Filter Banks - IV

Half-Band Filters, QMF Banks and Multiresolution Analysis

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Half-Band filters, QMF banks and Multiresolution analysis

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1. Mth band filters (Nyquist filters)

Consider an M-fold interpolation filter, the output is given as $Y(z) = X(z^M)H(z)$. Representing $H(z)$ in terms of polyphase components with $E_0(z)$ a constant c, we have

$$H(z) = c + z^{-1}E_1(z^M) + \ldots + z^{-(M-1)}E_{M-1}(z^M) \tag{1}$$

then $$Y(z) = cX(z^M) + \sum_{l=1}^{M-1} z^{-l}E_l(z^M)X(z^M) \tag{2}$$

This means $y(Mn) = cx(n)$. A filter with this property is known as *Nyquist filter*

Then the Impulse response h(n) satisfies

$$\begin{aligned} h(Mn) &= c, \quad n = 0 \\ &= 0, \quad otherwise \end{aligned} \tag{3}$$

Generalized definition

$$H(z) = E_0(z^M) + z^{-1}E_1(z^M) + \ldots + cz^{-k}z^{-Mn_k} + \ldots + z^{-(M-1)}E_{M-1}(z^M) \tag{4}$$

$$\begin{aligned} h(Mn+k) &= c \quad n = n_k \\ &= 0 \quad otherwise \end{aligned} \tag{5}$$

$$Y(z) = cz^{-k}z^{-Mn_k}X(z^M) + \sum_{l \neq k} z^{-l}E_l(z^M)X(z^M) \tag{6}$$

$$y(Mn + Mn_k + k) = cx(n) \tag{7}$$

Manifestation in frequency domain

If *H(z)* satifies *(1)*, then

$$\sum_{k=0}^{M-1} H(zW^k) = Mc = 1 \text{ (assuming } c = 1/M) \qquad (8)$$

The frequency response of $H(zW^k)$ is the shifted version of $H(zW^{kj(w-2pik/M)})$.

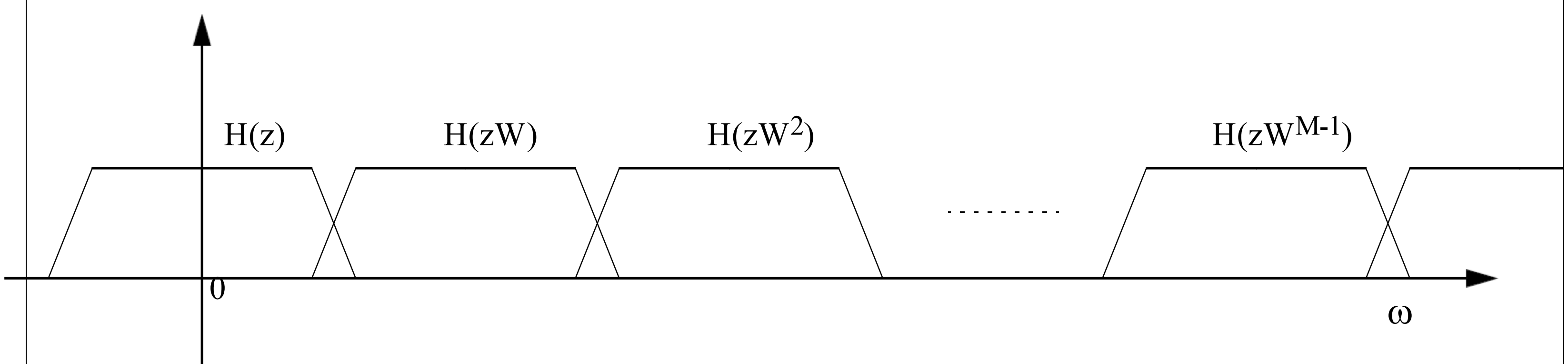


fig 1 For an Mth band filter H(z), the responses $H(e^{j(w-2pik/M)})$ add up to a constant

a) Half-Band Filters

A half-band filter *H(z)* is an *Mth* band filter with *M = 2*.

$$H(z) = c + z^{-1}E_1(z^2) \qquad (9)$$

In terms of impulse response *h(n)* this means

$$h(2n) = c \quad n = 0 \qquad (10)$$
$$\quad = 0 \quad \text{otherwise}$$

The condition in frequency domain is

$$H(z) + H(-z) = 1 \quad (\text{ assuming } c = .5) \qquad (11)$$

If H(z) has real coefficients, then $H(-e^{j\omega}) = H(e^{j(\pi - \omega)})$ and the equation

becomes $H(e^{j\omega}) + H(e^{j(\pi - \omega)}) = 1$ (12)

From (11), we observe that the two factors $H\left(e^{j\left(\frac{\Pi}{2} - \theta\right)}\right)$ and $H\left(e^{j\left(\frac{\Pi}{2} + \theta\right)}\right)$ add up to unity for all θ . i.e. we have a symmetry with respect to the half band frequency $\frac{\Pi}{2}$, justifying the name *"half-band filters"*.

Design of zero-phase FIR equiripple half-band filters

i) Suppose $G(z) = \sum_{n=0}^{N} g(n)z^{-n}$ is a Type 2 linear phase filter.

This means that N is odd and $g(n)$ is real, with $g(n) = g(N-n)$.

This also means that there is a zero at $\omega = \Pi$

We can write $G(e^{j\omega}) = e^{-j\omega\frac{N}{2}} G_R(\omega)$ where $G_R(\omega)$ is real.

ii) Design $G_R(\omega)$ with passband as $0 \le \omega \le \theta_p$ and transition band to be $\theta_p \le \omega \le \pi$. There is no stop band.

Such filters with one equiripple passband and no stopband can be designed using the Parks-McClellan algorithm.

iii) Define the transfer function $F(z) = [z^{-N} + G(z^2)] / 2$.

This is Type 1 linear phase filter.

$F(z)$ is a half-band filter.

iv) Define $\hat{H}(z) = z^N F(z)$

$\hat{H}(z)$ is a zero-phase half-band filter. Its length is $2N+1$

2. Complementary transfer functions

A) Strictly Complementary (SC) functions

A set of transfer functions $[H_0(z), H_1(z), \ldots, H_{M-1}(z)]$ is said to be strictly complementary if they add up to a delay, that is

$$\sum_{k=0}^{M-1} H_k(z) = cz^{-n_0} \qquad c \neq 0 \tag{13}$$

If we split a signal *x(n)* into *M* subband signals using the SC analysis filters $H_k(z)$, then we can just add the subband signals to get back the original signal *x(n)* with no distortion, except a delay.

B) Power Complementary (PC) Functions

A set of M transfer functions is said to be power-complementary if

$$\sum_{k=0}^{M-1} \left|H_k(e^{j\omega})\right|^2 = c \qquad \text{for all } \omega \tag{14}$$

where c>0 is a constant.

l This property is equivalent to $\sum_{k=0}^{M-1} \tilde{H}_k(z) H_k(z) = c$ for all z (15)

where

$$\tilde{H}(z) = H_*(z^{-1}) \quad (16)$$

C) Allpass Complementary (AC) Functions

A set of transfer functions is said to be allpass complementary if

$$\sum_{k=0}^{M-1} H_k(z) = A(z) \qquad \text{where } A(z) \text{ is allpass.} \quad (17)$$

Note - SC functions are also AC but not PC.

