial \rho_i}f(\rho) = \frac{\partial}{\partial \rho_i}f(\rho_i)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i = \rho_k(\sum_{i=1}^n\alpha_i\rho_i) + \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. Thus, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. If $\sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i = 0$, we denote $\rho_i(\alpha_k\rho_i - \alpha_i\rho_k)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i = \rho_k(\sum_{i=1}^n\alpha_i\rho_i) + \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. We denote $\frac{\partial}{\partial \rho_i}f(\rho) = \frac{\partial}{\partial \rho_i}f(\rho_i)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. If $\sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i = 0$, we denote $\rho_i(\alpha_k\rho_i - \alpha_i\rho_k)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. We denote $\frac{\partial}{\partial \rho_i}f(\rho) = \frac{\partial}{\partial \rho_i}f(\rho_i)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. If $\sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i = 0$, we denote $\rho_i(\alpha_k\rho_i - \alpha_i\rho_k)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. We denote $\frac{\partial}{\partial \rho_i}f(\rho) = \frac{\partial}{\partial \rho_i}f(\rho_i)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. If $\sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i = 0$, we denote $\rho_i(\alpha_k\rho_i - \alpha_i\rho_k)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. We denote $\frac{\partial}{\partial \rho_i}f(\rho) = \frac{\partial}{\partial \rho_i}f(\rho_i)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. If $\sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i = 0$, we denote $\rho_i(\alpha_k\rho_i - \alpha_i\rho_k)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. We denote $\frac{\partial}{\partial \rho_i}f(\rho) = \frac{\partial}{\partial \rho_i}f(\rho_i)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$. If $\sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i = 0$, we denote $\rho_i(\alpha_k\rho_i - \alpha_i\rho_k)\forall i \in I$, we have $\sum_{i=1}^n\alpha_i\rho_i > 0, \sum_{i=1}^n\alpha_i\rho_i = \sum_{i=1}^n(\alpha_k\rho_i - \alpha_i\rho_k)\rho_i$.
as \( L(p_i, \lambda_0, \alpha_i, \gamma_i, \rho_i) = \sum_i \rho_i^2 + \sum_i \gamma_i(p_i - 1) + \sum_i \gamma_i(1 - p_i) + \lambda_0 \left( \lambda_{R,i} - \sum_i \alpha_i \rho_i \right) \). We have the following KKT conditions:

\[
2\rho_i - \lambda_0 \alpha_i + \gamma_i - \rho_i^* = 0, \quad \gamma_i(p_i - 1) = 0, \\
\gamma_i \rho_i = 0, \quad \lambda_0(\lambda_{R,i} - \sum_i \alpha_i \rho_i^*) = 0,
\]

\( \xi^* \geq 0, \quad \gamma^* \geq 0, \quad \lambda_0^* > 0, \quad 0 \leq \rho_i^* \leq 1, \quad \sum_i \alpha_i \rho_i^* \geq \alpha_{R,i} \).

Assume \( \gamma^*_i > 0 \) for some \( i \). Then \( \rho_i^* = 0 \) and thus \( \xi^*_i = 0 \). But \( \rho_i^* = \frac{\lambda_0}{\alpha_i} - \frac{\xi}{\gamma_i} = \frac{\lambda_0 \alpha_i}{\gamma_i} + \frac{\lambda_0}{\gamma_i} > 0 \), resulting in a contradiction. Therefore, \( \gamma_i^* = 0, \forall i \). Assume \( \lambda_0^* = 0 \). Then \( \rho_i^* = \frac{\lambda_0^*}{\alpha_i} = 0, \) and hence \( \sum_i \alpha_i \rho_i^* = 0 < \alpha_{R,i}, \) resulting in a contradiction. Thus \( \sum_i \alpha_i \rho_i^* = \alpha_{R,i} \). If \( \frac{\lambda_0^*}{\alpha_i} > 1 \) then \( \xi^*_i > 0 \) which in turn implies that \( \rho_i^* = 1 \). If \( \frac{\lambda_0^*}{\alpha_i} < 1 \), then \( \xi^*_i = 0 \) and hence \( \rho_i^* = \frac{\lambda_0^*}{\alpha_i} \). Thus, \( \rho_i^* = \min(\{\lambda_{R,i}, 1\}) \), where \( \lambda = \frac{\lambda_0^*}{\alpha_i} > 0 \) is such that \( \sum_i \min(\lambda_{R,i}, 1) \alpha_i = \alpha_{R,i} \).

**REFERENCES**


