Finite Impulse Response (FIR)

Digital Filters (I)

<u>Types of linear phase FIR filters</u>

Yogananda Isukapalli

Key characteristic features of FIR filters

1. The basic FIR filter is characterized by the following two equations:

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k) \qquad (1) \qquad\qquad H(z) = \sum_{k=0}^{N-1} h(k)z^{-k} \qquad (2)$$

   where h(k), k=0,1,…,N-1, are the impulse response coefficients of the filter, H(z) is the transfer function and N the length of the filter.

2. FIR filters can have an exactly linear phase response.

3. FIR filters are simple to implement with all DSP processors available having architectures that are suited to FIR filtering.

Linear phase response

• Consider a signal that consists of several frequency components passing through a filter.

1. The **phase delay** (T_p) of the filter is the amount of time delay each frequency component of the signal suffers in going through the filter.
2. The **group delay** (T_g) is the average time delay the composite signal suffers at each frequency.
3. Mathematically,

$$T_p = -\theta(\omega) / \omega \qquad (3)$$

$$T_g = -d\theta(\omega) / d\omega \qquad (4)$$

where θ(ω) is the phase angle.

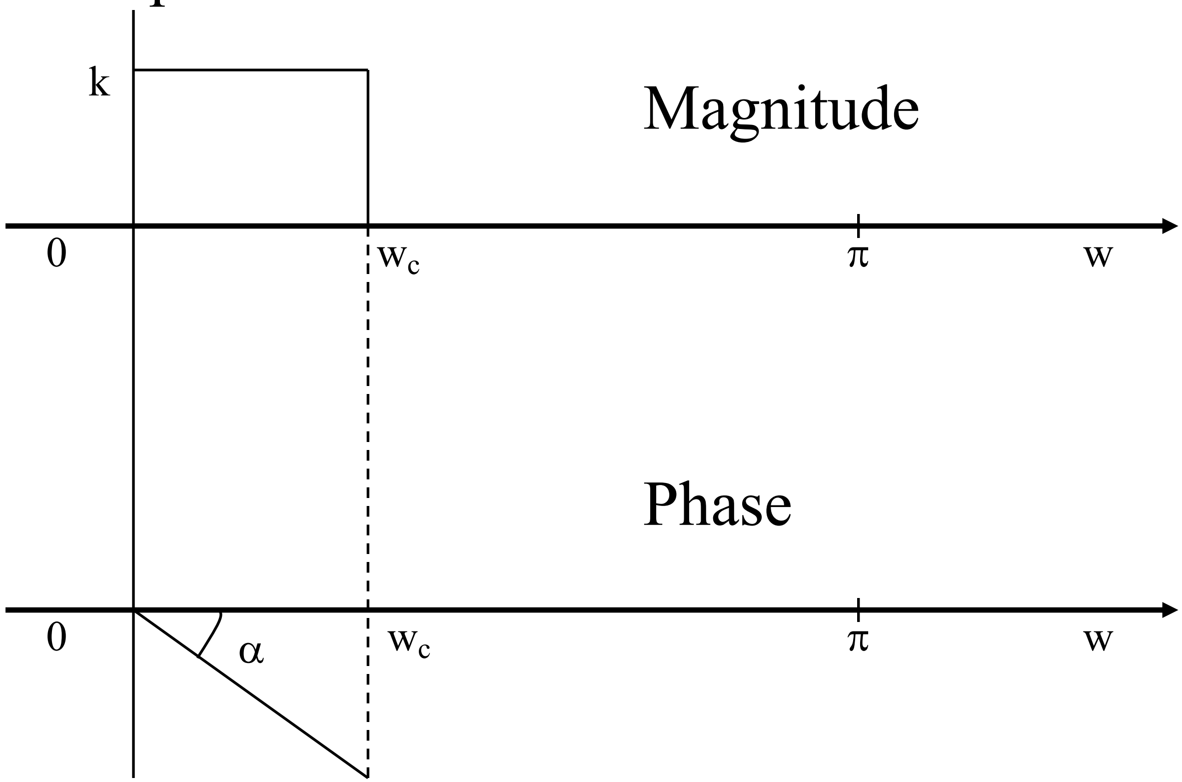
- A filter is said to have a **linear phase response** if,

$$\vartheta(\omega) = -\alpha\omega \qquad (5)$$

$$\vartheta(\omega) = \beta - \alpha\omega \qquad (6)$$

where α and β are constants.