Relation between Nyquist(M) Filters and Power Complementary Filters

Consider a transfer function $H(z)$ represented in the M-component polyphase form

$$H(z) = \sum_{l=0}^{M-1} z^{-l} E_l(z^M)$$ where $E_l(z)$ are Type1 polyphase components.

Define the new transfer function $G(z) = \tilde{H}(z)H(z)$. Then the set $[E_0(z), E_1(z), \ldots, E_{M-1}(z)]$ is power complementary if and only if $G(z)$ is an Mth band filter. (**Extra credit - Prove this result**).

3. Quadrature Mirror Filter (QMF) Banks

a) M-channel filter bank

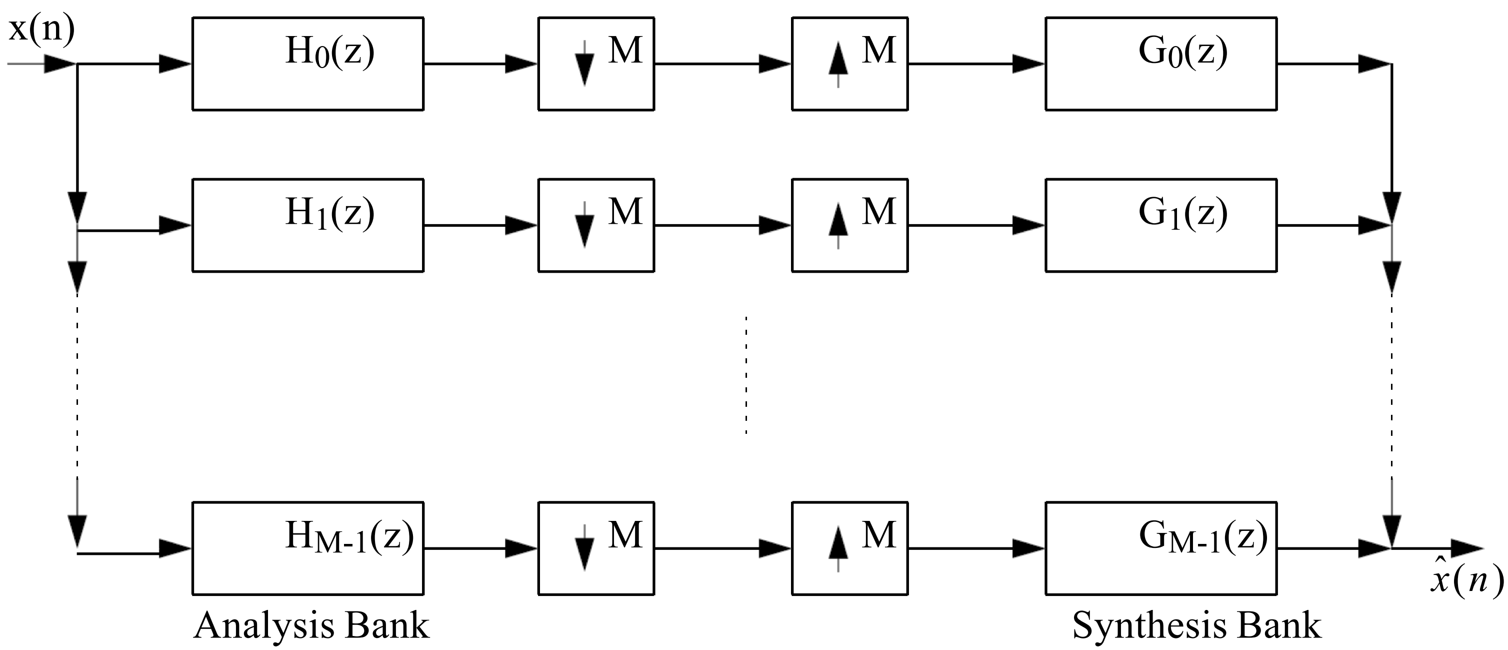


fig 2. The M-channel QMF bank

Subband coding involves spliting the input signal *x(n)* into subsequences using a bank of filters $H_0(z), H_1(z), \ldots, H_{M-1}(z)$ as shown in fig 2. This would produce M times as many samples as the original. To preserve the overall number of samples, the outputs of the filters are decimated by a factor of M. Such filter banks are said to be maximally decimated. The output of the i_{th} filter can be written as

$$y_i(m) = \sum_n x(n)h_i(m-n) \tag{18}$$

$$y_i(m) = \langle x(n), h_i(m-n)\rangle \tag{19}$$

where $\langle x, y\rangle$ denotes the inner product of x and y.

After decimation, we have

$$y_i(mM) = \langle x(n), h_i(mM-n)\rangle \tag{20}$$

Thus the analysis filter bank computes the inner products of the signal with the basis functions

$$a_k(n) = h_i(mM-n) \qquad i = 0, 1, \ldots, M-1, m \in Z \quad \text{where } k = mM + i. \tag{21}$$

Reconstruction of the signal is achieved by using a synthesis filter bank, where the decimated outputs are upsampled, filtered by $G_0(z), G_1(z), \ldots, G_{M-1}(z)$ and summed.

When the output is identically equal to the input we say that the overall system is perfectly reconstructing.

The synthesis filter output is given as

$$z_i(n) = \sum_m y_i(mM) g_i(n - mM) \qquad i = 0, 1, \ldots, N-1. \tag{22}$$

summing over all the channels we get

$$x(n) = \sum_{i=0}^{M-1} z_i(n) \tag{23}$$

$$x(n) = \sum_{i=0}^{M-1} \sum_m y_i(mN) g_i(n - mM) = \sum_k \langle x(n), a_k(n) \rangle b_k(n) \tag{24}$$

where $b_k(n) = g_i(n-mM)$ and $k = mM + i$.

The analysis basis functions are $a_k(n)$ and the synthesis basis functions are $b_k(n)$.

The above equations can be written in matrix notation as

$X = BAX$, thus $B = A^{-1}$. If $B = A^*$, then A is *unitary*, $b_k(n) = a_k(n)$ and the expansion in (24) is orthonormal.

b) Two channel filter bank.

Consider the specific case of a maximally decimated two channel multirate filter bank, shown in fig. 3.

