Discrete - Time Signals and Systems

Continuous-Time signals & systems
Introduction

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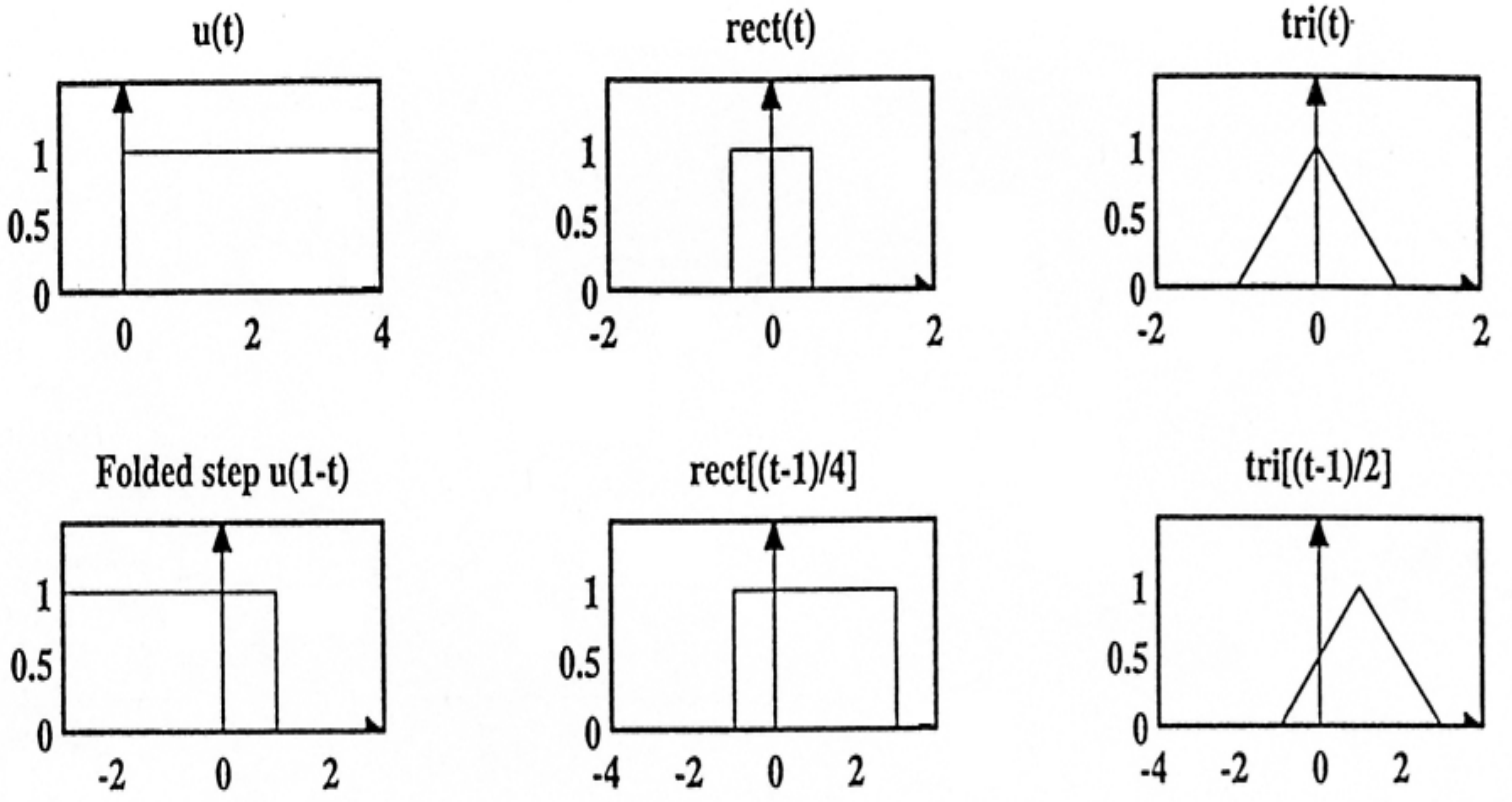
Mathematical Representation of Signals

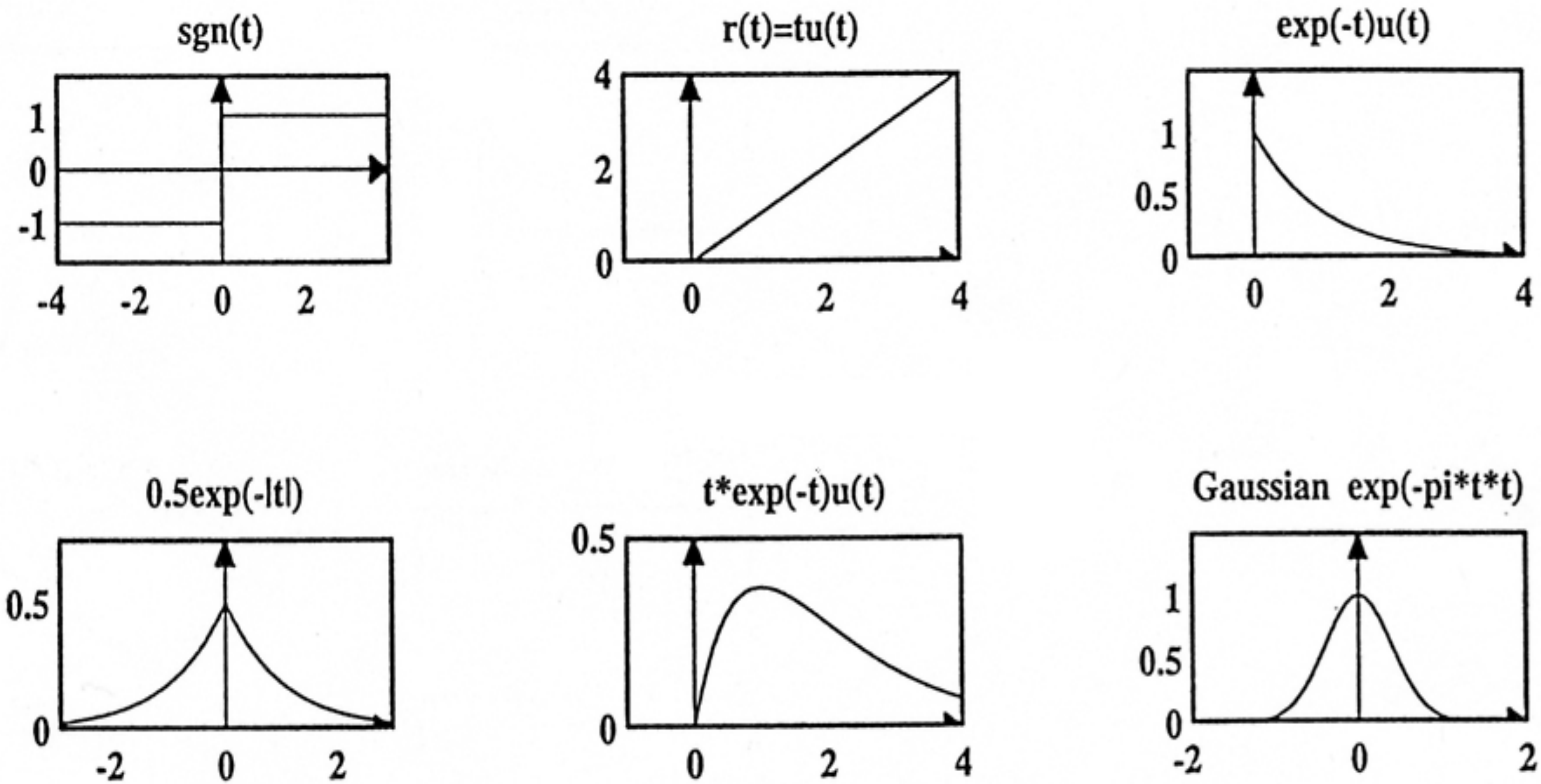
Function of an independent variable, in Mathematical Sense

- **Independent variable can be**
  - ***'Time'***, **ex. Speech signal**
  - **'spatial co-ordinates', ex. Image**
  - **'time and space', ex. Video**
  - **'numerical index'**
- **Function can be,**
  - **Of many Dimensions**
  - ***Continuous,*** **Notation "( )", as in x(t)**
  - **Discrete, Notation "[ ]", as in x[n]**

