Finite Impulse Response (FIR)

Digital Filters (V)

Special FIR filter design topics

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Half-band FIR filters

• Half-band filters are a special type of FIR filters.

• Causal half-band filters are characterized by the following features:

   1. Equal stop band and pass band ripples, i.e.

$$\delta_p = \delta_s = \delta \qquad (1)$$

   2. The pass band and stop band edge frequencies are related as:

$$f_s = \frac{F_s}{2} - f_p \qquad (2)$$

3. The frequency response is symmetrical about a quarter of the sampling frequency, i.e. at $f = F_s / 4$

$$H\left(\frac{F_s}{4} + f\right) = 1 - H\left(\frac{F_s}{4} - f\right)$$

Also at this frequency, the normalized frequency response is reduced by a factor of 2, i.e.

$$|H(f)| = 0.5 \quad \text{at } f = F_s / 4$$

4. In the unit impulse response, for N odd, every other coefficient is zero except $h(N-1)/2$:

$$h(2n) = 0, \quad n = 0,1,..., (N-1)/4$$
$$= 0.5, \quad n = (N-1)/2$$

• The coefficients can be obtained using the FIR methods like window and optimal methods.

• The constraints given in eqn (1) and (2) must be imposed while calculating the coefficients.

• Nearly half the filter coefficients of the half-band filters are zero, which leads to a reduction in computational effort by a factor of 2.

• Half-band FIR filters are used in applications like multirate processing where there is a need for efficient anti-aliasing filtering and/or anti-imaging filtering in order to alter the sampling rate.

Frequency transformations of ideal filters

• The coefficients of high pass, band pass and band stop filters can also be obtained indirectly from the coefficients of low pass filters by noting the relations between the corresponding frequency responses.

High pass Filter

The ideal low filter frequency response is shown below. $h_{LP}[n]$ represents the non-causal impulse response of $H_{LP}(e^{jw})$
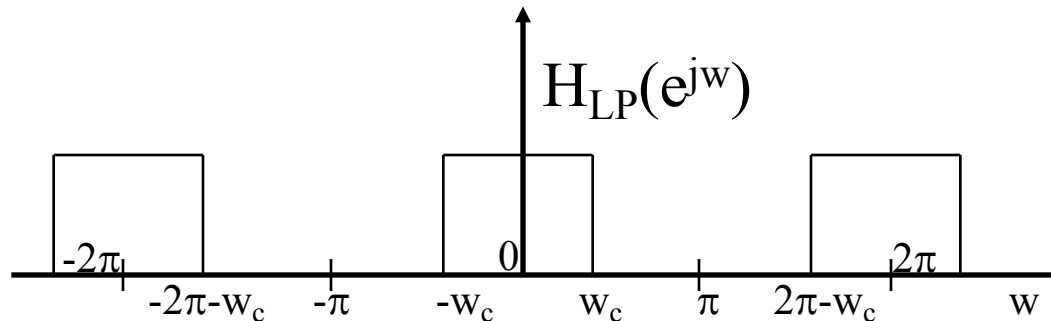
Shifting the low pass filter frequency by π, we get
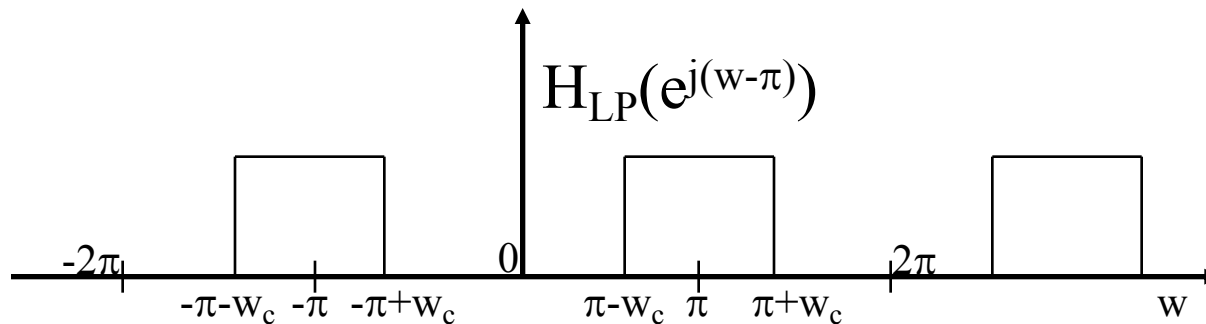
The above figure is the frequency response of an ideal high pass filter with a cutoff frequency of π-w_c ($H_{HP}(e^{jw})$).Hence,

