Analog Filters

(I)
Introduction

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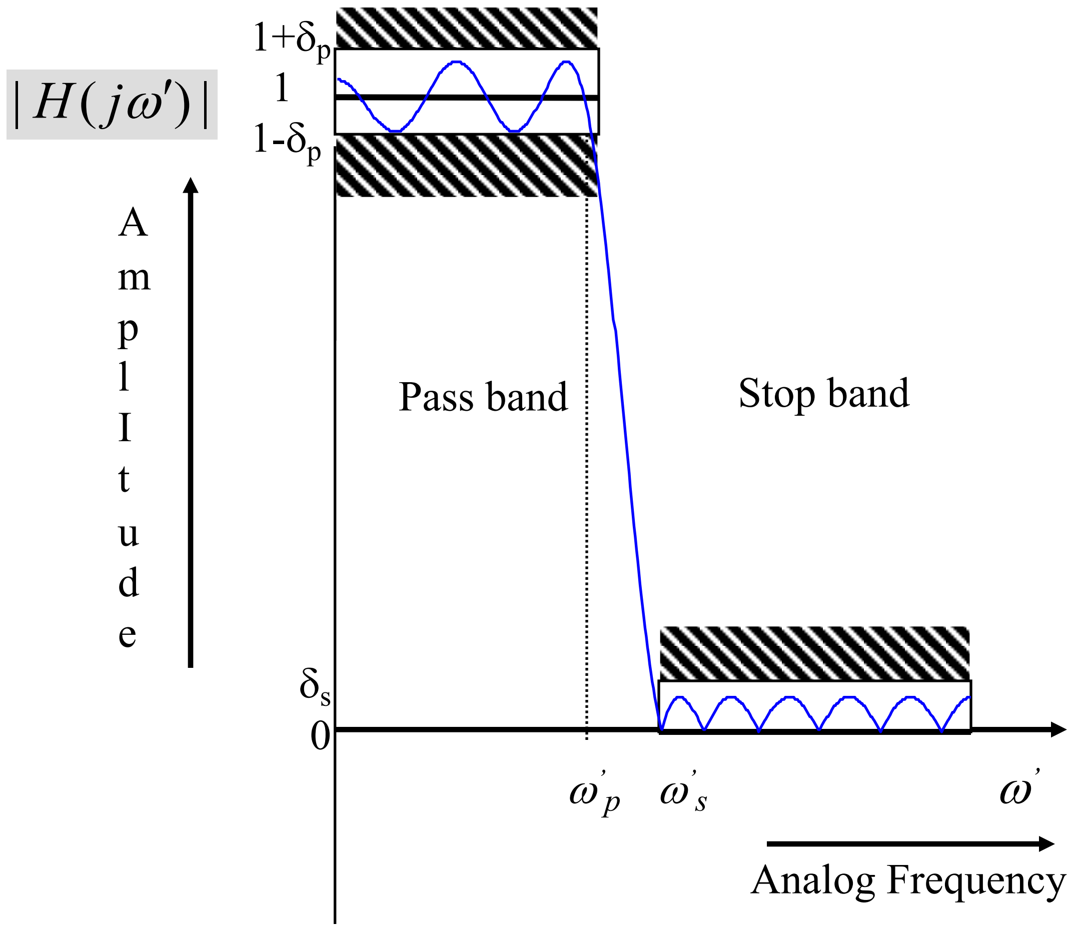
Analog Filter Specifications



$$1-\delta_p \le |H(j\omega')| \le 1+\delta_p \quad \text{for } |\omega'| \le \omega'_p,$$

$$|H(j\omega')| \le \delta_s \quad \text{for } \omega'_s \le |\omega'| \le \infty,$$

$$\alpha_p = -20\log_{10}(1-\delta_p)dB,$$
$$\alpha_s = -20\log_{10}(\delta_s)dB,$$

Where α_p and α_s are the peak pass band ripple and minimum stop band attenuation in dB respectively

Review :

$$y(t) = \sum_{k=0}^{N} a_k \frac{d^k x(t)}{dt^k} + \sum_{k=0}^{M} b_k \frac{d^k y(t)}{dt^k}$$

Example : a_1= 1 and all other “a’s” and “b’s” are zero.