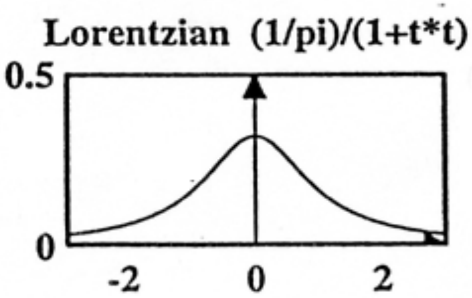
Examples of common Continuous signals

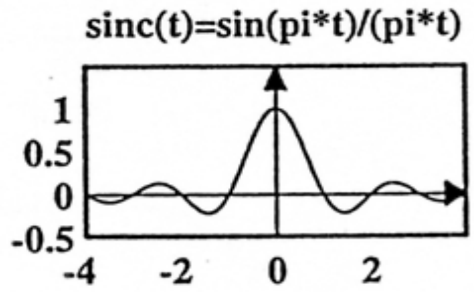


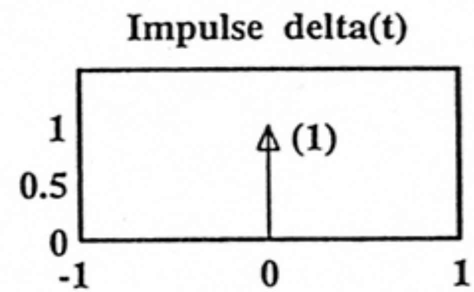
Continued…. *Examples*

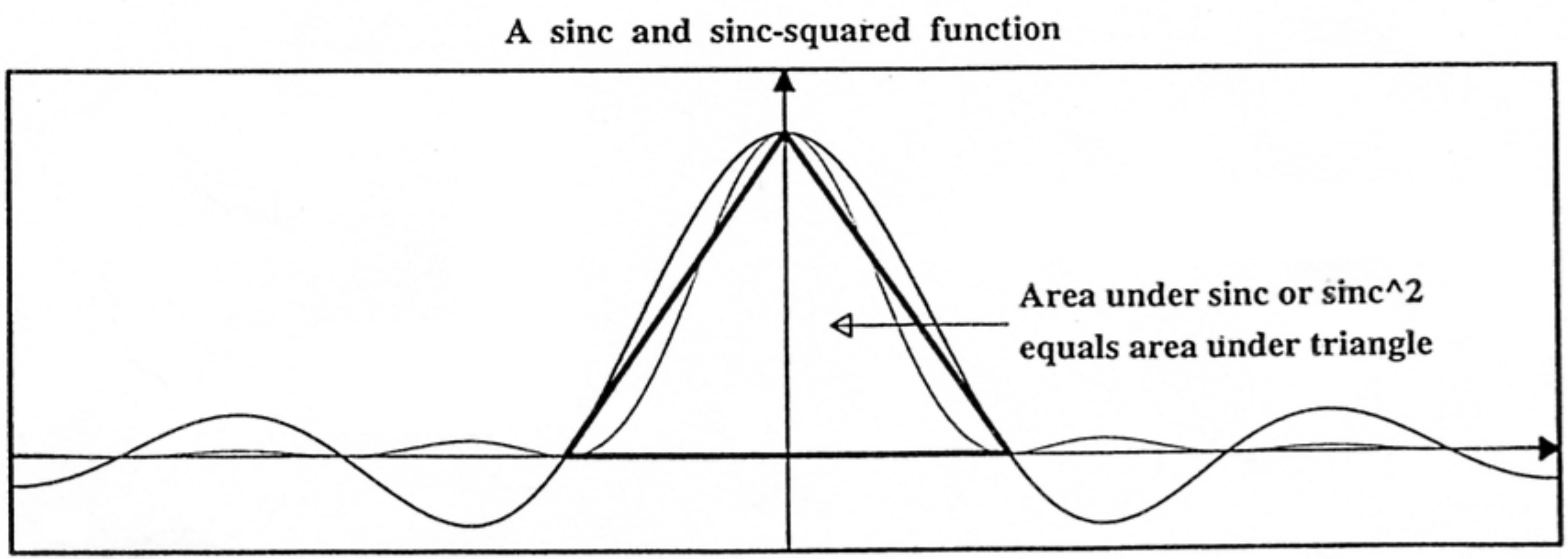


Continued…. *Examples*









Classification of Continuous-Time Signals

A) Energy Signals

$$E = \lim_{L\to\infty} \int_{-L}^{L} |x(t)|^2 \, dt$$

B) Average Power

$$P = \lim_{L\to\infty} \left[\frac{1}{2L} \int_{-L}^{L} |x(t)|^2 \, dt \right]$$

C) Periodic Signals; Average Power

$$P = \frac{1}{T_0} \int_{0}^{T_0} |x(t)|^2 \, dt$$

D) Area

$$\mathrm{A}\left[|x(t)| \right] = \int_{-\infty}^{\infty} |x(t)| \, dt$$

Common Continuous-Time Signals.

Function	Notation	Duration	Symmetry	$\mathbb{A}[\lvert x(t)\rvert]$	Energy/Power
Step	$u(t)$	Causal	None	∞	$P = \frac{1}{2}$
Signum	$\text{sgn}(t)$	Twosided	Odd	∞	$P = 1$
Ramp	$r(t)$	Causal	None	∞	Neither
Rect	$\text{rect}(t)$	Timelimited	Even	1	$E = 1$
Tri	$\text{tri}(t)$	Timelimited	Even	1	$E = \frac{2}{3}$
Exponential	$\exp(-t)u(t)$	Causal	None	1	$E = \frac{1}{2}$
Laplace	$\frac{1}{2}\exp(-\lvert t\rvert)$	Twosided	Even	1	$E = \frac{1}{4}$

Common Continuous-Time Signals.

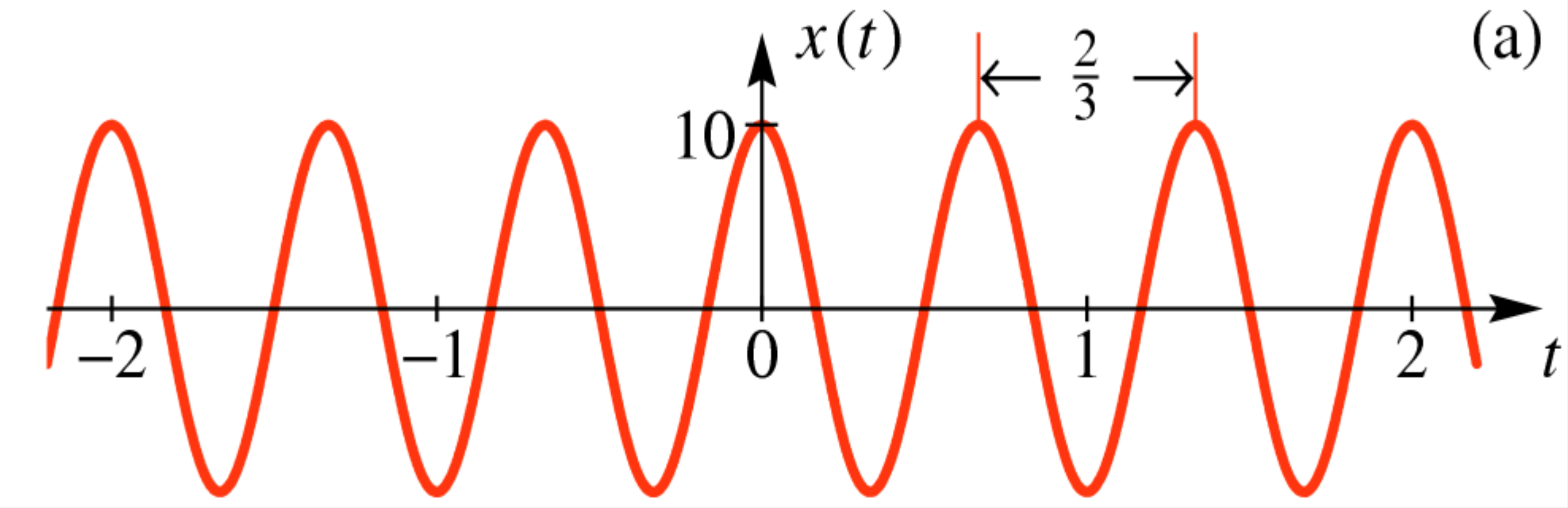
Laplace	$\frac{1}{2}\exp(-\lvert t\rvert)$	Twosided	Even	1	$E = \frac{1}{4}$
Damped exp	$t\exp(-t)u(t)$	Causal	None	1	$E = \frac{1}{4}$
Cosine	$\cos(2\pi f_0 t)$	Periodic	Even	∞	$P = \frac{1}{2}$
Sine	$\sin(2\pi f_0 t)$	Periodic	Odd	∞	$P = \frac{1}{2}$
Sinc	$\sin(\pi t)/\pi t$	Twosided	Even	∞	$E = 1$
Gaussian	$\exp(-\pi t^2)$	Twosided	Even	1	$E = 1/\sqrt{2}$
Lorentzian	$1/[\pi(1+t^2)]$	Twosided	Even	1	$E = 1/(2\pi)$
Impulse	$\delta(t)$	Zero	Even	1	Neither
Doublet	$\delta'(t)$	Zero	Odd	∞	Neither