Example) Ideal Lowpass filter

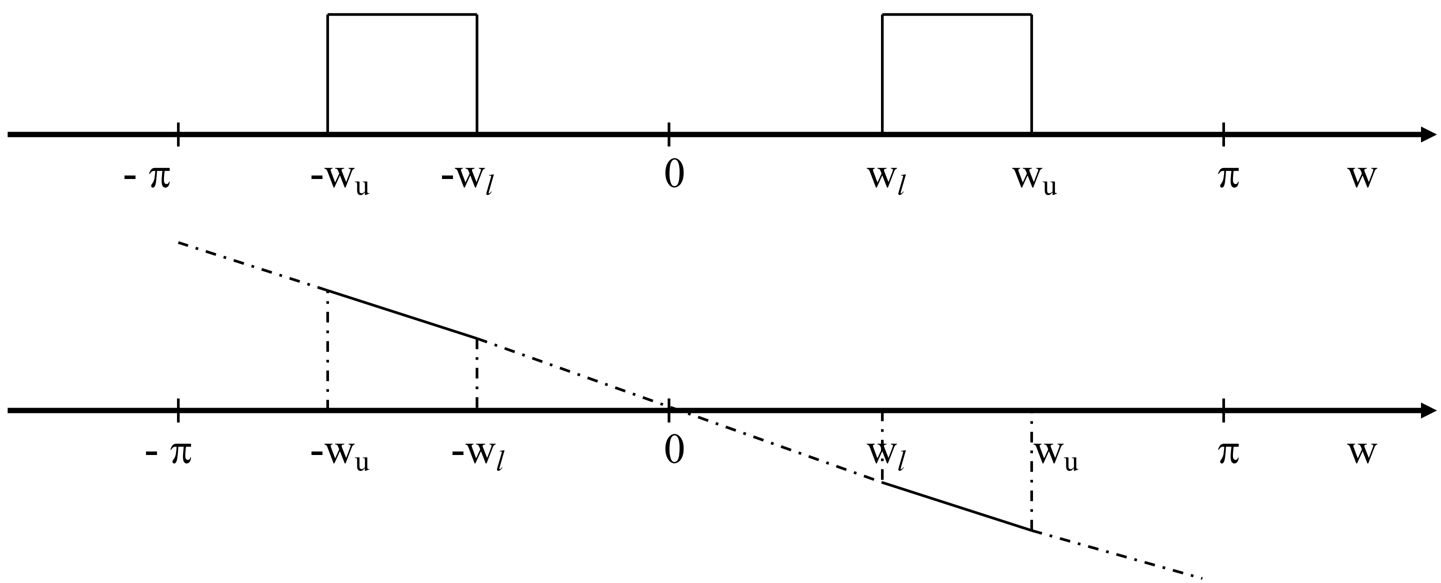


$$H(e^{jw}) = \begin{cases} k\, e^{-jw\alpha} & \text{passband} \\ 0 & \text{otherwise} \end{cases}$$

Magnitude response $= |H(e^{jw})| = k$

Phase response $(\theta(\omega)) = \angle H(e^{jw}) = -w\alpha$

Follows: $y[n] = kx[n-\alpha]$: Linear phase implies that the output is a replica of x[n] {LPF} with a time shift of α



Linear phase FIR filters

- Symmetric impulse response will yield linear phase FIR filters.