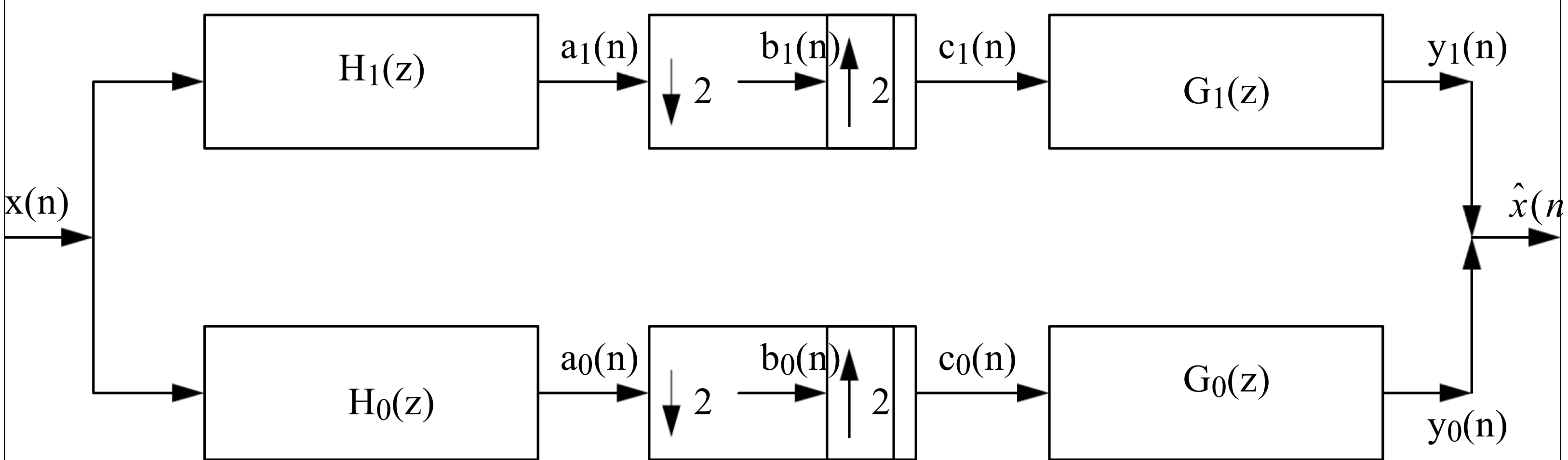


fig 3. A 2-channel QMF bank

Tracing the signals through the lower branch gives

$$A_0(z) = H_0(z)X(z) \qquad (25)$$

$$Y_0(z) = G_0(z)C_0(z) \qquad (26)$$

The outputs of the downsampler and upsampler are given as

$$B(z) = \frac{1}{2}\left[A_0\left(z^{\frac{1}{2}}\right) + A_0\left(-z^{\frac{1}{2}}\right)\right] \qquad (27)$$

$$C_0(z) = B_0(z^2) \qquad (28)$$

combining these equations we get

$$Y_0(z) = 1/2G_0(z)[H_0(z)X(z) + H_0(-z)X(-z)] \qquad (29)$$

Similarly, for the upper branch we can write

$$Y_1(z) = 1/2G_1(z)[H_1(z)X(z) + H_1(-z)X(-z)] \qquad (30)$$

$$\hat{X}(z) = 1/2[H_0(z)G_0(z) + H_1(z)G_1(z)]X(z) + 1/2[H_0(-z)G_0(z) + H_1(-z)G_1(z)]X(-z) \qquad (31)$$

For perfect reconstruction, the factor multiplying $X(z)$ should be equal to *1* and that multiplying $X(-z)$ should be equal to *0*. *i.e.*

$$H_0(z)G_0(z) + H_1(z)G_1(z) = 2, \qquad (32)$$

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \qquad (33)$$

In matrix notation (31) can be written as

$$\hat{X}(z) = \left(\frac{1}{2}\right)\begin{bmatrix} G_0(z) & G_1(z) \end{bmatrix}\begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix}\begin{bmatrix} X(z) \\ X(-z) \end{bmatrix} \quad (34)$$

Let the *2 X 2* matrix be $H_m(z)$ (**Alias component matrix**).
solving equations (32) and (33) we get

$$\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad (35)$$

$$\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \frac{2}{\nabla_m(z)}\begin{bmatrix} H_1(-z) \\ -H_0(-z) \end{bmatrix} \text{ where } \nabla_m(z) = det\, H_m(z). \quad (36)$$

$$\text{Let } P(z) = \frac{2H_0(z)H_1(-z)}{\nabla_m(z)} = H_0(z)G_0(z) \text{ then} \quad (37)$$

$$P(-z) = \frac{2H_1(z)H_0(-z)}{\nabla_m(-z)} = H_1(z)G_1(z) \quad (\text{since } \nabla_m(z) = -\nabla_m(-z)) \qquad (38)$$

The condition for perfect reconstruction in (32) can be written as

$$P(z) + P(-z) = 2 \qquad (39)$$

To have FIR solutions for the above equation, it is necessary and sufficient that

$$\nabla_m(z) = 2z^{-2l-1}. \text{ where } l \in Z \qquad (40)$$

To construct an FIR filter bank, we should find a valid $P(z)$ and factor it into its FIR factors. We could initially choose a factor of $P(z)$ and solve for the complementary part. Then we could refactor P(z) into $H_0(z)$ and $G_0(z)$ or $H_1(-z)$ and $G_1(-z)$ respectively.

4. Errors created in the QMF bank

a) Aliasing and Imaging. (ALD)

The analysis filters have non-zero transition bandwidth and stopband gain.
The signals $x_k(n)$ are not bandlimited, and their decimation results in aliasing (ALD).

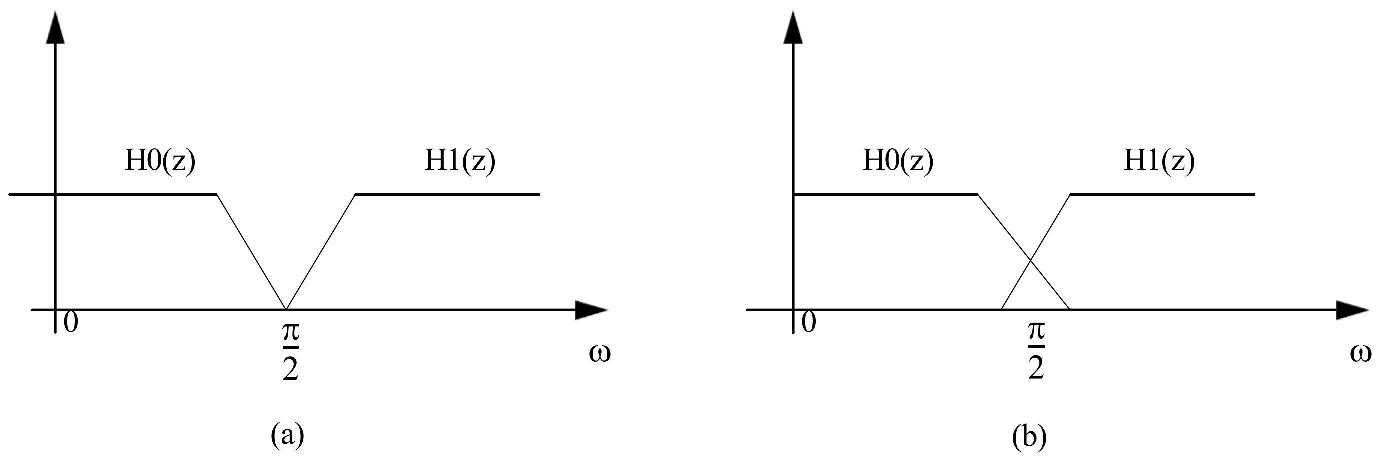


fig 4. Two possible magnitude responses for the analysis filters. (a) Nonoverlapping, and (b) overlapping.

In equation (31) the term containing $X(-z)$ takes into account aliasing due to the decimators and imaging due to the expanders.

We can cancel aliasing by choosing the filters such that the quantity

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \quad \text{or} \tag{41}$$

$$G_0(z) = H_1(-z),\ G_1(z) = -H_0(-z). \tag{42}$$

Thus, given $H_0(z)$ and $H_1(z)$, it is possible to completely cancel aliasing by this choise of synthesis filters.