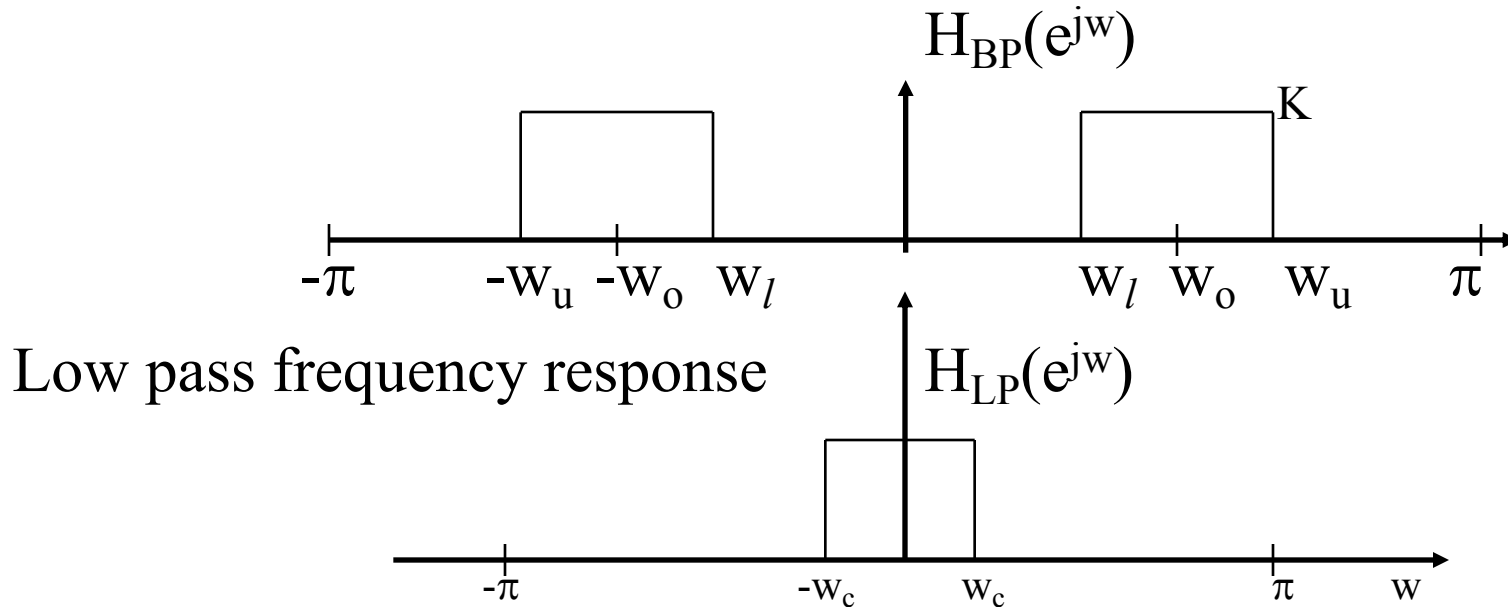$$H_{HP}(e^{jw}) = H_{LP}(e^{j(w-\pi)})$$