1. Positive symmetry of impulse response:

$$h(n) = h(N-n-1), \quad \begin{array}{ll} n = 0,1,\ldots,(N-1)/2 & (N \text{ odd}) \\ n = 0,1,\ldots,(N/2)-1 & (N \text{ even}) \end{array}$$

$\alpha = (N\text{-}1)/2$ in eqn (5)

1. Negative symmetry of impulse response:

$$h(n) = -h(N-n-1), \quad \begin{array}{ll} n = 0,1,\ldots,(N-1)/2 & (N \text{ odd}) \\ n = 0,1,\ldots,(N/2)-1 & (N \text{ even}) \end{array}$$

$\alpha = (N\text{-}1)/2$

$\beta = \pi/2$ in eqn (6)

Example) For positive symmetry and N = 11 odd length

$$h[n] = h[N-1-n] \quad n = 0,1,\ldots.(N-1)/2$$

$$h[0] = h[10]$$

$$h[1] = h[9]$$

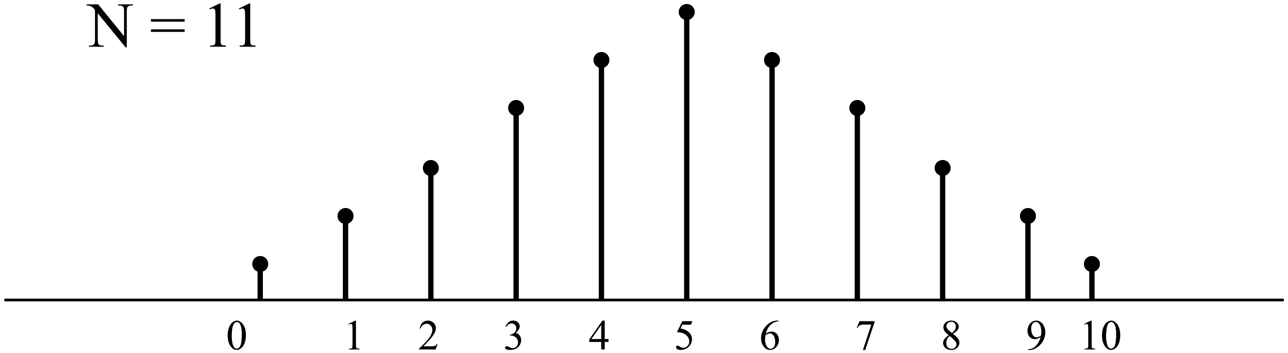
$$h[2] = h[8]$$

$$h[3] = h[7]$$

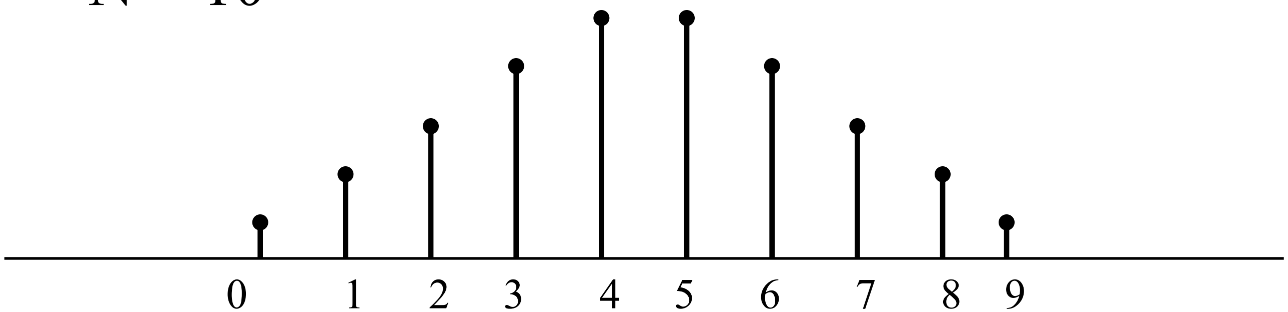
$$h[4] = h[6]$$

$$h[5] = h[5]$$

N = 11



N = 10



$$h[n] = h[10-1-n] = h[9-n]$$

$$h[0] = h[9]$$

$$h[1] = h[8]$$

$$h[2] = h[7]$$

$$h[3] = h[6]$$

$$h[4] = h[5]$$

Example : Proof of Linear Phase :

$$H(z) = \sum_{n=0}^{4} h[n] z^{-n}$$

$$H(z) = h[0] + h[1]z^{-1} + h[2]z^{-2} + h[3]z^{-3} + h[4]z^{-4}$$

For N = 5; Symmetric impulse Response Implies :

h[n] = h[5-1-n] = h[4-n]

h[0] = h[4]

h[1] = h[3]

h[2] = h[2]