So, in a QMF bank we permit aliasing in the analysis bank and then choose synthesis filters so that the alias component in the upper branch is cancelled by that in the lower branch.

fig. 5 shows the alias cancelation mechanism in the QMF bank.

b) Amplitude and phase distortions

With the above choice of synthesis filters, we have

$$\hat{X}(z) = T(z)X(z) \tag{43}$$

where $T(z) = 1/2[H_0(z)G_0(z) + H_1(z)G_1(z)]$, is called the distortion transfer function substituting the alias free condition, we get

$$T(z) = 1/2[H_0(z)H_1(-z) - H_1(z)H_0(-z)] \tag{44}$$

letting $T(e^{j\omega}) = |T(e^{i\omega})|e^{j\varphi(\omega)}$, we have

$$\hat{X}(e^{j\omega}) = |T(e^{i\omega})|e^{j\varphi(\omega)}X(e^{j\omega}) \tag{45}$$

Unless $T(z)$ is allpass, we say that $\hat{X}(e^{j\omega})$ suffers from amplitude distortion (AMD).

Similarly unless $T(z)$ has linear phase, $\hat{X}(e^{j\omega})$ suffers from phase distortion (PHD)

Note - If a QMF bank is free from ALD, AMD and PHD, it is said to have perfect reconstruction property. This is equivalent to the condition $T(z) = c^{z-n_0}$.

Let $H_1(z) = H_0(-z)$. Then $H_1(e^{j\omega})$ is a mirror image of $H_0(e^{j\omega})$ with repect to the quadrature frequency $\frac{2\Pi}{4}$, and therefore the name QMF.

With these filters we have

$$T(z) = 1/2(H_0^2(z) - H_0^2(-z)). \quad (46)$$

Polyphase representation

$$H_0(z) = E_0(z^2) + z^{-1}E_1(z^2) \quad \text{(Type1 representation)} \quad (47)$$

$$H_1(z) = E_0(z^2) - z^{-1}E_1(z^2) \text{ i.e} \quad (48)$$

$$\begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} E_0(z^2) \\ z^{-1}E1(z^2) \end{bmatrix} \quad (49)$$

similarly the synthesis filters can be represented as

$$\begin{bmatrix} G_0(z) & G_1(z) \end{bmatrix} = \begin{bmatrix} z^{-1}E1(z^2) & E_0(z^2) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (50)$$

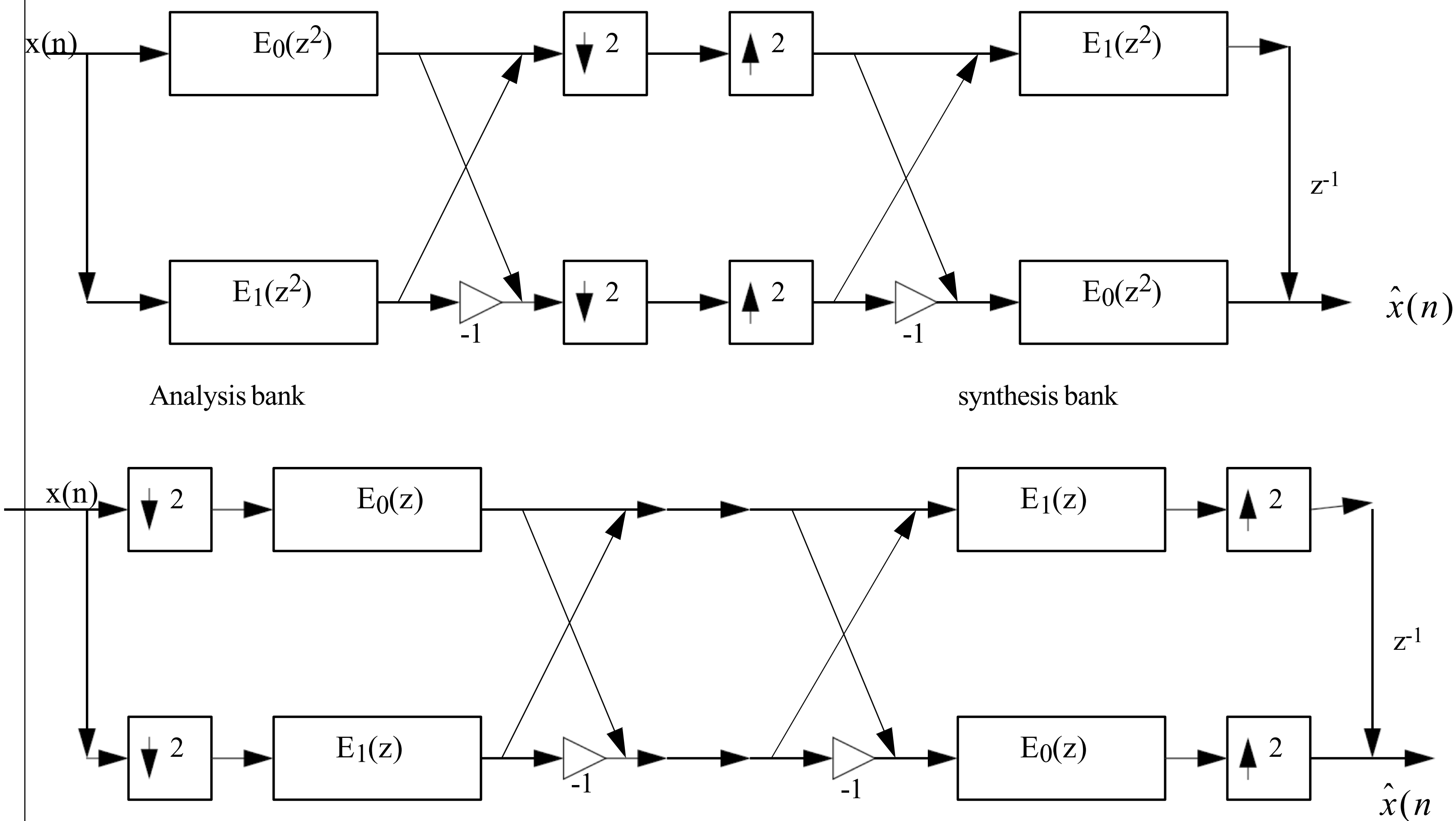


fig 6 (a) The complete QMF bank in polyphase form (b) Rearrangement using noble identities

With the analysis filters related as $H_1(z) = H_0(-z)$
and the synthesis filters chosen to cancel aliasing

$$G_0(z) = H_1(-z), \quad G_1(z) = -H_0(-z)$$

the distortion function T(z) given in (46) can be written as

$$T(z) = 2z^{-1}E_0(z^2)E_1(z^2). \qquad (51)$$

When $H_0(z)$ is a FIR filter, then the amplitude distortion can be eliminated if and only if the FIR functions $E_0(z)$ and $E_1(z)$ is a delay . *i.e.*

$$E_0(z) = c_0 z^{-n_0} \qquad (52)$$

$$E_1(z) = c_1 z^{-n_1} \qquad (53)$$

. This means

$$H_0(z) = cz^{-2n_0} + cz^{-(2n_1+1)}, \quad H_1(z) = cz^{-2n_0} - cz^{-(2n_1+1)} \qquad (54)$$

That is the filters cannot have sharp cutoff and good stopband attenuation.
We cannot, therefore, obtain useful FIR perfect reconstruction systems under the constraint $H_1(z) = H_0(-z)$.