Infinite length signals: Example 1

$$a)\ x(t) = A\cos(\omega_0 t + \phi) \qquad -\infty < t < \infty$$

$$with \quad \omega_0 = 3\pi \ \ rad/s, \ \ A = 10 \ and \ \phi = 0$$

$$x(t) = 10\cos(3\pi t)$$



Examples 2&3: Infinite length signals

$b)$ *Periodic signals,* $x(t) = x(t + T_0)$ $\quad -\infty < t < \infty$

Square wave with fundamental frequency of $3Hz$

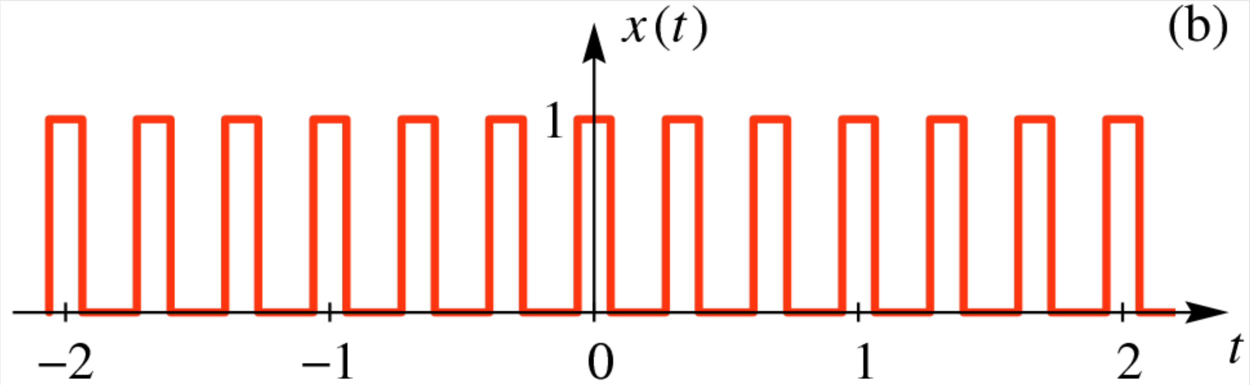


Fig 16.5

$c)$ *Two sided decaying exponential* $x(t) = 5e^{-|t|}$ $\quad -\infty < t < \infty$

