Finite Impulse Response (FIR)

Digital Filters (IV)

Impulse Response Coefficients calculation with the Optimal method

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Optimum Design of Filters

- Design of FIR filters by windowing is straightforward. However, we wish to design a filter that is the "best" that can be achieved for a given value of 'M'

- We know that the rectangular window provides the best mean-square approximation to the desired-frequency for a given value of M. However this approximation shows adverse behavior at discontinuities of $H_d(e^{jw})$.

- The window method although simple does not provide for individual control over approximation errors in different bands.

Optimum Design of Filters contd...

- Frequency selective filters designed by windowing often have the property that the *error is greatest on either side of a discontinuity* of the ideal frequency response, and the error becomes smaller for frequencies away from the discontinuity as shown in Fig 1a.

- But, if the ripples are distributed more evenly over the pass band and stop band, for example as shown in Fig 1b, a better approximation of the desired frequency response can be achieved.

- *The optimal method is hence based on the concept of equiripple pass band and stop band.*

Optimum Design of Filters contd...

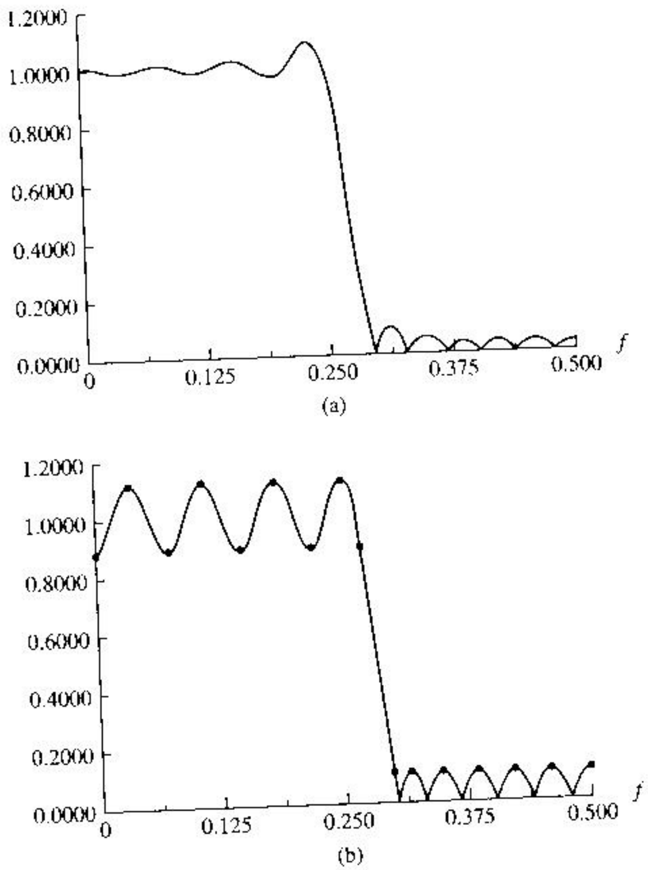


Fig 1[1]. (a) In the window filter, the ripples are largest near the band edge
(b) In the optimal filter, the ripples have the same peaks (equiripple) in the pass band or stop band.

Optimum Design of Filters contd...

• Consider the low pass filter frequency response shown in Fig 2a.

• In the pass band , the practical response oscillates between 1- δ_p and 1+ δ_p.

• In the stop band, the filter response lies between 0 and δ_s.

• The difference between the ideal filter and the practical response can be viewed as an error function (Fig 2b):

$$E(\omega) = W(\omega)[H_D(\omega) - H(\omega)]$$

where $H_D(\omega)$ is the ideal or desired response and $W(\omega)$ is a weighting function that allows the relative error of approximation between different bands to be defined.

Optimum Design of Filters contd...