Eliminating phase distortion with FIR filters

From linear phase constraint we have $h_0(n) = \mp h_0(N-n)$ Since $H_0(z)$ has to be lowpass, the only possibility is $h_0(n) = h_0(N-n)$. With this choice

$$H(e^{j\omega}) = e^{-j\omega\frac{N}{2}} R(\omega) \text{ where } R(\omega) \text{ is real for all w.} \quad (55)$$

The expression for distortion function is thus,

$$T(e^{j\omega}) = \left(\frac{1}{2}\right)\left(\left|H_0(z)\right|^2 - (-1)^N \left|H_0(z^{-1})\right|^2\right) \quad (56)$$

which implies that:

$$T(e^{j\omega}) = e^{-\frac{jN\omega}{2}}\left(\left|H_0(e^{j\omega})\right|^2 - (-1)^N\left|H_0(e^{j(\pi-\omega)})\right|^2\right)$$

5. Power symmetric QMF banks

Power symmetric property

From (49) we have

$$\tilde{E}(z)E(z) = 2 \qquad R^{\dagger}R = 0.5I \qquad (57)$$

where $E(z) = \begin{bmatrix} E_0(z^2) \\ z^{-1}E_1(z^2) \end{bmatrix}$ $R = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ so that

$$\tilde{h}(z)h(z) = 1 \ . \qquad (58)$$

where $h(z) = \begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix}$ In terms of ω this can be written

as $\left|H_0(e^{j\omega})\right|^2 + \left|H_1(e^{j\omega})\right|^2 = 1$. **(power complementary condition)** *(59)*

so $H_1(z)$ is related to $H_0(z)$ in two ways

1) $H_1(z) = H_0(-z)$, (*60*) 2) power complementary condition above .

Combining these two, $\tilde{H}_0(z)H_0(z) + \tilde{H}_0(-z)H_0(-z) = 1$ **(power symmetry condition)** *(61)*

FIR PR system with power symmetric filters

Let the synthesis filters be chosen so as to cancel aliasing then we have

$$\hat{X}(z) = \frac{1}{2}[H_0(z)H_1(-z) - H_1(z)H_0(-z)]X(z) \qquad (62)$$

For perfect reconstruction, we require this to be a delay. Assume that $H_0(z)$ is power symmetric. If the filter $H_1(z)$ is chosen as

$$H_1(z) = -z^{-N}\tilde{H}_0(-z)$$ then equation (62) reduces to (63)

$$\tilde{X}(z) = 0.5z^{-N}X(z)$$ for odd N. (64)

This satisfies the PR system condition.

Then the synthesis filters are given as

$$G_0(z) = z^{-N}\tilde{H}_0(z) \qquad G_1(z) = z^{-N}\tilde{H}_1(z) \qquad (65)$$

In time domain these can be written as

$$h_1(n) = (-1)^N h_0^*(N-n) \qquad (66)$$

$$g_0(n) = h_0^*(N-n) \qquad g_1(n) = h_1^*(N-n) \qquad (67)$$

Design of FIR PR system with power symmetric filters

i) From above discussion we see that only the filter $H_0(z)$ is to designed.
The power symmetric property means that the zero-phase filter

$$H(z) = \hat{H}_0(z)H_0(z) \quad \text{is a half-band filter.}$$

$$H(e^{j\omega}) \quad \text{is nonnegative.}$$

ii) First design a zero-phase FIR half band filter

$$G(z) = \sum_{n=-N}^{N} g(n)z^{-n}$$ of order 2N using Parks-McClellan algorithm.

The half-band property can be achieved by constraining the bandedges to be such that $\omega_p + \omega_s = \pi$, and the peak ripples in the passband and stopband to be identical.

iii) Define $H(z) = G(z) + \delta$, where δ is the peak stopband ripple of $G(e^{j\omega})$

This ensures that $H(e^{j\omega}) \ge 0$.

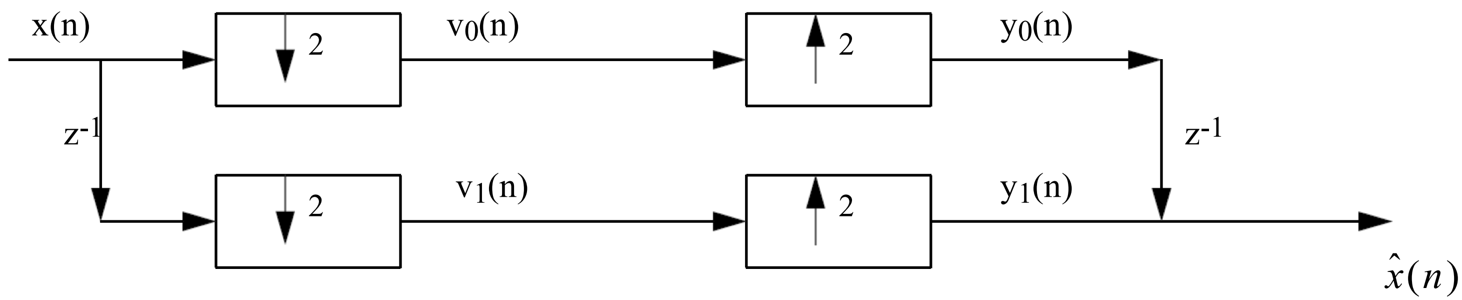
iv) Compute the spectral factor $H_0(z)$ of the filter $H(z)$. This can be done by computing the zeros of $H(z)$ and assigning the appropriate subset to $H_0(z)$.

v) Once $H_0(z)$ has been computed, the remaining three filters can be obtained by using (66) and (67).

6. Perfect Reconstruction (PR) systems

The delay chain perfect reconstruction system

$$H_0(z) = 1,\ H_1(z) = z^{-1},\ G_0(z) = z^{-1},\ G_1(z) = 1.$$



x(n):	*x(0)*	*x(1)*	*x(2)*	*x(3)*	*x(4)*	*x(5)*	*x(6)*	. . .
v0(n):	*x(0)*		*x(2)*		*x(4)*		*x(6)*	. . .
v1(n):	*x(-1)*		*x(1)*		*x(3)*		*x(5)*	. . .
y0(n):	*x(0)*	*0*	*x(2)*	*0*	*x(4)*	*0*	*x(6)*	. . .
y1(n):	*x(-1)*	*0*	*x(1)*	*0*	*x(3)*	*0*	*x(5)*	. . .
x^(n):	*x(-1)*	*x(0)*	*x(1)*	*x(2)*	*x(3)*	*x(4)*	*x(5)*	. . .

fig7. (a) The delay chain perfect reconstruction QMF bank and (b) its operation in the time domain

- Refering to fig. 17 from handout on polyphase representation(filter banks(II))

  we have $\hat{x}(n) = x(n)$ if $\boldsymbol{R}(z)\boldsymbol{E}(z) = \boldsymbol{I}$ (68)

- Generalizing the above condition, we have

$$\boldsymbol{R}(z)\boldsymbol{E}(z) = cz^{-m_0}\boldsymbol{I} \text{ with } \boldsymbol{T}(z) = cz^{-(Mm_0 + M - 1)} \qquad (69)$$

- Condition for FIR perfect reconstruction systems

$$det\ \boldsymbol{E(z)} = \alpha z^{-K},\ \alpha \neq 0,\ K = integer. \qquad (70)$$

- **R(z)** should also satisfy similar condition.