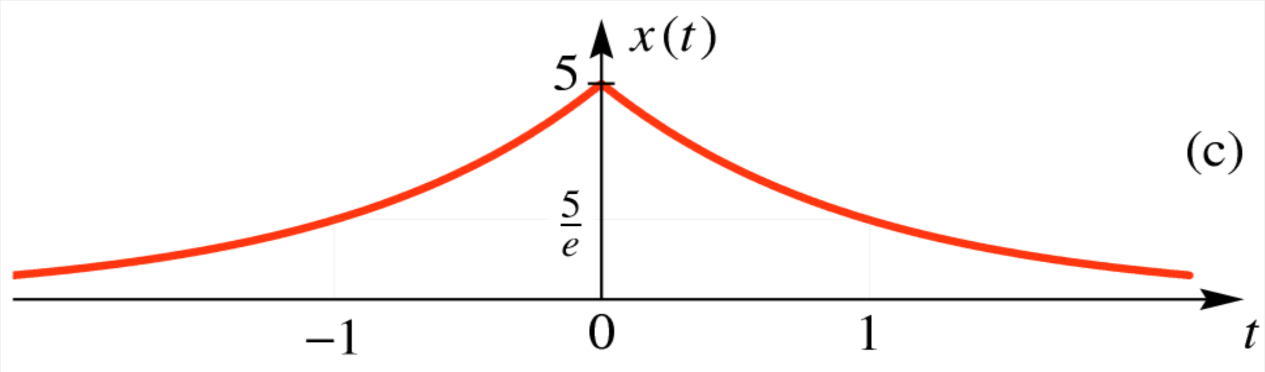


Fig 16.6

Finite length signals

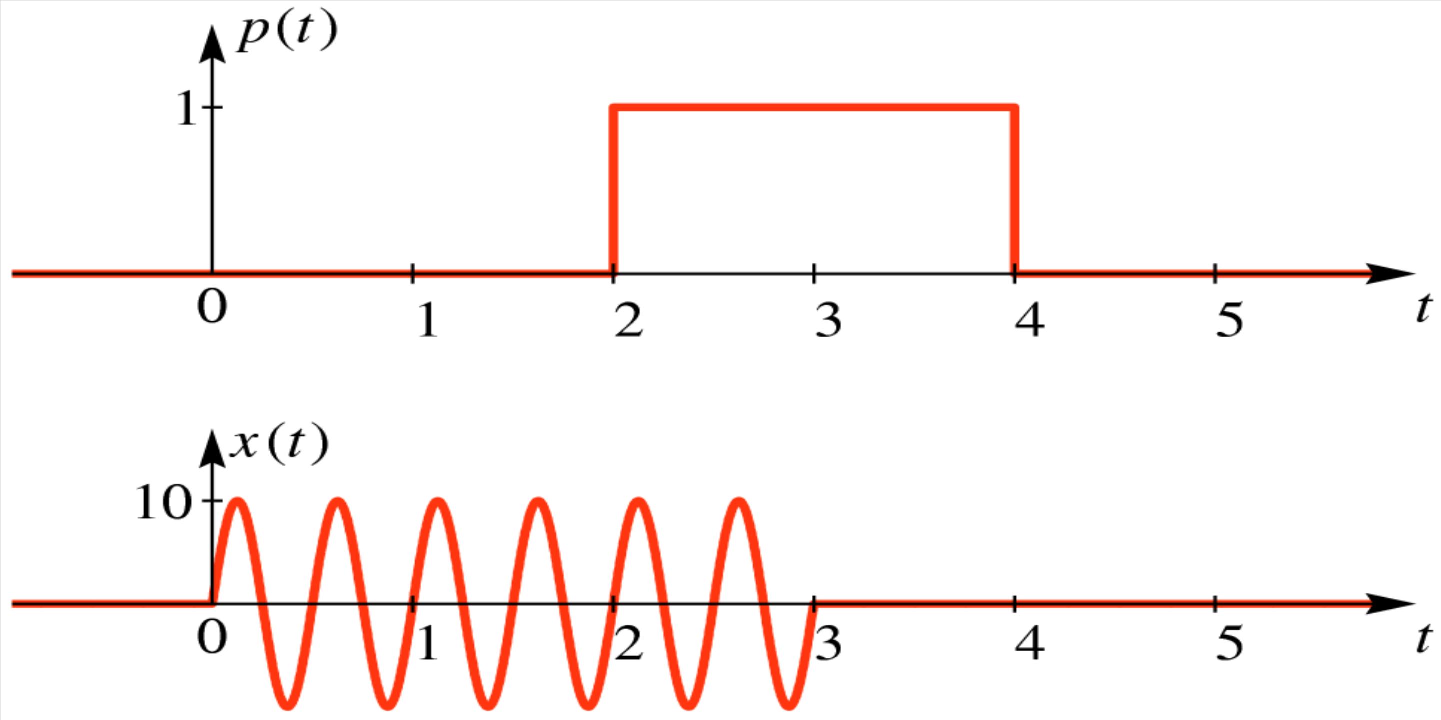


Fig 16.7

$p(t) = u(t-2) - u(t-4)$, $u(t)$... *unit step signal*, *next section*

$x(t) = \sin(4\pi t)\left[u(t) - u(t-3)\right]$

Unit Step Signal

Fig 16.8

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Finite continuous pulse

Fig 16.9

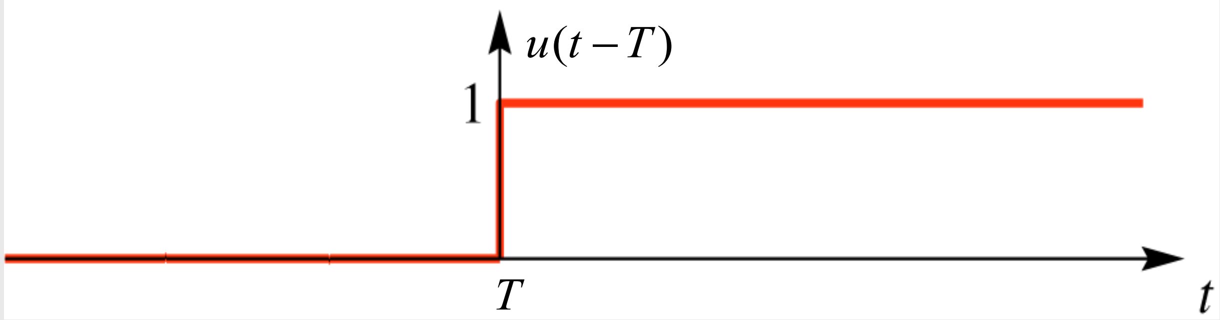


Fig 16.10

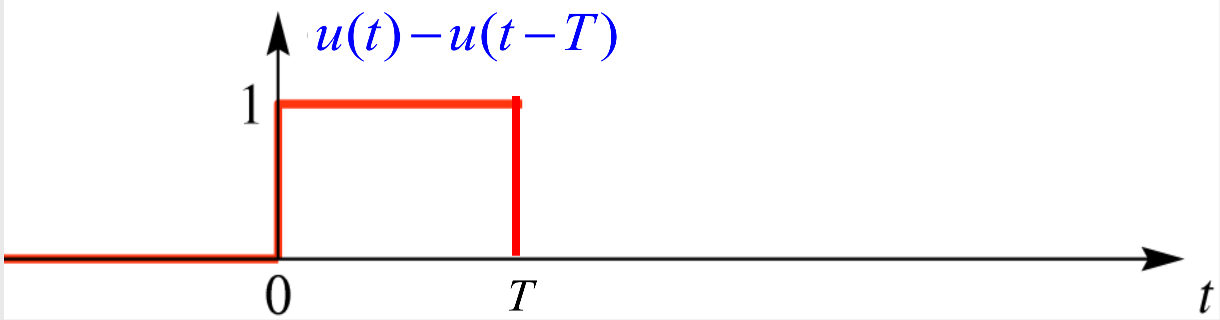


Fig 16.11

One Sided Signals: Application of Unit step signal

Unit step signal operates on the ideal non-causal infinite length signal into a causal one

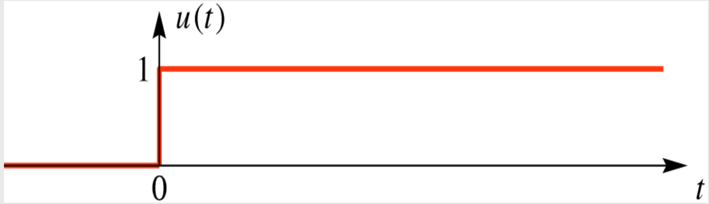
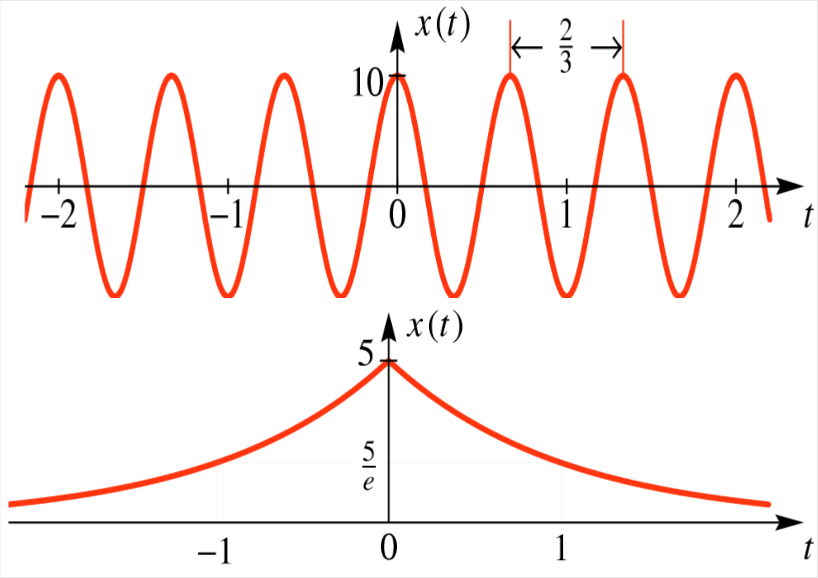