$$y(t) = \frac{dx(t)}{dt}$$

Ideal Differentiator

Example : Second order Analog Filter

$$y(t) = a_0 x(t) + a_1 \frac{dx}{dt} + b_1 \frac{dy}{dt} + b_2 \frac{d^2 y}{dt^2}$$

Transfer Function :

$$H(s) = \frac{\sum_{i=0}^{N} a_i s^i}{1 - \sum_{i=0}^{M} b_i s^i}$$

Example:

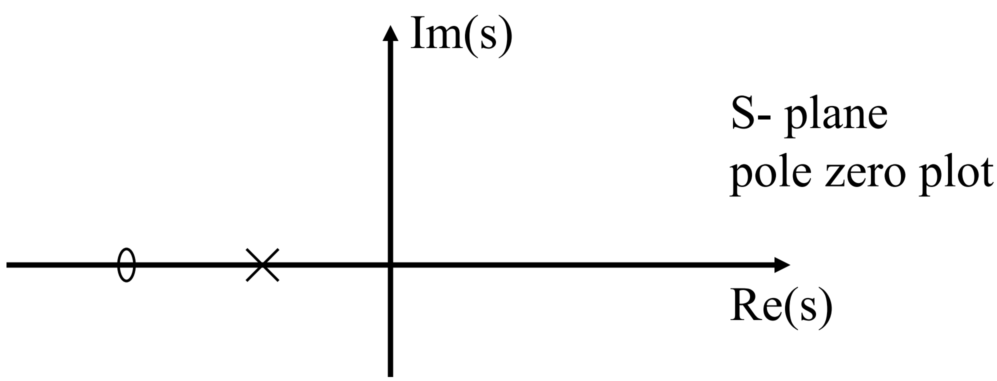
$$y(t) = x(t) + \frac{dx}{dt} - 2\frac{dy}{dt}$$

$$H(s) = \frac{1+s}{1+2s}$$

$$H(s) = \frac{1}{2} . \frac{s+1}{s+\frac{1}{2}}$$

Pole: s = -1/2 Zero: s = -1

Gain = 1/2



Impulse Response :

$$h(t) = L^{-1}H(s)$$

Example :

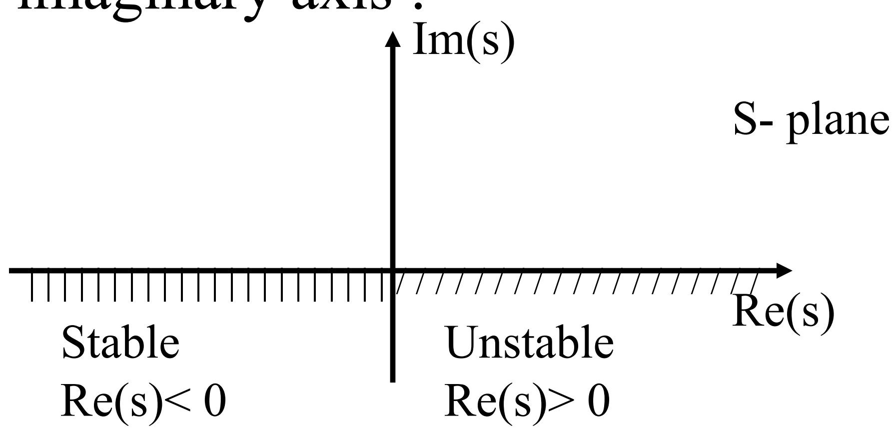
$$H(s) = \frac{a_0}{1 - b_1 s}$$

$$h(t) = \begin{cases} \frac{a_0}{b_1} e^{\frac{1}{b_1}t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Stable Analog Filter :

Define h(t) $\to 0$ as $t \to \infty$

Implies that all poles must lie to the left of the imaginary axis :



Stability and Pole Locations :

Analog Filters with M poles:

$$H(s) = G\frac{(s-z_1)(s-z_2)\ldots\ldots\ldots\ldots\ldots(s-z_N)}{(s-p_1)(s-p_2)\ldots\ldots\ldots\ldots\ldots(s-p_M)}$$