- *Example1* - Consider the two channel system shown in fig 8

  (a) Find whether the system satisfies PR condition and also

  (b) find the analysis and synthesis filters when c = 2 and $T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

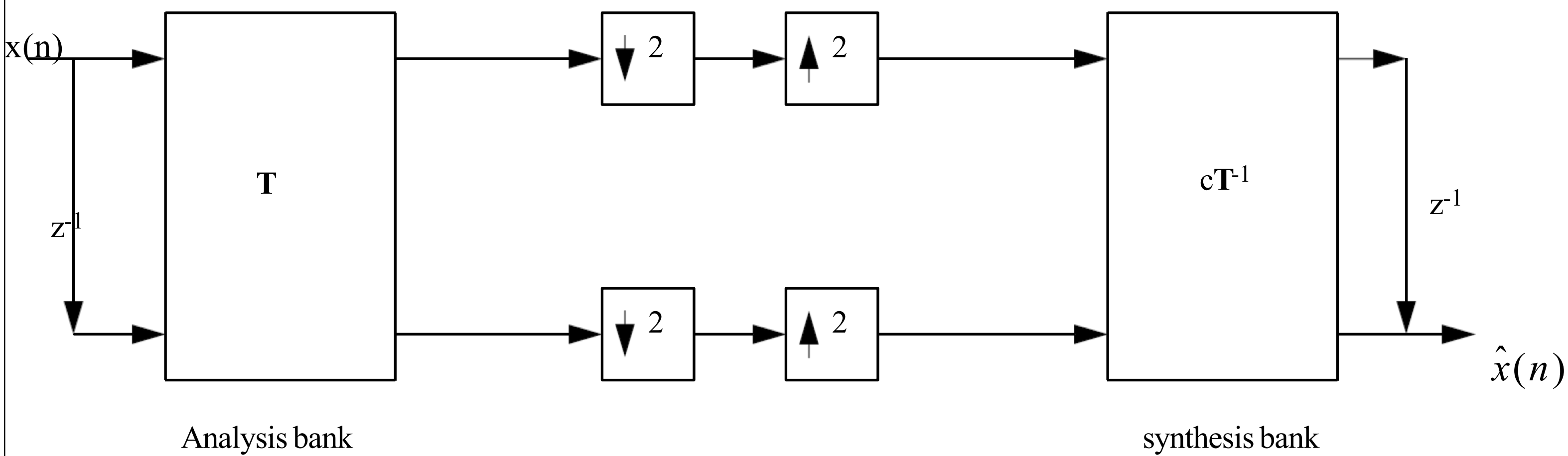


fig 8. Example of perfect reconstruction filter.

a) By comparing with fig 7 we have

E(z) = T and ***R(z)*** $= c\mathbf{T}^{-1}$ and ***E(z)R(z)*** $= c$.

Therefore the system is PR system with output given as

$$\hat{x}(n) = cx(n-1)$$

b) With $T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, we have $cT^{-1} = T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = T$.

So the above figure can be redrawn as shown in fig 9

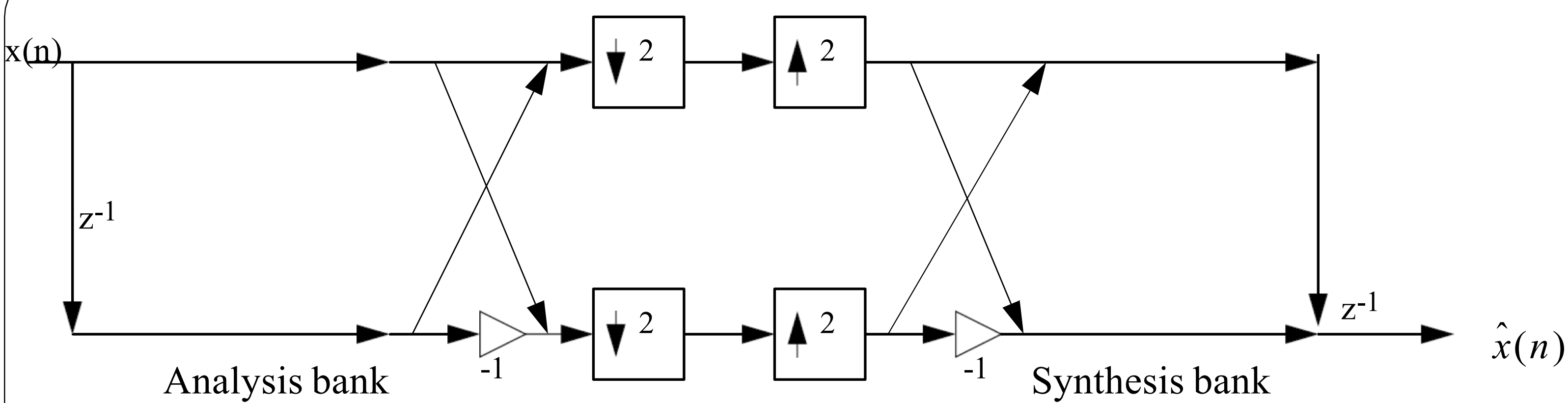


fig 9. For specific choice of T, fig 8 redrawn

so the filters are

$$H_0(z) = 1+z^{-1}, \quad H_1(z) = 1-z^{-1}, \quad G_0(z) = 1+z^{-1}, \quad G_1(z) = -1+z^{-1}$$