Fig 16.12



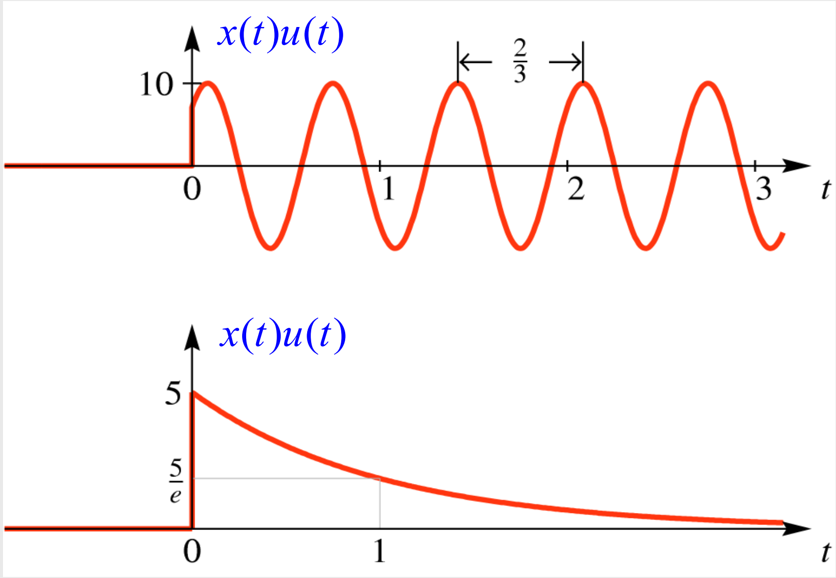


Fig 16.13

Rectangular Pulse

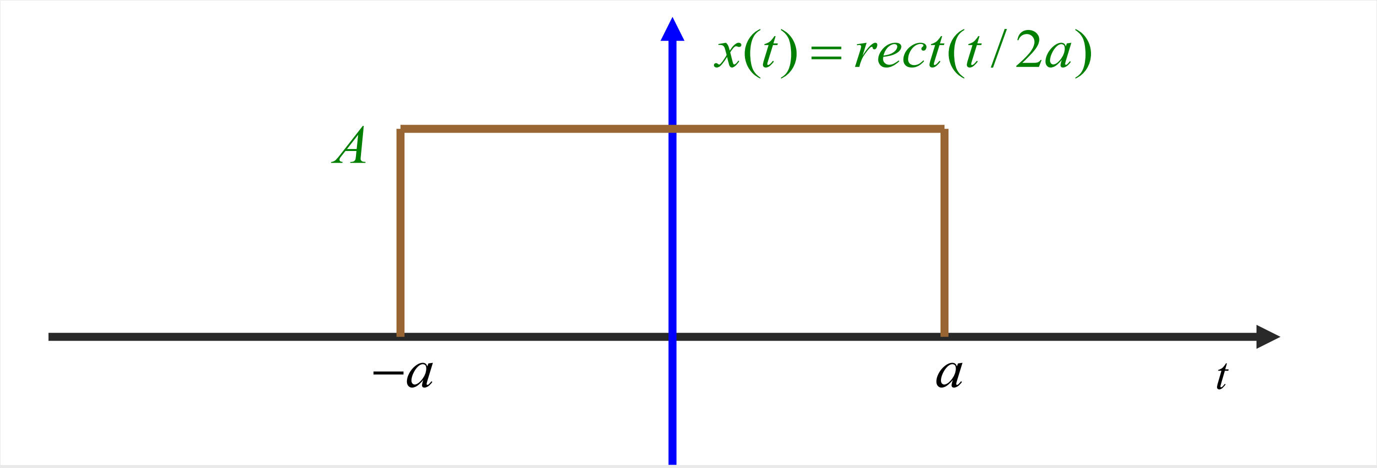


Fig 16.15

$x(t)\text{...is derived from 2 unit step functions}$

$$x(t) = A\left[u(t+a) - u(t-a)\right]$$

Unit Ramp Signal

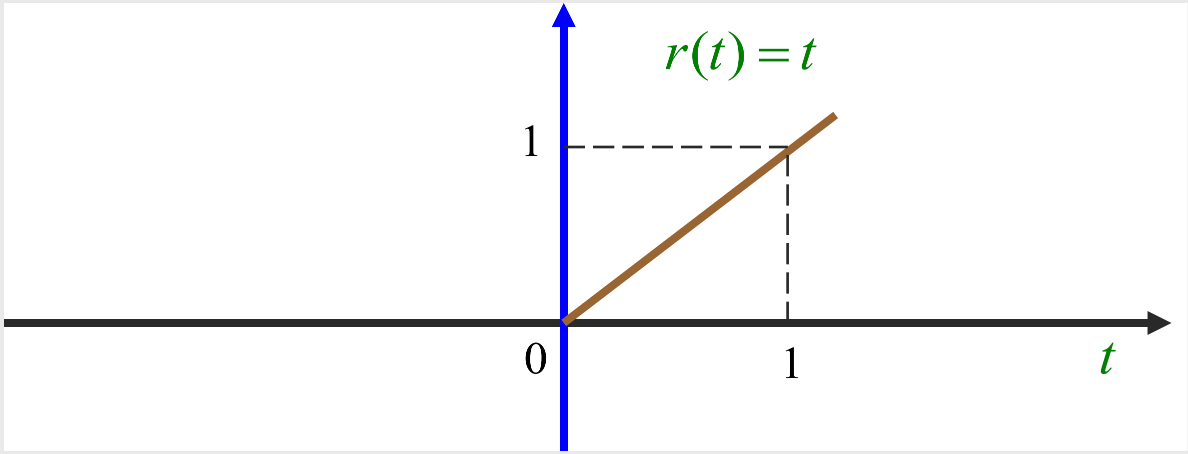


Fig 16.16

$$r(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

$u(\tau)$... *unit step signal*

Also note that, $u(\tau) = \dfrac{dr(t)}{dt} = \dfrac{dt}{dt} = 1$

Unit Impulse Signal

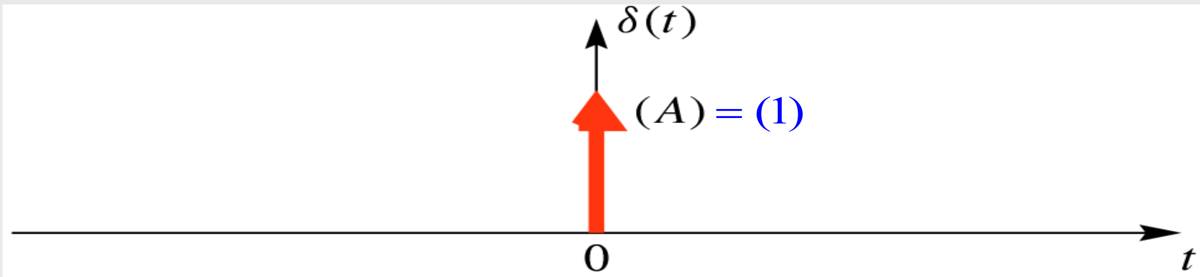


Fig 16.17

Unit – impulse signal is symbolized with an arrow

The area of the impulse is written in paranthesses next to the arrow head

$$a)\ \delta(t) = 0 \quad t \neq 0$$

$$b)\ \int_{t_1}^{t_2} \delta(t)dt = \begin{cases} 1 & t_1 < 0 < t_2 \\ 0 & otherwise \end{cases}$$

$$c)\ \delta(t)\ undefined\ at\ t = 0$$

$$d)\ \delta(t) = \delta(-t);\ \ even\ function$$

Unit Impulse & Unit Step Signals

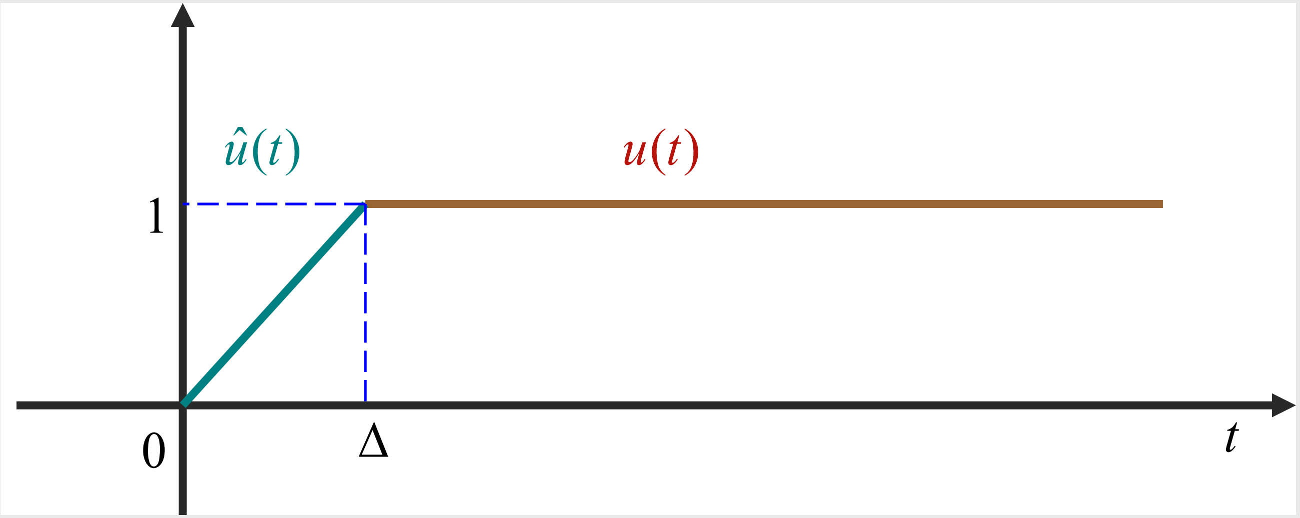


Fig 16.18

Δ is an infinitely small value

$$u(t) = \lim_{\Delta \to 0} \hat{u}(t)$$

$$\textit{Definition}: \quad \hat{\delta}(t) = \frac{d\hat{u}(t)}{dt}$$

Unit Impulse & Unit Step Signals, *contd..*

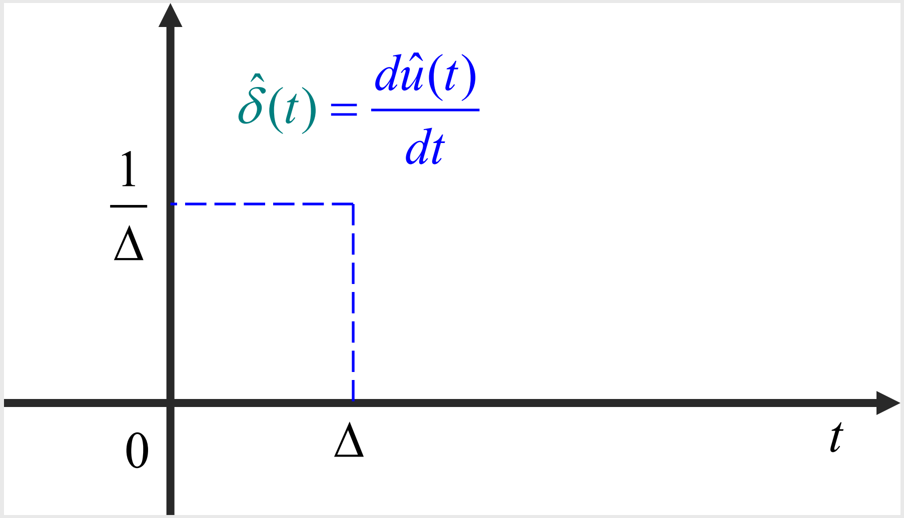


Fig 16.19

Thus unit - impulse can't be defined as a single point

It is an approximation of narrow pulses whose area is '1'

$$\delta(t) = \frac{du(t)}{dt}; \qquad u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