• From the shifting property of the Fourier transform, we get

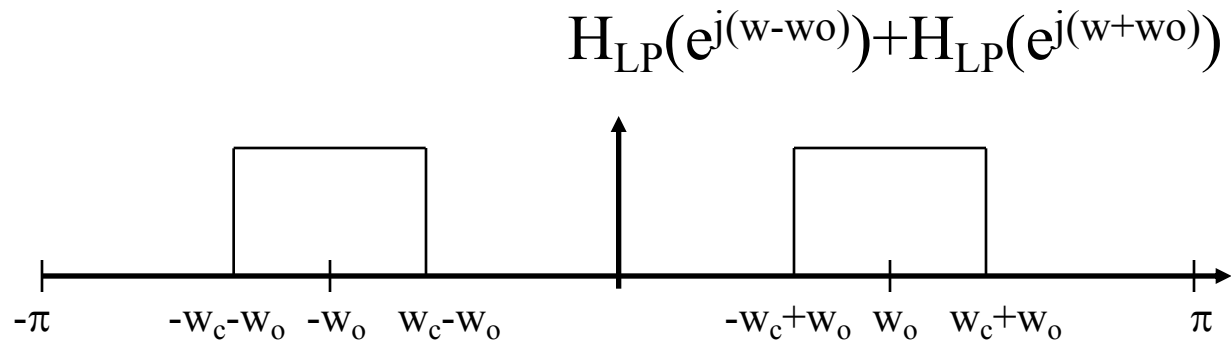$$h_{HP}[n] = h_{LP}[n]e^{j\pi n}$$

$$h_{HP}[n] = (-1)^n h_{LP}[n]$$

Band pass Filter

The frequency response for an ideal band pass filter is shown below. The center frequency is w_o.

$H_{BP}(e^{jw})$

K

$-\pi$ $-W_u$ $-W_o$ W_l W_l W_o W_u π

Low pass frequency response

$H_{LP}(e^{jw})$

$-\pi$ $-w_c$ w_c π w

Looking at the frequency responses of a band pass filter and a low pass filter, we can observe that a band pass filter is obtained by shifting the low pass filter to the left and to the right by w_o and adding the two shifted responses.

Shifting the low pass filter by w_o to the left and to the right and adding , we get

$$H_{LP}(e^{j(w-wo)}) + H_{LP}(e^{j(w+wo)})$$

$-\pi \qquad -w_c-w_o \quad -w_o \quad w_c-w_o \qquad\qquad -w_c+w_o \quad w_o \quad w_c+w_o \qquad \pi$

Comparing the ideal band pass filter and the above figure, we can observe that

$$w_l = -w_c + w_o$$

$$w_u = w_c + w_o$$

$$w_c = \frac{w_u - w_l}{2}$$

$$w_o = \frac{w_u + w_l}{2}$$

The frequency response of the band pass filter $H_{BP}(e^{jw})$ is

$$H_{BP}(e^{jw}) = H_{LP}(e^{j(w-w_o)}) + H_{LP}(e^{j(w+w_o)})$$

From the shifting property of the Fourier transform , we have
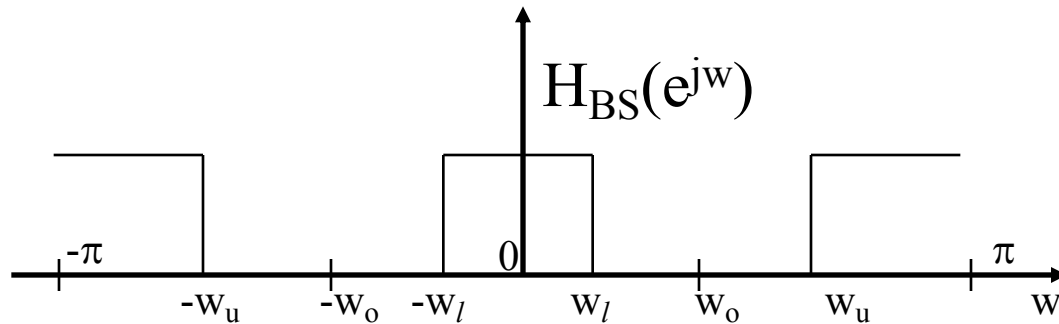
$$h_{BP}[n] = h_{LP}[n]e^{jw_o n} + h_{LP}[n]e^{-jw_o n}$$

$$h_{BP}[n] = h_{LP}[n]\{e^{jw_o n} + e^{-jw_o n}\}$$

$$h_{BP}[n] = h_{LP}[n]\{2\cos(nw_o)\}$$

Band stop Filter

The frequency response of an ideal band stop filter ($H_{BS}(e^{jw})$) is

Looking at the frequency responses of band stop and band pass filters, we observe that the band stop filter is obtained by subtracting the band pass from 1.

$$H_{BS}(e^{jw}) = 1 - H_{BP}(e^{jw})$$

$$\sum_{n=-\infty}^{\infty} h_{BS}[n]e^{-jnw} = 1 - \sum_{n=-\infty}^{\infty} h_{BP}[n]e^{-jnw}$$