Now :

$$H(z) = h[2]z^{-2} + [h[1]z + h[3]z^{-1}]z^{-2} +$$
$$+ [h[0]z^{2} + h[4]z^{-2}]z^{-2}$$

Consider Frequency Response :

$$H(e^{jw}) = H(z)\Big|_{z=e^{jw}} \quad (\mathrm{T}=1)$$

$$H(e^{jw}) = h[2]e^{-j2w} + [h[1]e^{jw} + h[3]e^{-jw}]e^{-j2w} +$$

$$[h[0]e^{j2w} + h[4]e^{-j2w}]e^{-j2w}$$

$$= e^{-j2w}\big[[h[2] + 2h[1]\cos w + 2h[0]\cos 2w\big]$$

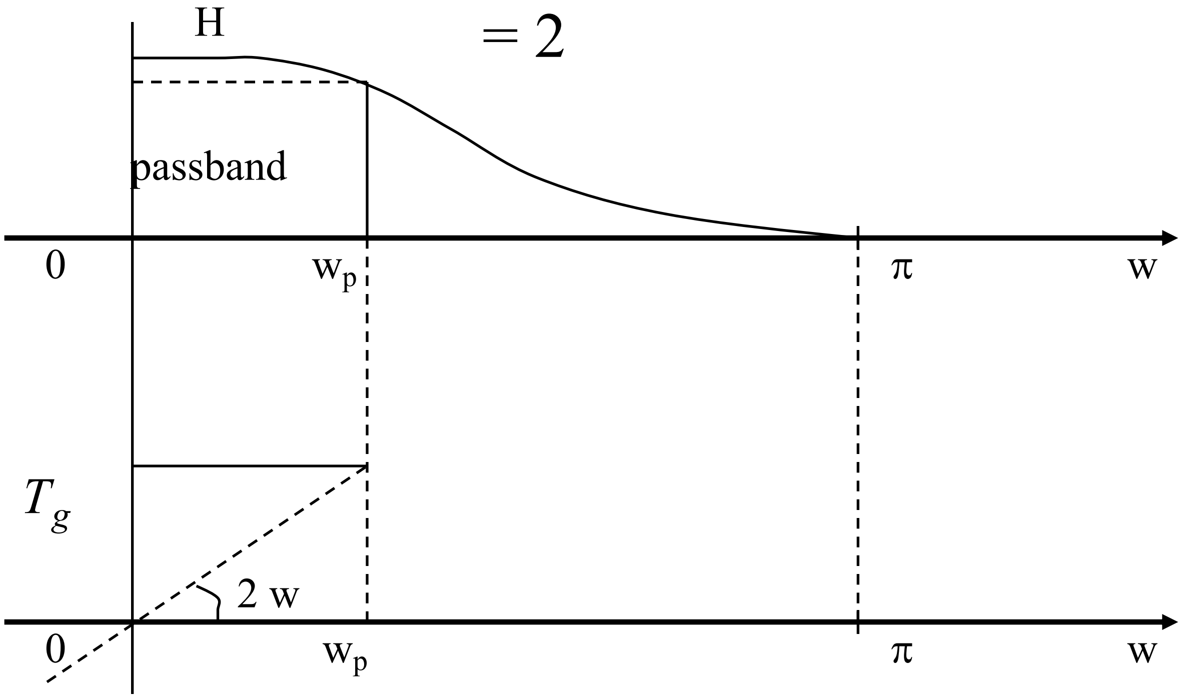
$$= e^{-j2w}\left[[h[2] + \sum_{n=0}^{1} 2h[n]\cos(w(n-2))\right]$$

$$= e^{-j2w}\,|\,H(e^{j\theta w})\,| \qquad (\text{ Linear Phase form})$$

Phase = -2w

Group Delay :

$$T_g = \frac{-d}{dw}(phase)$$



Group delay is constant over the passband for linear phase filters.