Unit Impulse: Approximation with pulses

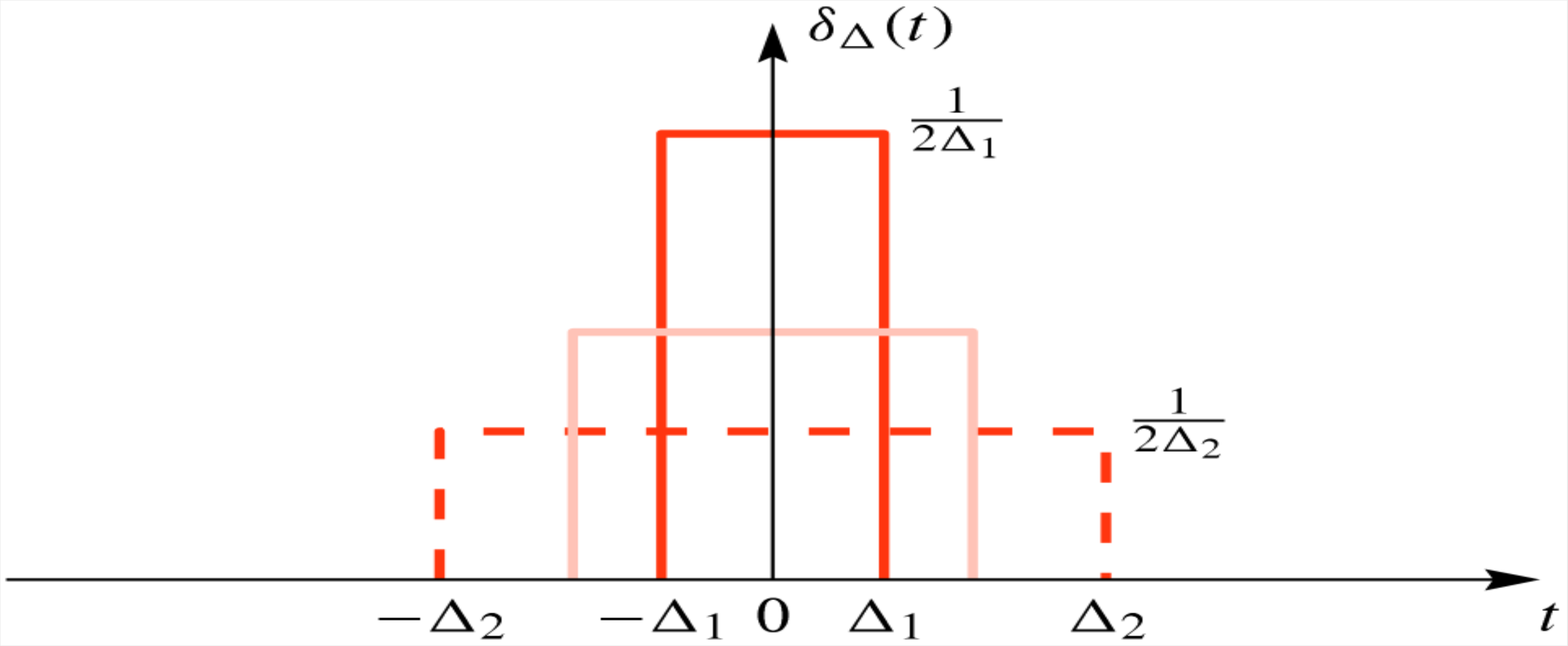


Fig 16.20

Notice that the area of any pulse is '1'

Impulse results at the derivative of any discontinuous point

Unit Impulse: Derivative at discontinuity

$$\delta(t) = \frac{du(t)}{dt}$$

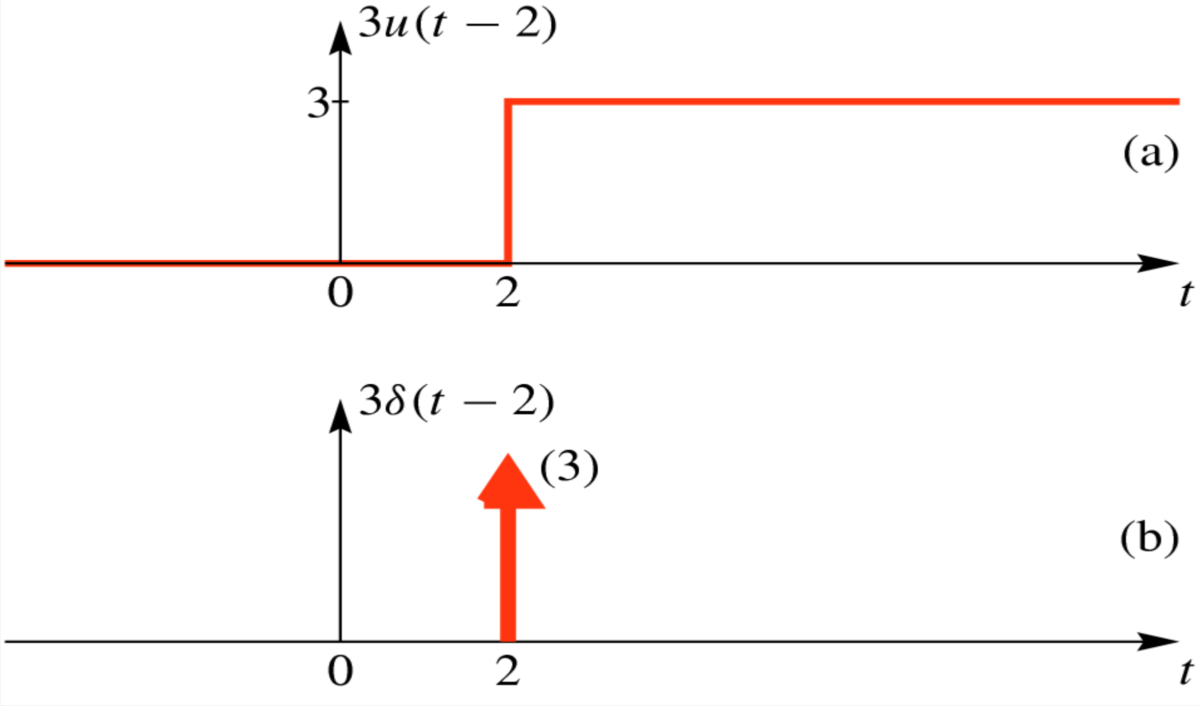


Fig 16.21

Notice the location and magnitude of impulse

Unit Impulse- Derivative of a discontinuous function

$$x(t) = e^{-2(t-1)}u(t-1)$$

$$\frac{dx(t)}{dt} = e^{-2(t-1)}\frac{du(t-1)}{dt} + u(t-1)\frac{de^{-2(t-1)}}{dt}$$