If we work out the first few terms, we get

$$......+ h_{BS}[-2]e^{j2w} + h_{BS}[-1]e^{jw} + h_{BS}[0] + h_{BS}[1]e^{-jw} + h_{BS}[2]e^{-j2w}$$

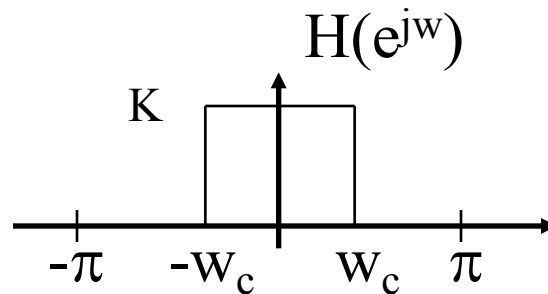$$= ......+ h_{BP}[-2]e^{j2w} + h_{BP}[-1]e^{jw} + 1 - h_{BP}[0] + h_{BP}[1]e^{-jw} + h_{BP}[2]e^{-j2w}$$

Hence, the impulse response coefficients for a band stop filter are obtained as

$$h_{BS}[0] = 1 - h_{BP}[0] = 1 - 2h_{LP}[0]$$

$$h_{BS}[n] = -h_{BP}[n] = -h_{LP}[n]\{2\cos(nw_o\} \qquad n \neq 0$$

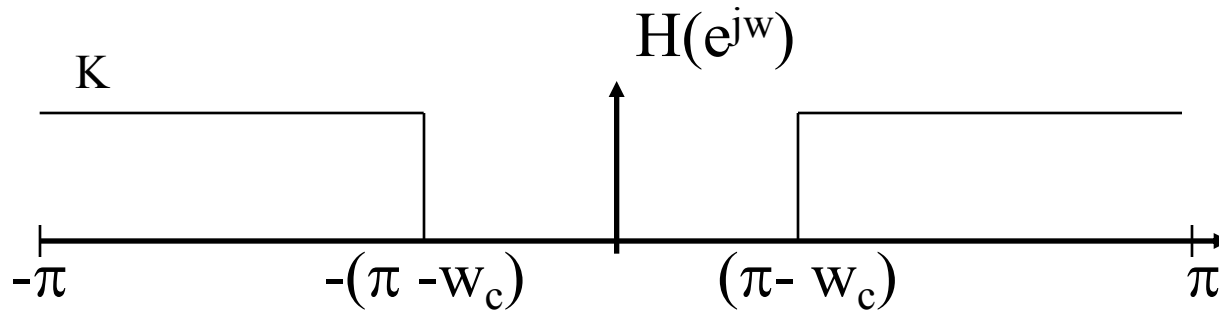# Low pass

$$h_{LP}(n) = \frac{2f_c \sin(n\omega_c)}{n\omega_c}, \quad n \neq 0, -\infty \leq n \leq \infty$$
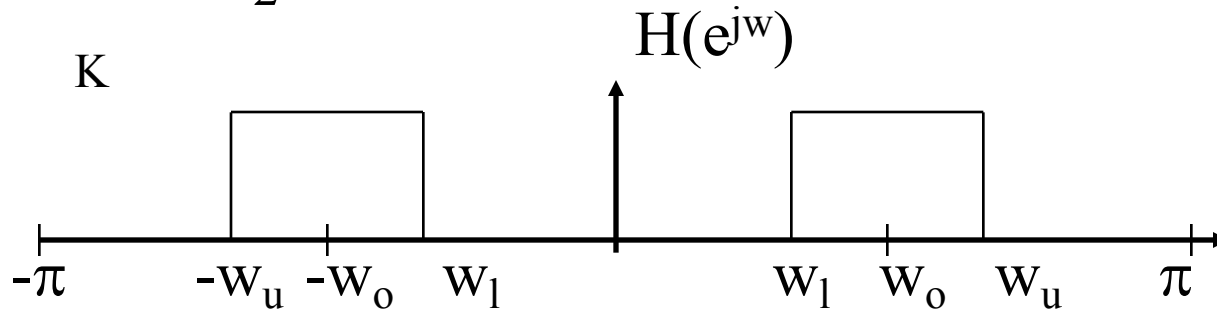
$$= 2f_c, \qquad n = 0$$