and the system can be redrawn as in fig 10

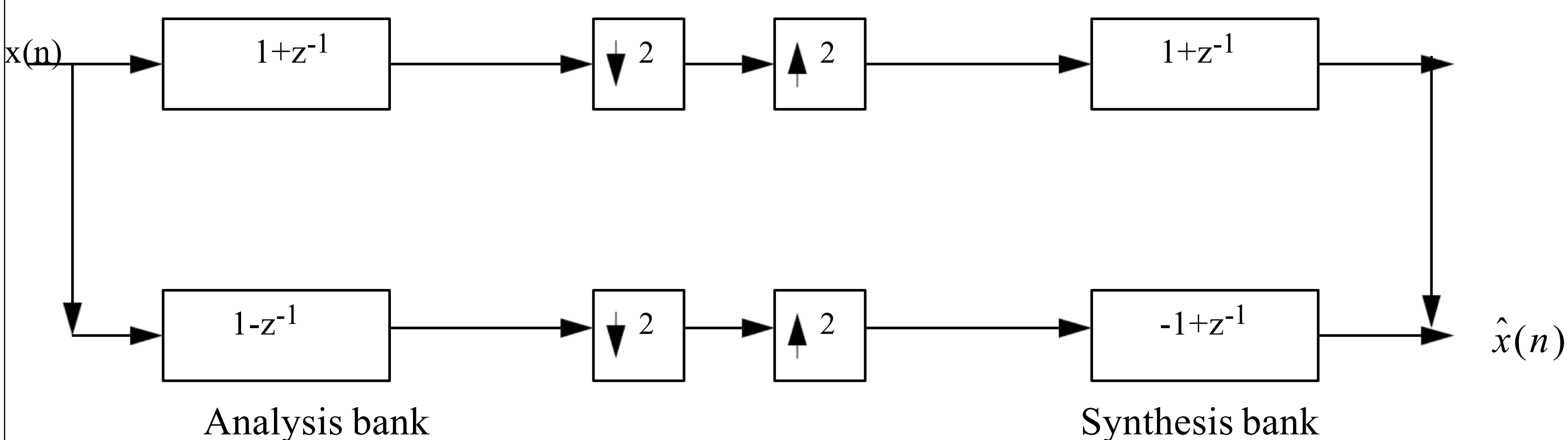


fig 10. redrawing fig 9 in conventional form

7. Tree Structured Filter Banks

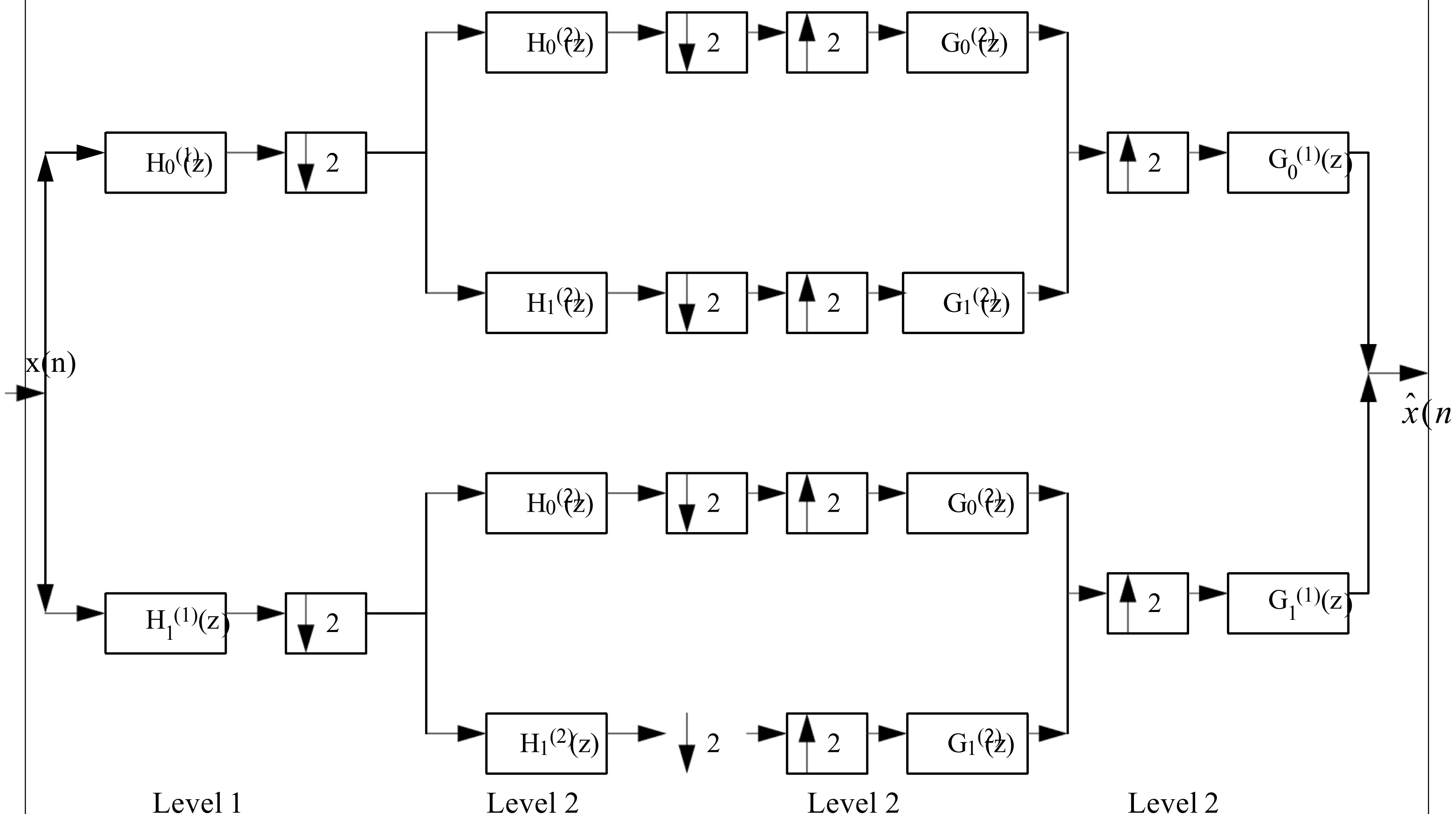


fig11. A two level maximally decimated tree structured filter bank

- In a tree structured fiiter bank, the input signal is split into two subbands, and after decimation, each subband is again split into two and decimated. The subbands are then recombined, two at a time, by use of two channel synthesis bank. This system is said to be maximally decimated binary tree structured filter bank.

- Prove that the system shown in fig 11 can be redrawn as in fig 2 with $M = 4$. Also express the resulting filters $H_m(z)$ and $G_m(z)$ for $m = 0,1,2,3$ in terms of the filters $H_i^{(k)}(z)$ and $G_i^{(k)}(z)$. **(Extra credit)**

8. Multiresolution Analysis Algorithm.

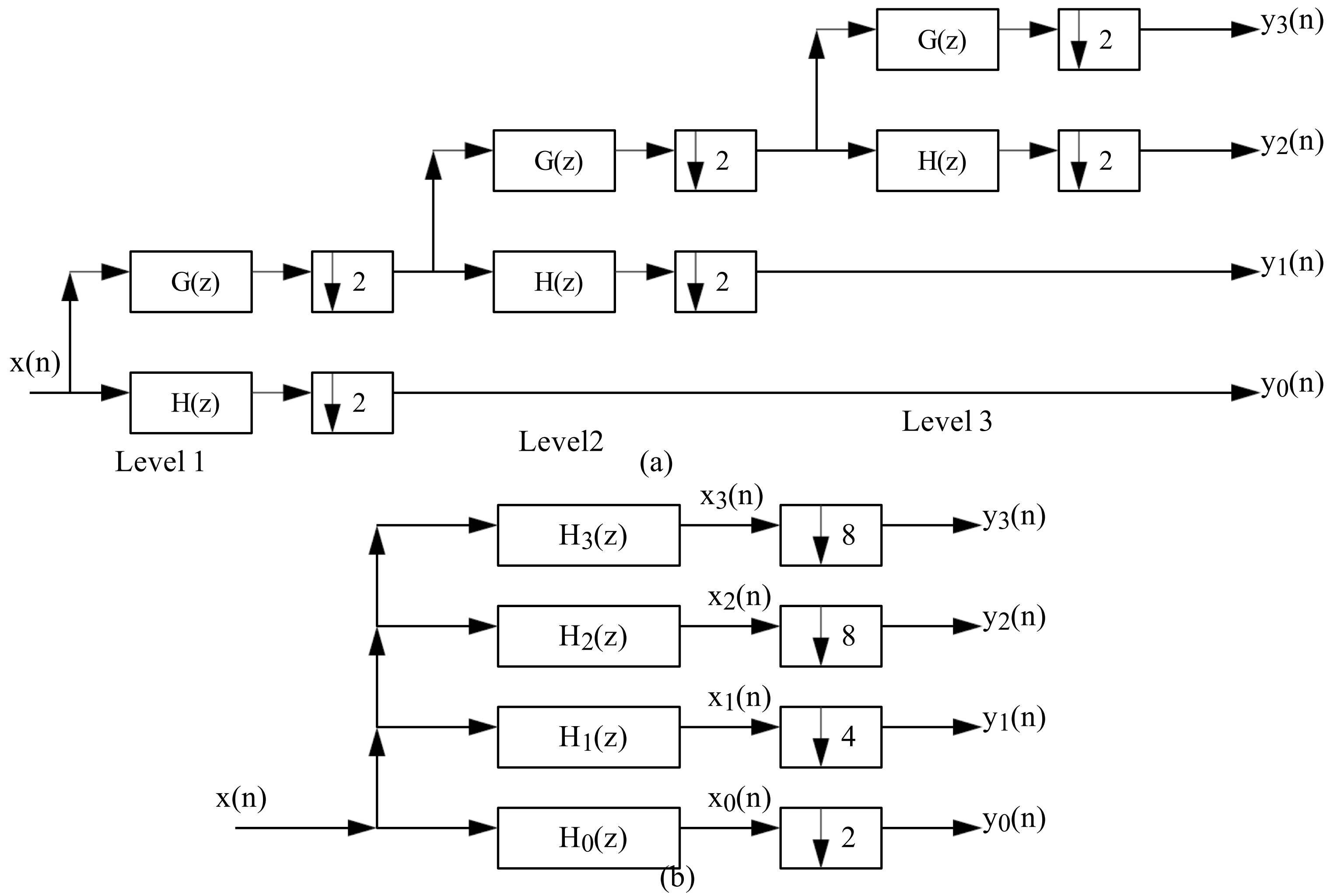


fig12(a) A 3-level binary tree structured QMF bank, and (b) the equivalent four channel system.