$$= e^{-2(t-1)}\delta(t-1) - 2e^{-2(t-1)}u(t-1)$$

$$= e^{0}\delta(t-1) - 2e^{-2(t-1)}u(t-1)$$

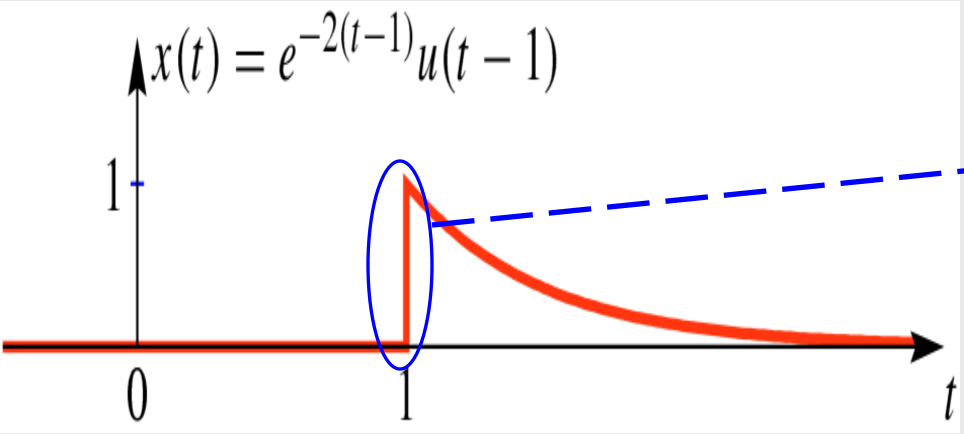


Fig 16.22

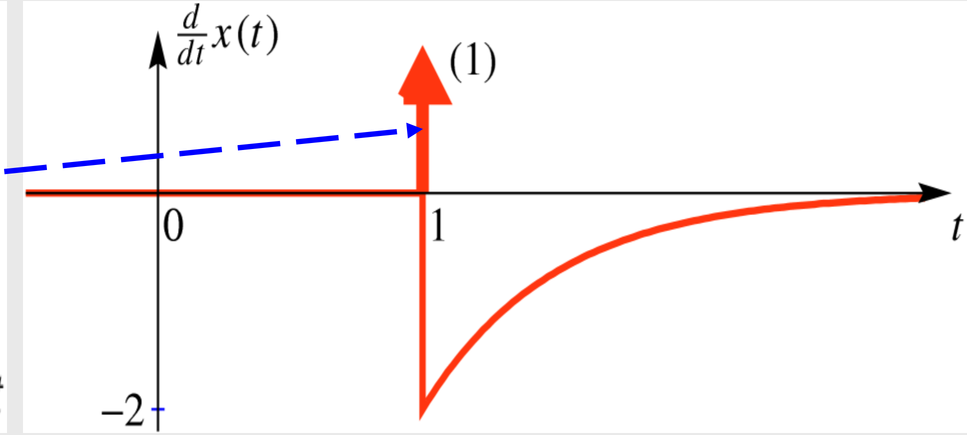
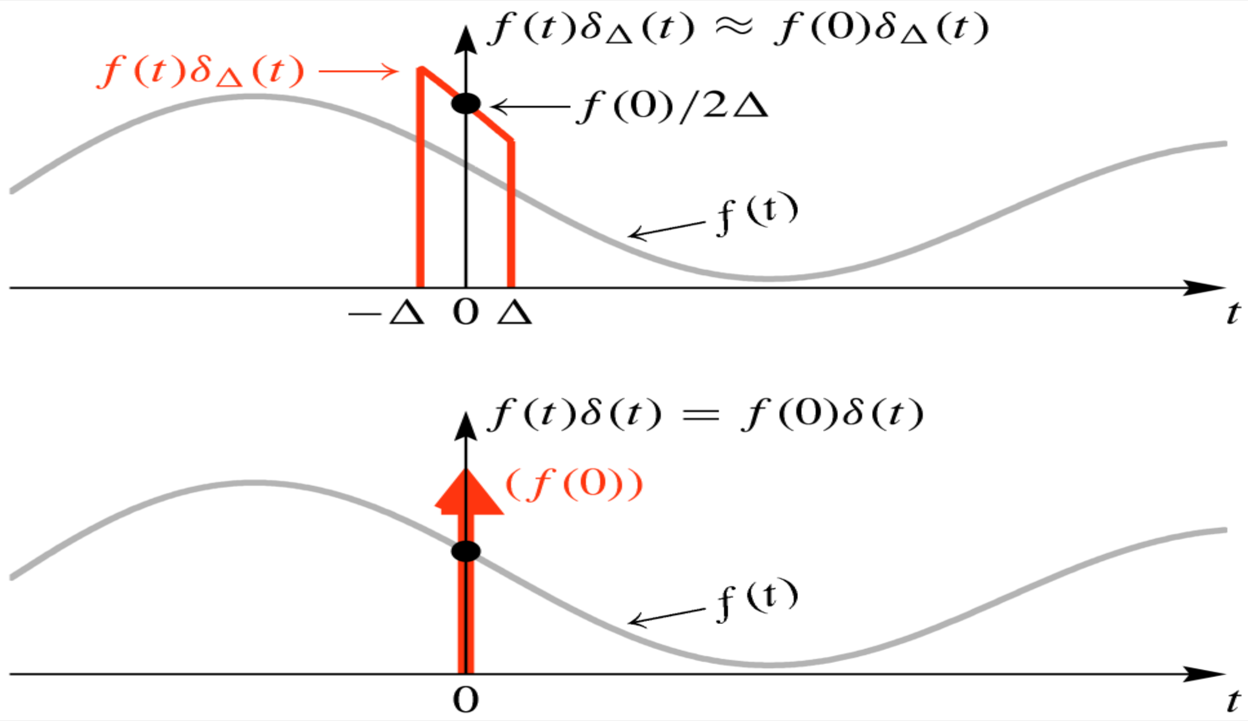


Fig 16.23

Unit Impulse - Scaling property

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$$



The impulse is shown as pulse

Unit Impulse - Scaling property

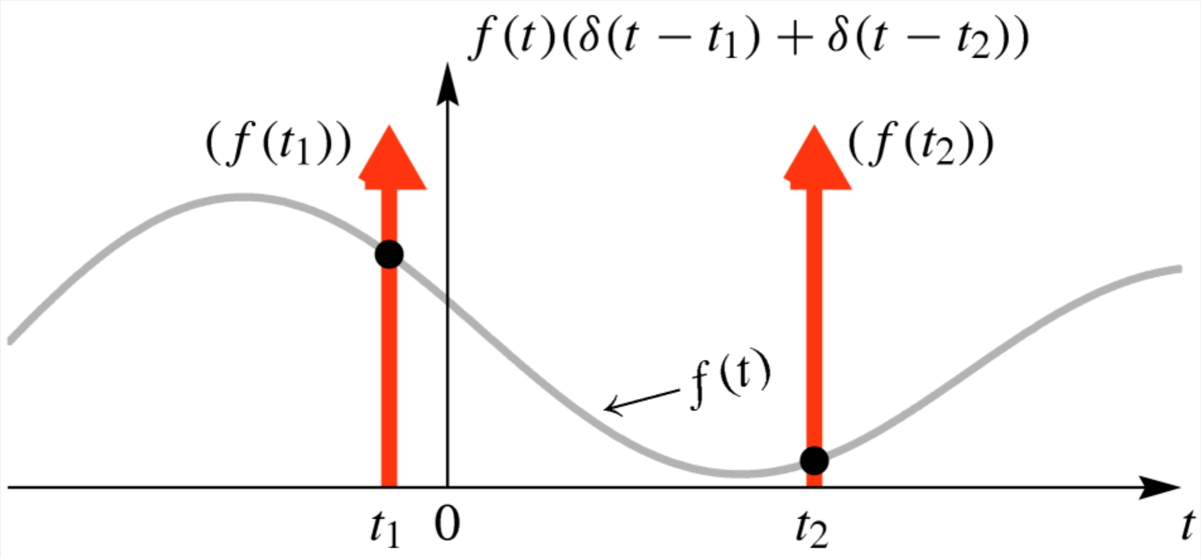


Fig 16.25

signal $f(t)$ multiplied by two shifted unit impulses

$$f(t)\big(\delta(t-t_1)+\delta(t-t_2)\big) = f(t_1)\delta(t-t_1)+f(t_2)\delta(t-t_2)$$

$$\delta(at+b) = \frac{1}{|a|}\delta(t+\frac{b}{a}); \qquad \delta\big(a(t-b)\big) = \frac{1}{|a|}\delta(t-b)$$

Unit Impulse - Example

Evaluate the expression, $x(t) = \sin(20\pi t)\delta(t - 1/80)$

$$x(t) = \sin(20\pi t)\delta(t - 1/80)$$
$$= \sin(20\pi 1/80)\delta(t - 1/80)$$
$$= 0.707\delta(t - 1/80)$$

the continuous function multiplied by an impulse becomes an impulse with a size depending only on the value of continuous function evaluated at the time location of impulse

$$\int_{-\infty}^{\infty} x(t)dt = \int_{-\infty}^{\infty} \sin(20\pi t)\delta(t - 1/80)dt = \int_{-\infty}^{\infty} 0.707\delta(t - 1/80)$$
$$= 0.707$$

Unit Impulse - Examples

$$a)\int_{-\infty}^{\infty}\delta(t-t_0)e^{-j\omega t}dt = e^{-j\omega t_0}$$

$$b)\int_{-\infty}^{\infty}t\delta(t-3)dt = \int_{-\infty}^{\infty}3\delta(t-3)dt$$