H(e^{jw})

K

-π   -w_c   w_c   π

High pass

$$h_{HP}[n] = (-1)^n h_{LP}[n], \qquad n = 0, \pm 1, \pm 2, ..., \pm I$$

H(e^{jw})

K

-π   -(π - w_c)   (π - w_c)   π

Band pass

$$h_{BP}[n] = [2\cos(nw_o)]h_{LP}[n], \qquad n = 0, \pm 1, \pm 2, ..., \pm I$$
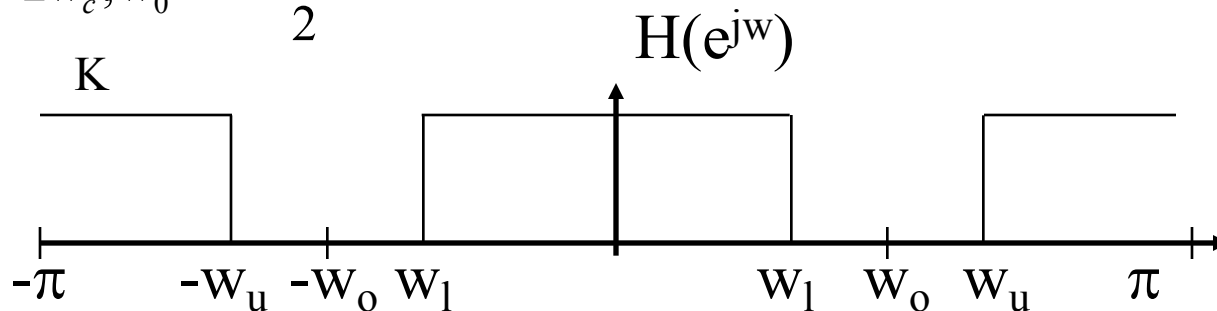
$$w_u - w_l = 2w_c, \, w_0 = \frac{w_u + w_l}{2}$$