Types of FIR linear phase systems

1. Type I FIR linear phase system

The impulse response is positive symmetric and N an odd integer

$$h[n] = h[N-1-n], \qquad 0 \le n \le (N-1)/2$$

The frequency response is

$$H(e^{jw}) = \sum_{n=0}^{(N-1)/2} h[n]e^{-jwn}$$

$$H(e^{jw}) = e^{-jw(N-1)/2} \sum_{n=0}^{(N-1)/2} a[n]\cos(wn)$$

$$\begin{aligned} &where \\ &a[0] = h[(N-1)/2], \\ &a[n] = 2h[((N-1)/2) - n], \end{aligned}$$

$$n = 1,2,\ldots(N-1)/2.$$

2. Type II FIR linear phase system

The impulse response is positive symmetric and N is an even integer

$$h[n] = h[N-1-n], \quad 0 \le n \le (N/2)-1$$

The frequency response is

$$H(e^{jw}) = \sum_{n=0}^{(N/2)-1} h[n]e^{-jwn}$$

$$H(e^{jw}) = e^{-jw(N-1)/2}\left\{\sum_{n=1}^{N/2} b[n]\cos[w(n-\frac{1}{2})]\right\},$$

where

$$b[n] = 2h[N/2-n],$$

$$n = 1,2,\ldots N/2.$$

3. <u>Type III FIR linear phase system</u>

The impulse response is negative-symmetric and N an odd integer.

$$h[n] = -h[N-1-n], \qquad 0 \le n \le (N-1)/2$$

The frequency response is

$$H(e^{jw}) = \sum_{n=0}^{(N-1)/2} h[n]e^{-jwn}$$

$$H(e^{jw}) = je^{-jw(N-1)/2}\left\{\sum_{n=1}^{(N-1)/2} a[n]\sin(wn)\right\},$$

where

$$a[n] = 2h[((N-1)/2)-n],$$

$$n = 1,2,\ldots(N-1)/2.$$

4. Type IV FIR linear phase system

The impulse response is negative-symmetric and N an even integer.

$$h[n] = -h[N-1-n], \qquad 0 \le n \le (N/2)-1$$

The frequency response is

$$H(e^{jw}) = \sum_{n=0}^{(N/2)-1} h[n]e^{-jwn}$$

$$H(e^{jw}) = je^{-jw(N-1)/2}\left\{\sum_{n=1}^{N/2} b[n]\sin[w(n-\frac{1}{2})]\right\},$$

where

$$b[n] = 2h[N/2-n],$$

$$n = 1,2,\ldots, N/2.$$

Impulse response symmetry	*Number of coefficients N*	*Frequency response $H(\omega)$*	*Type of linear phase*
Positive symmetry,	Odd	$e^{-j\omega(N-1)/2} \sum_{n=0}^{(N-1)/2} a(n) \cos(\omega n)$	1
$h(n) = h(N-1-n)$			
	Even	$e^{-j\omega(N-1)/2} \sum_{n=1}^{N/2} b(n) \cos[\omega(n-\frac{1}{2})]$	2
Negative symmetry,	Odd	$e^{-j[\omega(N-1)/2-\pi/2]} \sum_{n=1}^{(N-1)/2} a(n) \sin(\omega n)$	3
$h(n) = -h(N-1-n)$			
	Even	$e^{-j[\omega(N-1)/2-\pi/2]} \sum_{n=1}^{N/2} b(n) \sin[\omega(n-\frac{1}{2})]$	4

$a(0) = h[(N-1)/2]$; $a(n) = 2h[(N-1)/2-n]$
$b(n) = 2h(N/2-n)$

Fig: A summary of four types of linear phase FIR filters[1]