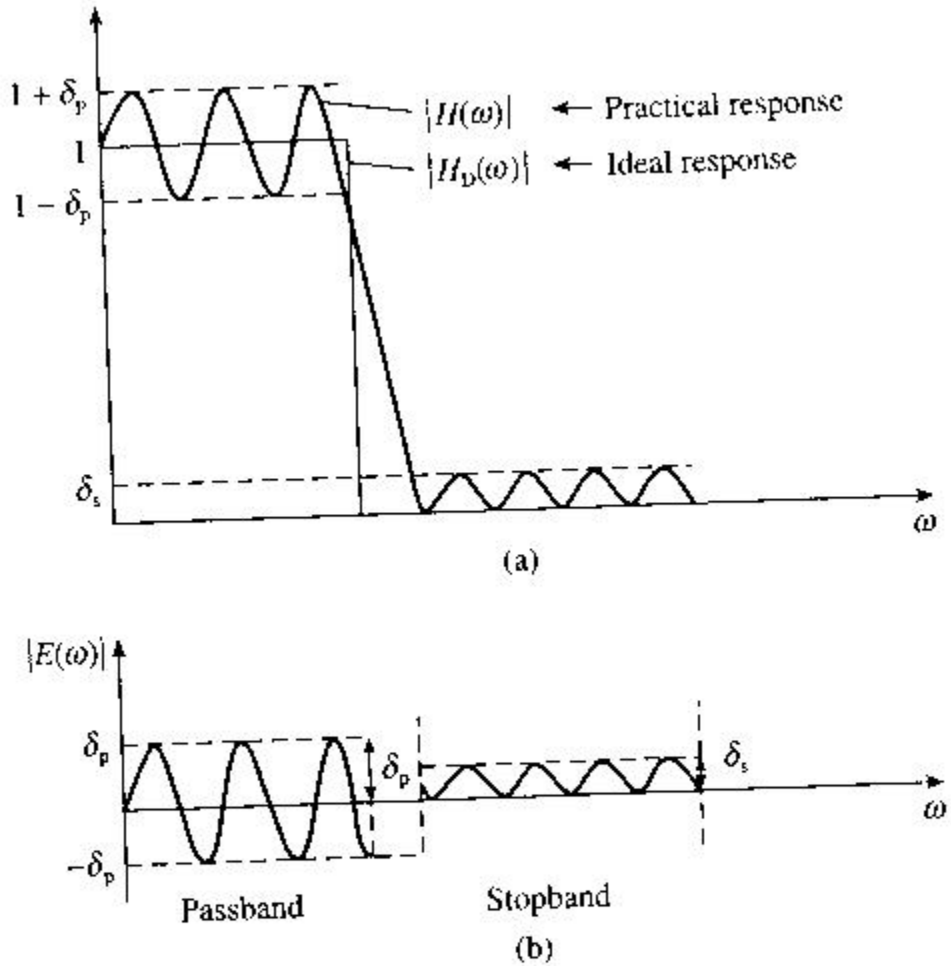


Fig 2[1]. (a) Frequency response of an optimal low pass filter (b) Response of the error between the ideal and practical responses $(\delta_p = 2\delta_s)$

Optimum Design of Filters contd...

• In the optimal method, the objective is to determine the filter coefficients, *h(n),* such that the value of the maximum weighted error, $|E(\omega)|$ is minimized in the pass band and stop band, i.e.

$$\min\left[\max|E(\omega)|\right] \quad \text{over the pass band and stop band.}$$

• Algorithmic techniques in designing filters to meet the above mentioned specifications must systematically vary the impulse response $h[n]$ to obtain the best results.

• Design algorithms have been developed in which some parameters (M, w_s, w_p, δ_s and δ_p) are fixed and an iterative procedure is used to obtain the optimum adjustments of others.

Parks-McClellan algorithm

- The most dominant of the optimum solution generation algorithms is the Parks-McClellan algorithm.
- The Parks-McClellan algorithm is based on reformulating the filter design problem as a problem in polynomial approximation.

Consider the design of a type I FIR filter having zero phase. Its frequency response is given by:

$$H(e^{jw}) = \sum_{n=0}^{N-1} h[n]e^{jw}$$

$$for\ M = N/2,$$

$$A(e^{jw}) = \sum_{n=-M}^{M} h[n]e^{jwn}$$

$$i.e.\ A(e^{jw}) = h[0] + \sum_{n=1}^{M} 2h[n]\cos(wn)$$

$$\sin ce\ h[n] = h[-n]\ for\ a\ zero\ phase\ filter.$$

Optimum Design of Filters contd...

This can be written as : $A(e^{jw}) = \sum_{k=0}^{M} a_k \cos(w)^k$

Thus here we are expressing $cos(wn)$ as a sum of powers of $cos(w)$ in the form: $cos(wn) = T_n \cos(w)$ where $T_n(x)$ is an n^{th} order polynomial. a_k's are constants that are related to $h[n]$.

We could re-write the equation $A(e^{jw}) = \sum_{k=0}^{M} a_k \cos(w)^k$

as :

$$A(e^{jw}) = P(x)\Big|_{x=\cos w}$$

$$P(x) = \sum_{k=0}^{M} a_k x^k$$

Optimum Design of Filters contd...

- Thus $A(e^{jw})$ can be expressed as a M^{th} degree polynomial.

- The key to arriving at an optimum design for the required filter is to fix w_s, and w_p and vary δ_s and δ_p.