K

$H(e^{jw})$

-π   -W$_u$ -W$_o$ W$_l$ W$_l$ W$_o$ W$_u$ π

Band stop

$$h_{BS}[0] = K - h_{BP}[0],$$

$$h_{BS}[n] = -h_{BP}[n], \qquad n = \pm 1, \pm 2, ..., \pm I$$

$$w_u - w_l = 2w_c, \, w_0 = \frac{w_u + w_l}{2}$$

K

$H(e^{jw})$

-π   -W$_u$ -W$_o$ W$_l$ W$_l$ W$_o$ W$_u$ π

Illustrations showing the functioning of different windows
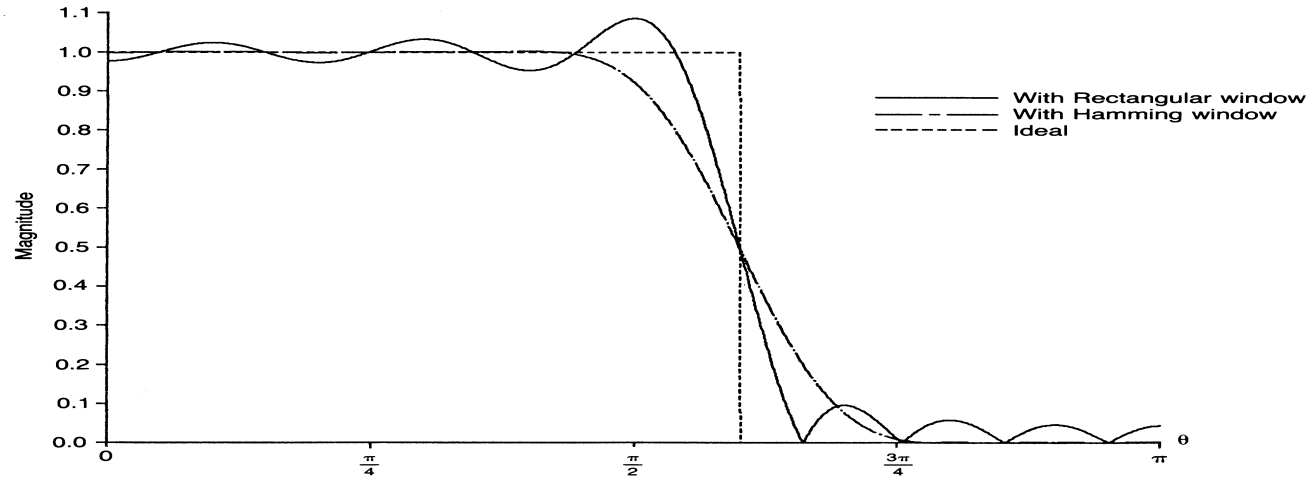
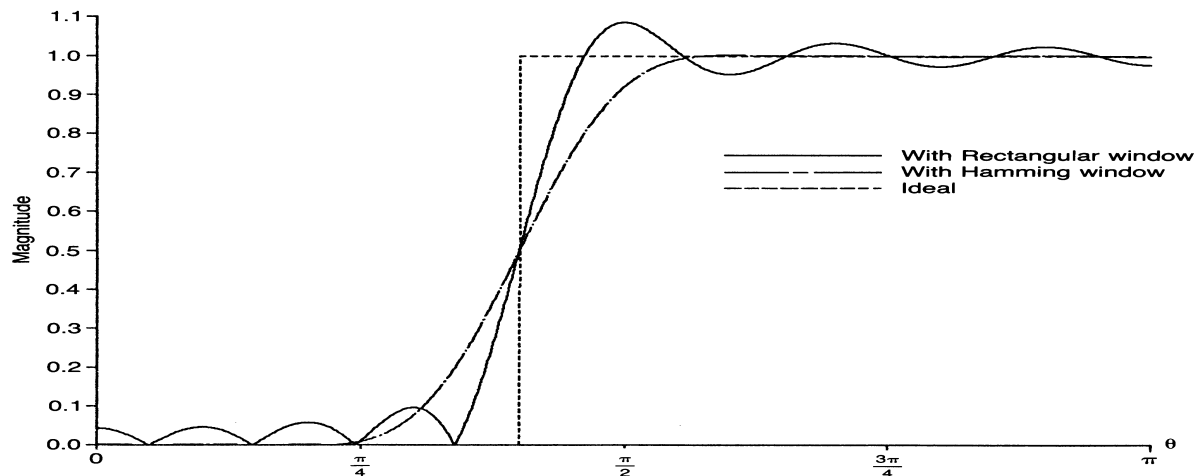**FIGURE 9.18(a)**   21-coefficient lowpass filter magnitude response with and without Hamming window