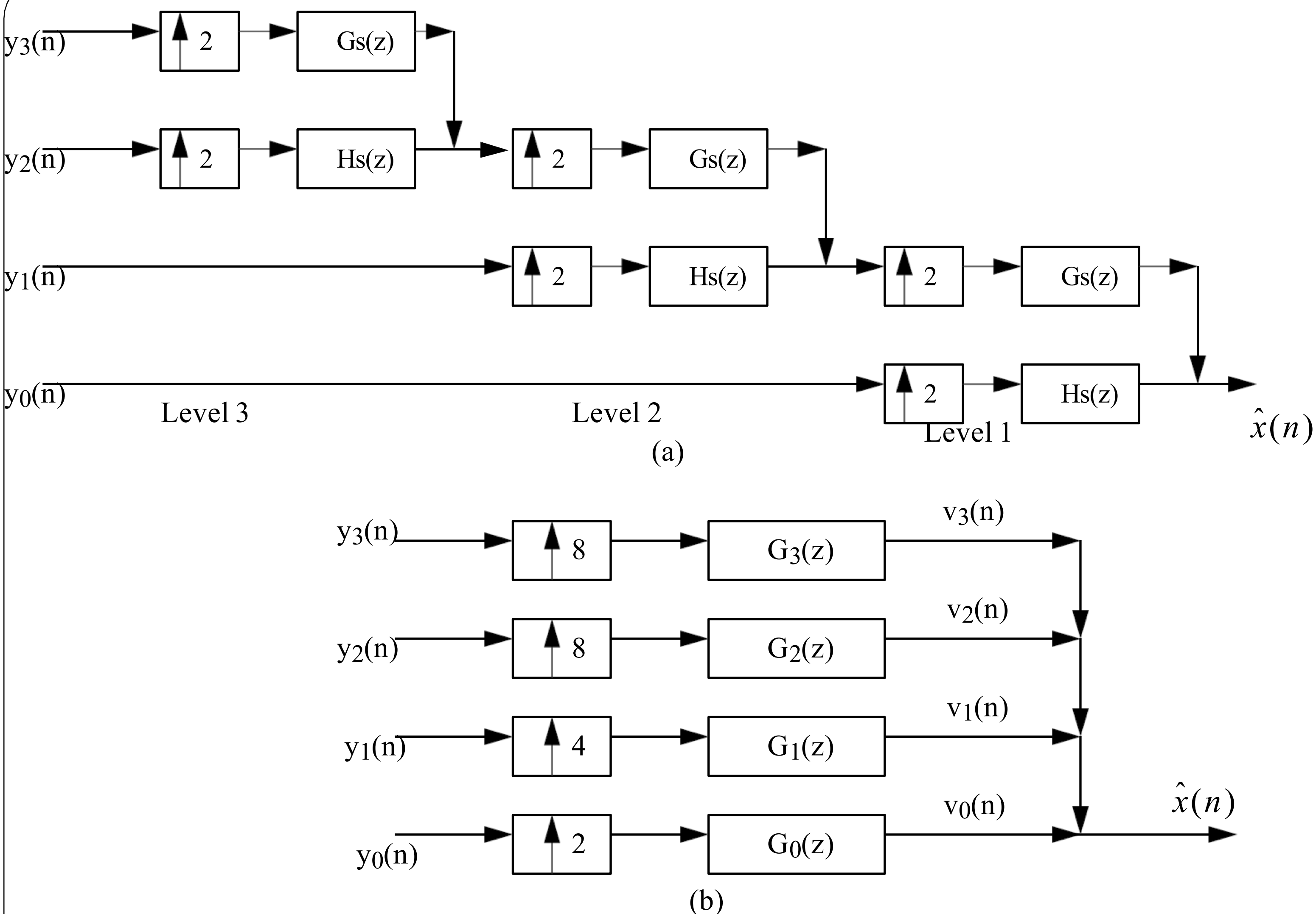


fig 13. (a) The synthesis bank corresponding to fig 12 (b) the equivalent four channel system

1 Assume that Gs(z) and Hs(z) are chosen such that the two channel QMF bank with filters G(z), H(z), Gs(z) and Hs(z) has perfect reconstruction, with unit gain and no delay. We then have $\hat{x}(n) = x(n)$. The signals $v_k(n)$ are called multiresolution components. This structure can be used in image compression and video compression.

Fig. 14 shows the frequency response of the analysis and synthesis fitlers

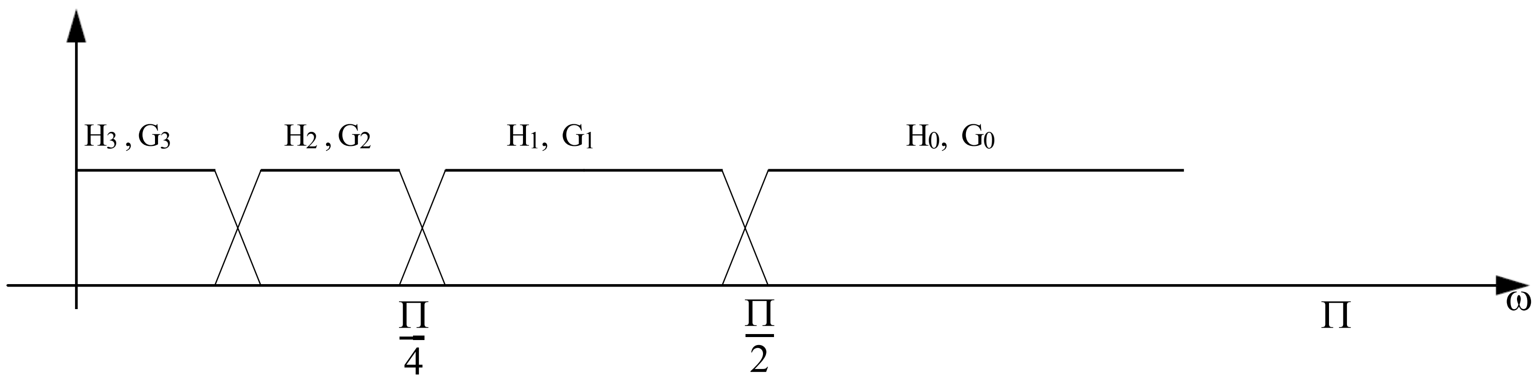


fig 14. Typical appreancances of magnitude responses of filters in the 3-level tree.