Partial Fraction (no repeated poles)

$$H(s) = \frac{g_1}{s-p_1} + \frac{g_2}{s-p_2} + \ldots\ldots + \frac{g_m}{s-p_m}$$

$$h(t) = g_1 e^{p_1 t} + g_2 e^{p_2 t} + \ldots\ldots\ldots + g_m e^{p_m t}$$

Since stable means $h(t) \to 0$ as $t \to \infty$

It follows that : $p_i < 0$

If p_i is a complex pole :

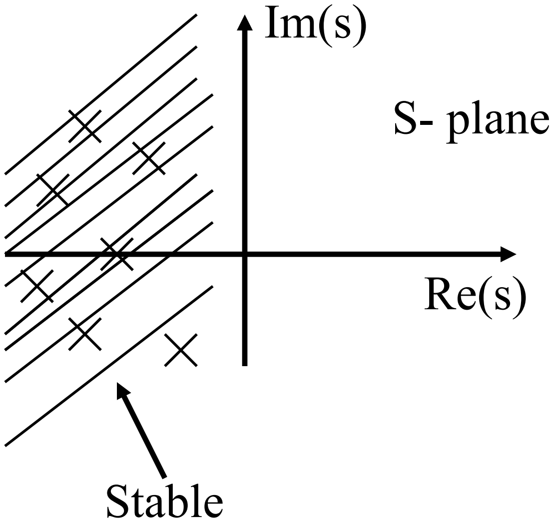
$$h_i(t) = g_i e^{\mathrm{Re}(p_i)t} e^{j\,\mathrm{Im}(p_i)t}$$

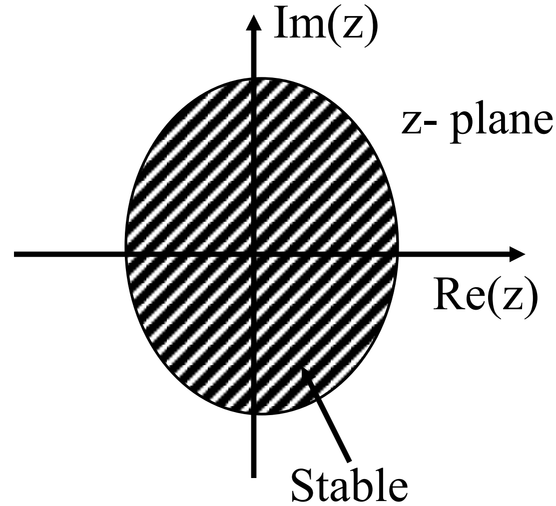
Follows:

$$\mathrm{Re}(p_i) < 0$$

$$|e^{j\,\mathrm{Im}(p_i)t}| = 1$$

Analog filters are stable when their poles are in the left half of the s-plane, where as digital filters are stable when their poles are confined within the unit circle.





Analog Filter : Magnitude

Function :

$$H(j\omega') = H(s)\,|_{s=j\omega'} = \left|H(j\omega')\right| < H(j\omega')$$

Magnitude Square Function :

$$\left|H(j\omega')\right|^2 = H(s)H(-s)\,|_{s=j\omega'}$$

Example :

$$H(s) = \frac{1}{s+1}$$

$$H(-s) = \frac{1}{-s+1}$$

Follows :

$$H(s)H(-s) = \frac{1}{1-s^2}$$

$$H(s)H(-s)\Big|_{s=j\omega'} = \frac{1}{1+\omega'^2}$$

$\therefore$

$$|H(j\omega')|^2 = \frac{1}{1+\omega'^2}$$

$$|H(j\omega')| = \frac{1}{(1+\omega'^2)^{1/2}}$$

Example :

$$|H(j\omega')|^2 = \frac{4+\omega'^2}{1+\omega'^6}$$

Find H(s) or H(-s)

$Follows:$

$$H(s)H(-s) = |H(j\omega')|^2$$

$$\omega'^2 = -s^2$$

$$= \frac{4-s^2}{1-s^6} = \frac{(2-s)(2+s)}{(1-s^3)(1+s^3)}$$

$$(1-s^3) = (1-s)(1+s+s^2)$$

$$(1+s^3) = (1+s)(1-s+s^2)$$

H(s) : poles in the left - half of the s - plane

$$H(s) = \frac{(s+2)}{(1+s)(1+s+s^2)}$$

H(-s) : poles in the right - half of the s - plane

$$H(-s) = \frac{(2-s)}{(1-s)(1-s+s^2)}$$