- Parks and McClellan showed that with M, w_s, and w_p fixed, the frequency selective filter design problem becomes a problem in Chebyshev approximation over disjoint sets.

- The error function is given by : $E(w)=W(w)[H_d(e^{jw}) - A(e^{jw})]$ where $W(w)$ is a weighting function which incorporates the approximation parameters into the design process.

Optimum Design of Filters using Matlab

- The M-file function *remez* in the Signal Processing Toolbox of Matlab implements the Parks-McClellan Algorithm, which uses the Remez exchange algorithm and Chebyshev approximation to arrive at the desired filter.

- More about the Remez exchange algorithm could be found in *"Digital Filter Design" by T.W. Parks and C.S Burrus.*

- The filters designed are optimal in the sense that they minimize the maximum error between the desired and actual frequency responses. They are sometimes called *minmax* filters.

Design example using Matlab

Specifications:

Filter Order = 20

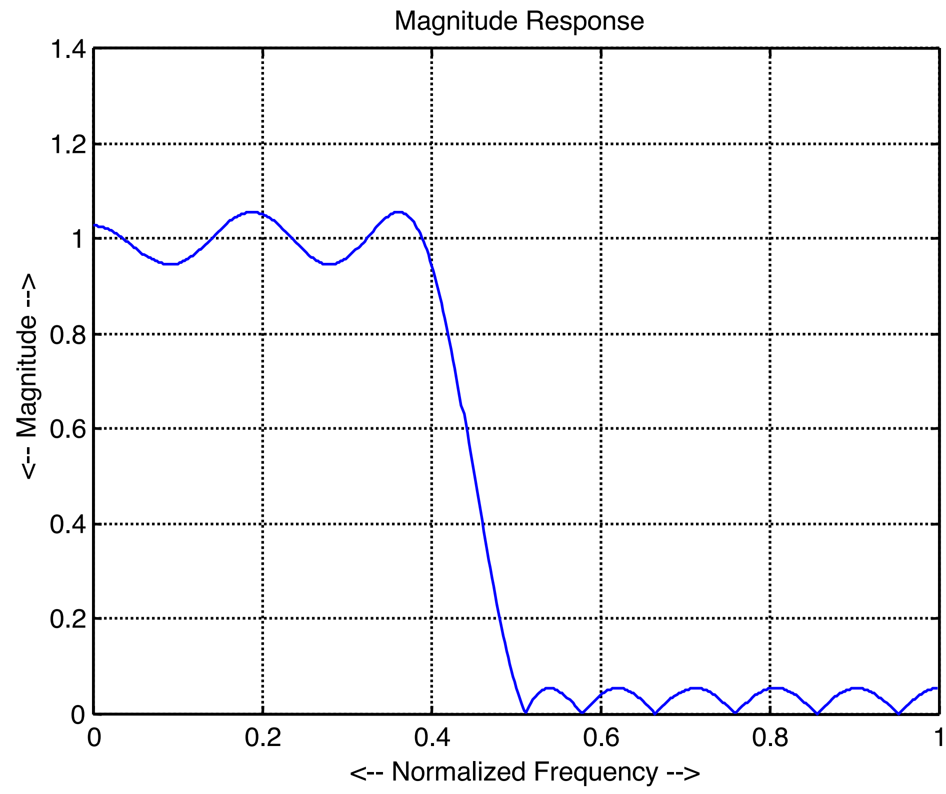
Band Edges : $w_p = 0.4$, $w_s = 0.5$ {normalized frequencies}

Matlab Code:

```
n=20;                    %filter order
f=[0 0.4 0.5 1];         %frequency band edges
a=[1 1 0 0];             %desired amplitudes
coeff=remez(n,f,a);
[H,w]=freqz(coeff);      %calculating the frequency response
plot(w/pi,abs(H));       %ploting the magnitude response
grid on;
xlabel('<-- Normalized Frequency -->');
ylabel('<-- Magnitude -->');
title('Magnitude Response');
```

Design Example contd..

Magnitude Response of the Designed Filter :



Other Matlab functions used for Optimal FIR filter Design

- Another very useful function from the Matlab Signal processing toolbox used for designing optimal FIR filters is the "*remezord*" function.
- This function gives the approximate order of the filter which we are designing based on the pass and stop band ripple and the cutoff frequencies.