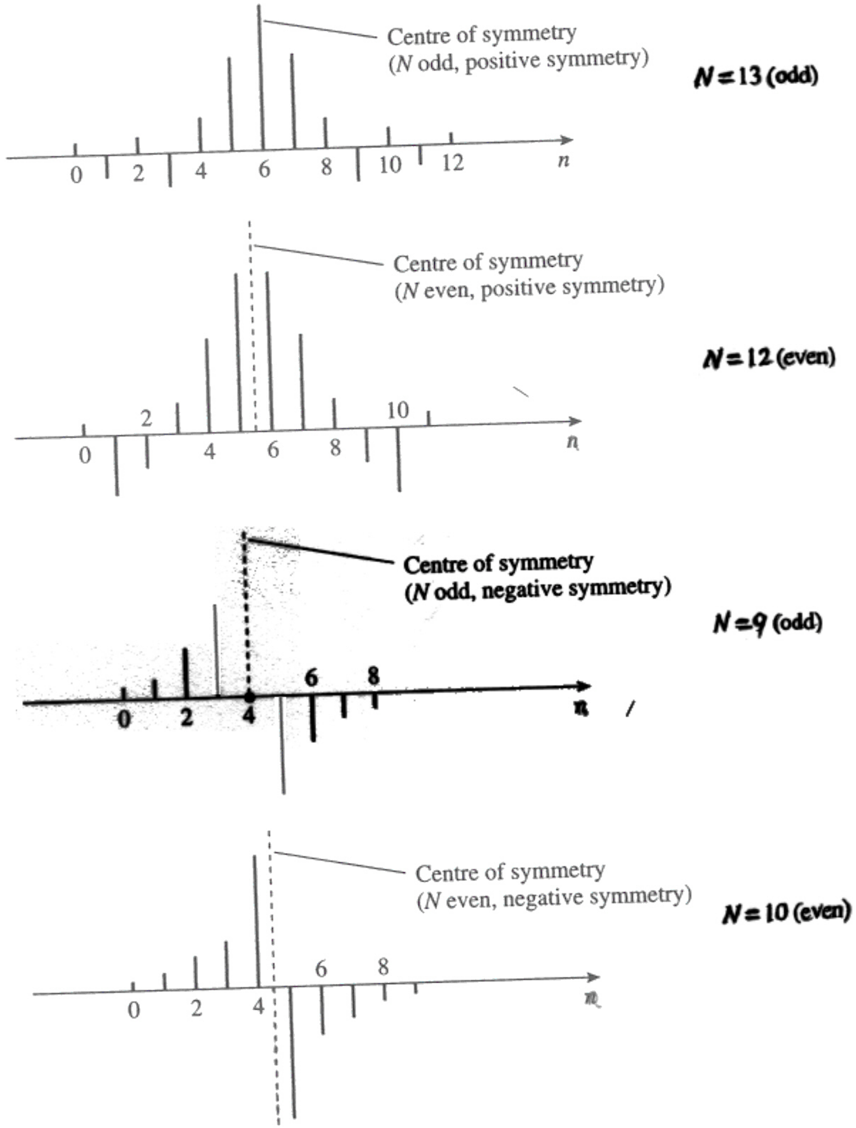


Fig: A comparison of the impulse of the four types of linear phase FIR filters[1]

Notes:

- The frequency response of a Type 2 filter is always zero at f=0.5 (half the sampling frequency as all the frequencies are normalized to the sampling frequency) and thus is unsuitable as a highpass filter.

- The frequency response of a Type 3 filter is always zero at f=0 and 0.5, while that of Type 4 filter is zero at f=0. Thus, Type 3 filter cannot be used as either a lowpass or high pass filter whereas Type 4 cannot be used as a lowpass filter.

- Both Type 3 and 4 filters introduce a 90^o phase shift and are often used to design differentiators and Hilbert transformers.

- Type 1 is the most versatile of the four.

• The phase delay (for type 1 and 2 filters) or group delay (for all four types) is expressible in terms of the number of coefficients of the filter.

• Thus they can be corrected to give a zero phase or group delay response.

• For example,

$$T_p = \left(\frac{N-1}{2} \right) T$$

Phase delay for types 1 and 2

$$T_g = \left(\frac{N-1-\pi}{2} \right) T$$

Group delay for types 3 and 4

where T is the sampling period.

References

1. "Digital Signal Processing – A Practical Approach" - Emmanuel C. Ifeachor and Barrie W. Jervis
Second Edition