FIGURE 9.18(b)   21-coefficient highpass filter magnitude response with and without Hamming window
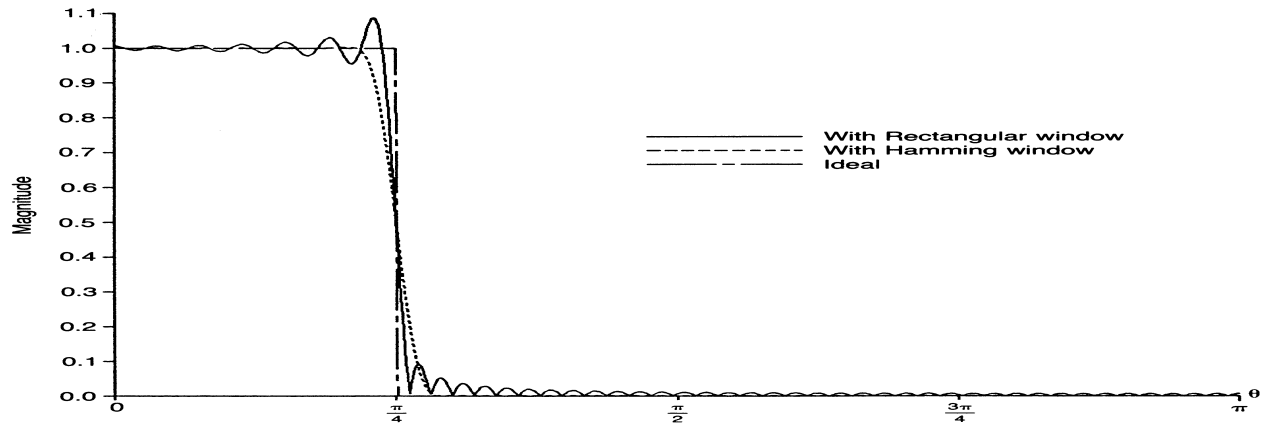
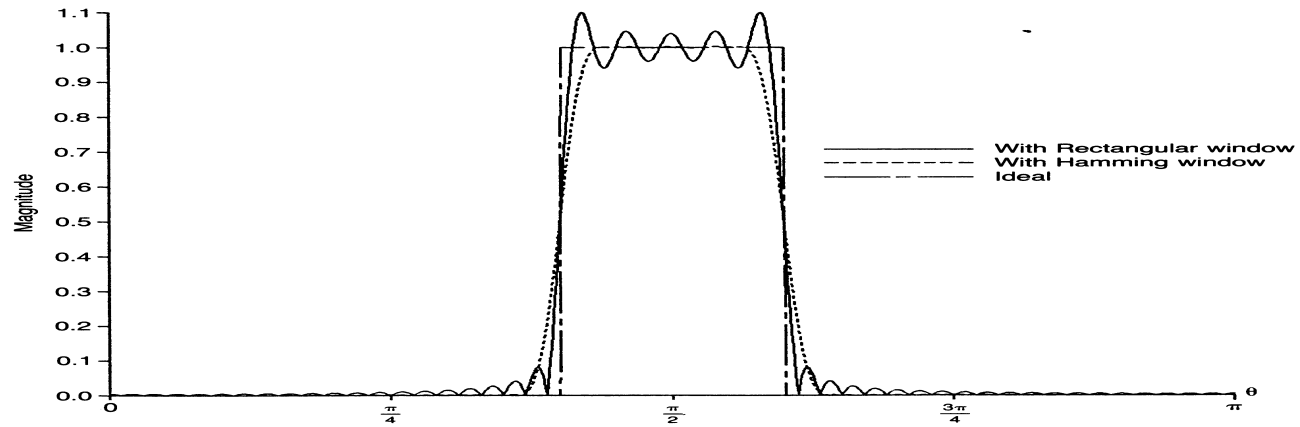**FIGURE 9.19(a)** Lowpass filter frequency response with and without Hamming window for 101-coefficient design

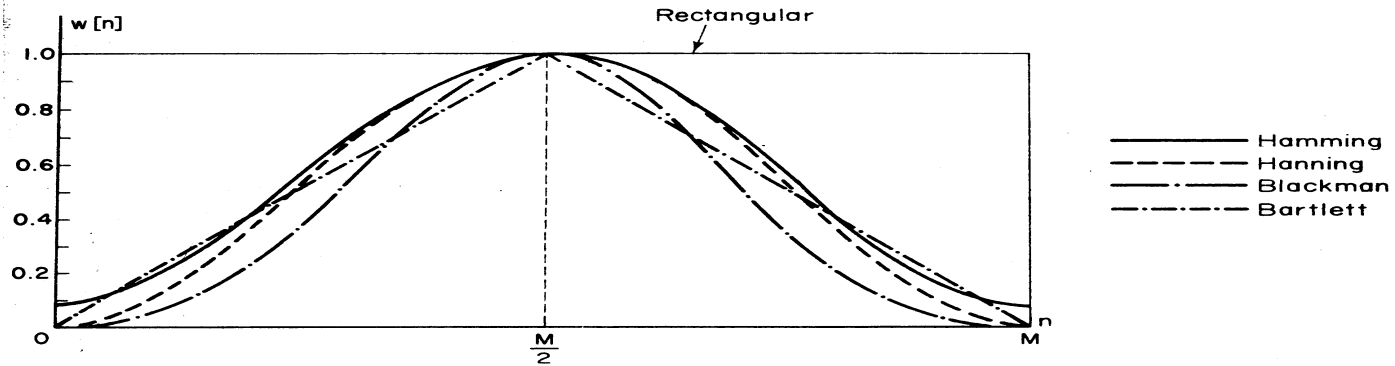**FIGURE 9.19(b)** Bandpass filter frequency response with and without Hamming window for 101-coefficient design

Figure 7.29   Commonly used windows.