Example:

Specifications of required filter:

Passband cutoff	= 500Hz
Stopband cutoff	= 600Hz
Sampling Frequency	= 2000Hz
Passband Ripple	= 3dB
Stopband Attenuation	= 40dB

Matlab Code:

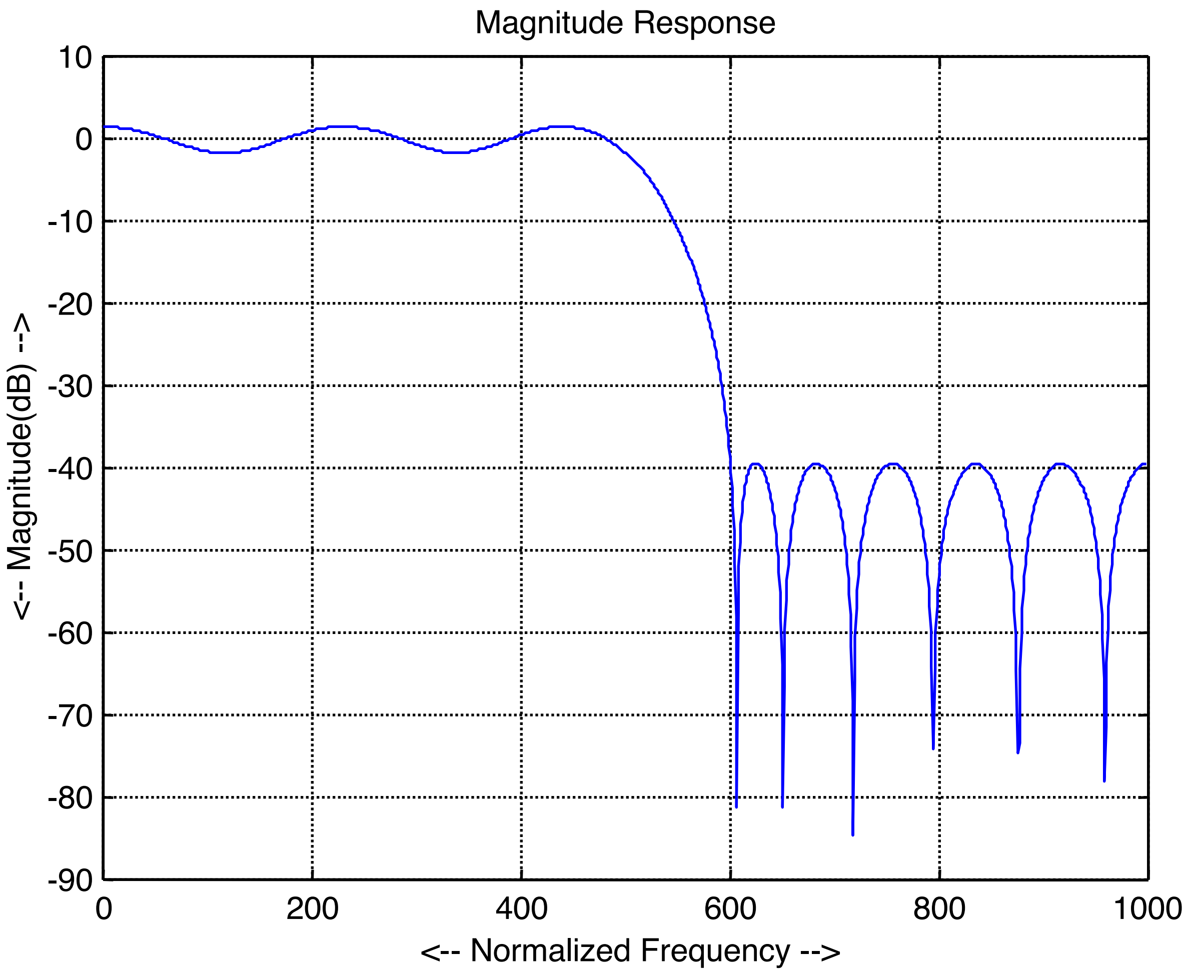
```
rp=3;                   %passband ripple;
rs=40;                  %stopband attenuation
Fs=2000;                %sampling fequency
f=[500 600];            %cutoff frequency
a=[1 0];                %desired amplitudes
d=[(10^(rp/20)-1)/(10^(rp/20)+1) 10^(-rs/20)];
[order, fo, ao, w]=remezord(f,a,d,Fs);
coeff=remez(order,fo,ao,w);

%calculating the frequency response
[H,w]=freqz(coeff,1,1024,Fs);

%plotting the magnitude response
plot(w,20*log10(abs(H)));
grid on;
xlabel('<-- Normalized Frequency -->');
ylabel('<-- Magnitude(dB) -->');
title('Magnitude Response');
```

Design Example contd..

Magnitude Response of the Designed Filter :



References

1. "Digital Signal Processing – A Practical Approach" - Emmanuel C. Ifeachor and Barrie W. Jervis
Second Edition