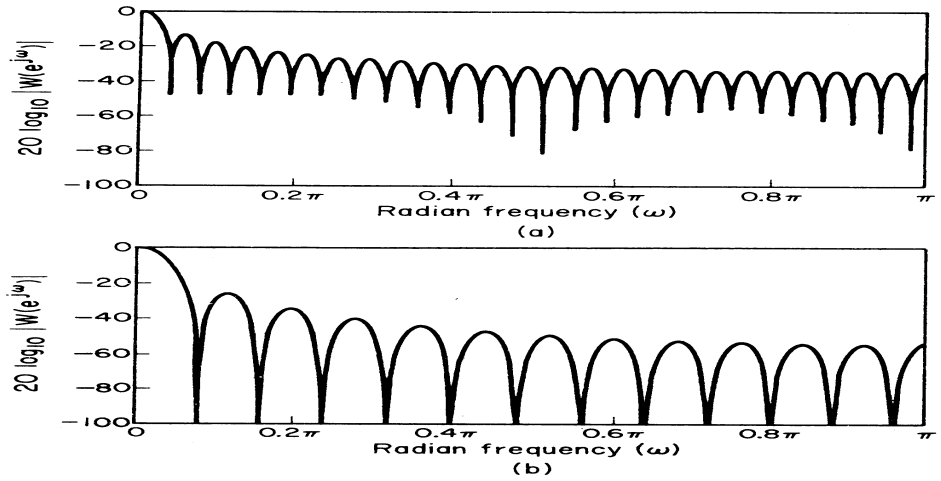| Window	Peak amplitude of sidelobe (dB)	Transition width of mainlobe	Minimum stopband attenuation (dB)
Rectangular	-13	$4\pi/N$	-21
Bartlett	-25	$8\pi/N$	-35
Hanning	-31	$8\pi/N$	-44
Hamming	-41	$8\pi/N$	-53
Blackman	-57	$12\pi/N$	-74

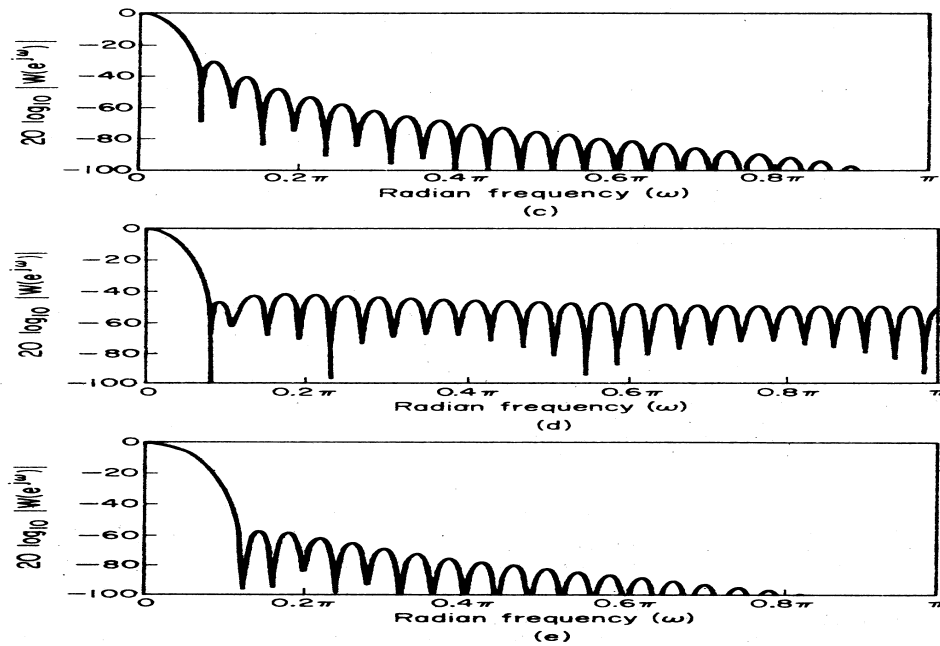**Figure 7.30** Fourier transforms (log magnitude) of windows of Fig. 7.29. (a) Rectangular. (b) Bartlett.

Figure 7.30 (*continued*) (c) Hanning. (d) Hamming. (e) Blackman.