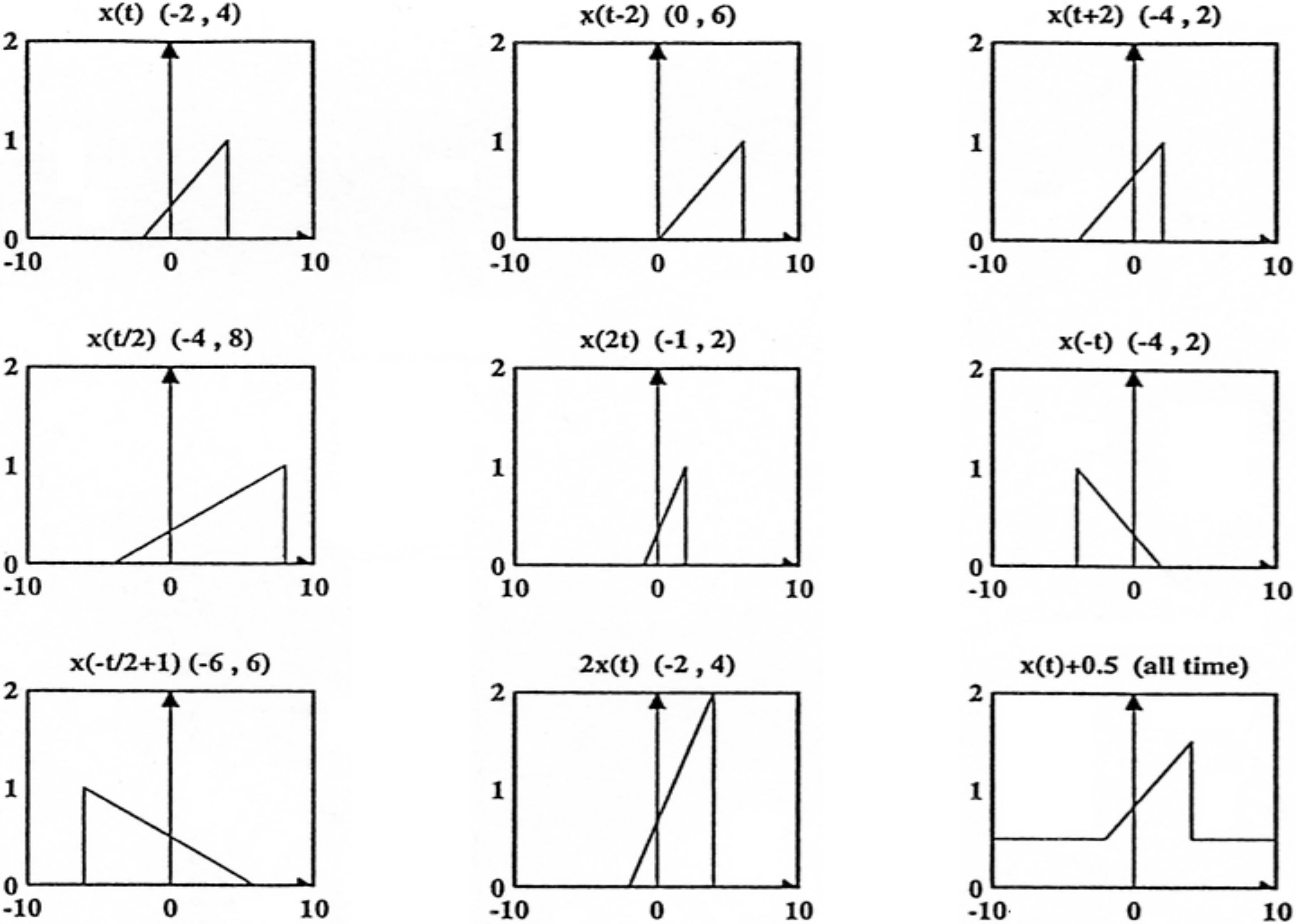
$$= 3\int_{-\infty}^{\infty}\delta(t-3)dt = 3$$

$$c)\int_{-3}^{3}(t+t^2)\delta(t-2)dt = \int_{-3}^{3}(2+2^2)\delta(t-2)dt$$

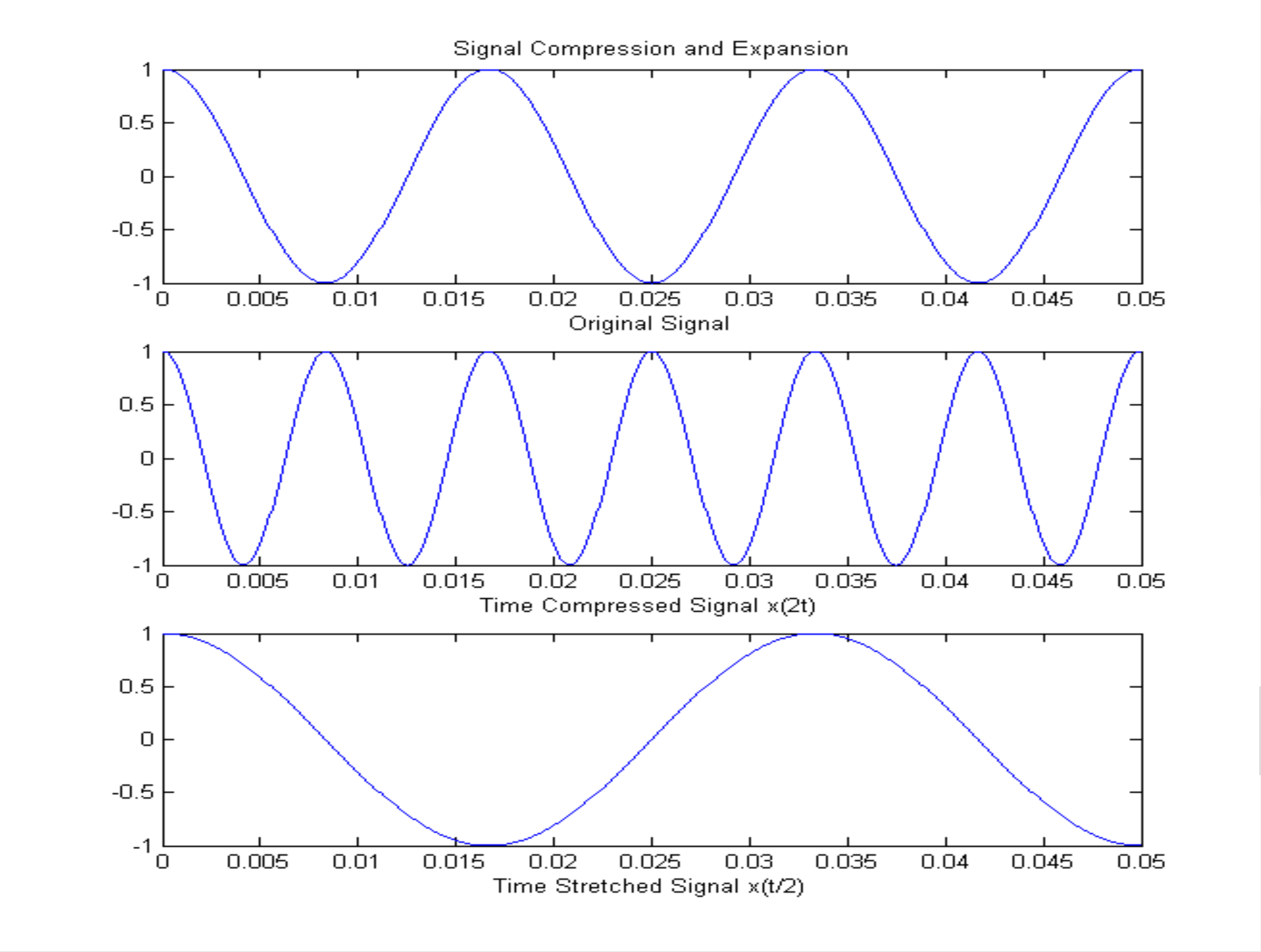
$$= 6\int_{-3}^{3}\delta(t-2)dt = 6$$

Operations on continuous-time signals

Operation	Example	Explanation
Time shift	$x(t-2)$	Shift $x(t)$ right by 2 (delay).
	$x(t+2)$	Shift $x(t)$ left by 2 (advance).
Time scale	$x(2t)$	Compress $x(t)$ by factor of 2 (speed up).
	$x(\frac{1}{2}t)$	Stretch $x(t)$ by factor of 2 (slow down).
Folding	$x(-t)$	Fold $x(t)$ about origin.
Combinations	$x(-\frac{1}{2}t+1)$	Shift $x(t)$ left by 1, fold and stretch by 2.
or	$x[-\frac{1}{2}(t-2)]$	Fold & stretch $x(t)$ by 2 and shift right by 2.
Amplitude scale	$2x(t)$	Multiply ordinate by factor of 2.
Amplitude shift	$x(t)+\frac{1}{2}$	Add dc offset of $\frac{1}{2}$ to $x(t)$ *everywhere.*


x(t) (-2 , 4)
x(t-2) (0 , 6)
x(t+2) (-4 , 2)
x(t/2) (-4 , 8)
x(2t) (-1 , 2)
x(-t) (-4 , 2)
x(-t/2+1) (-6 , 6)
2x(t) (-2 , 4)
x(t)+0.5 (all time)
2
1
0
-10
0
10

Operations on continuous-time signals, *contd…*



$x(t)$

$x(2t)$

$x(t/2)$

Operations on continuous-time signals, *contd…*

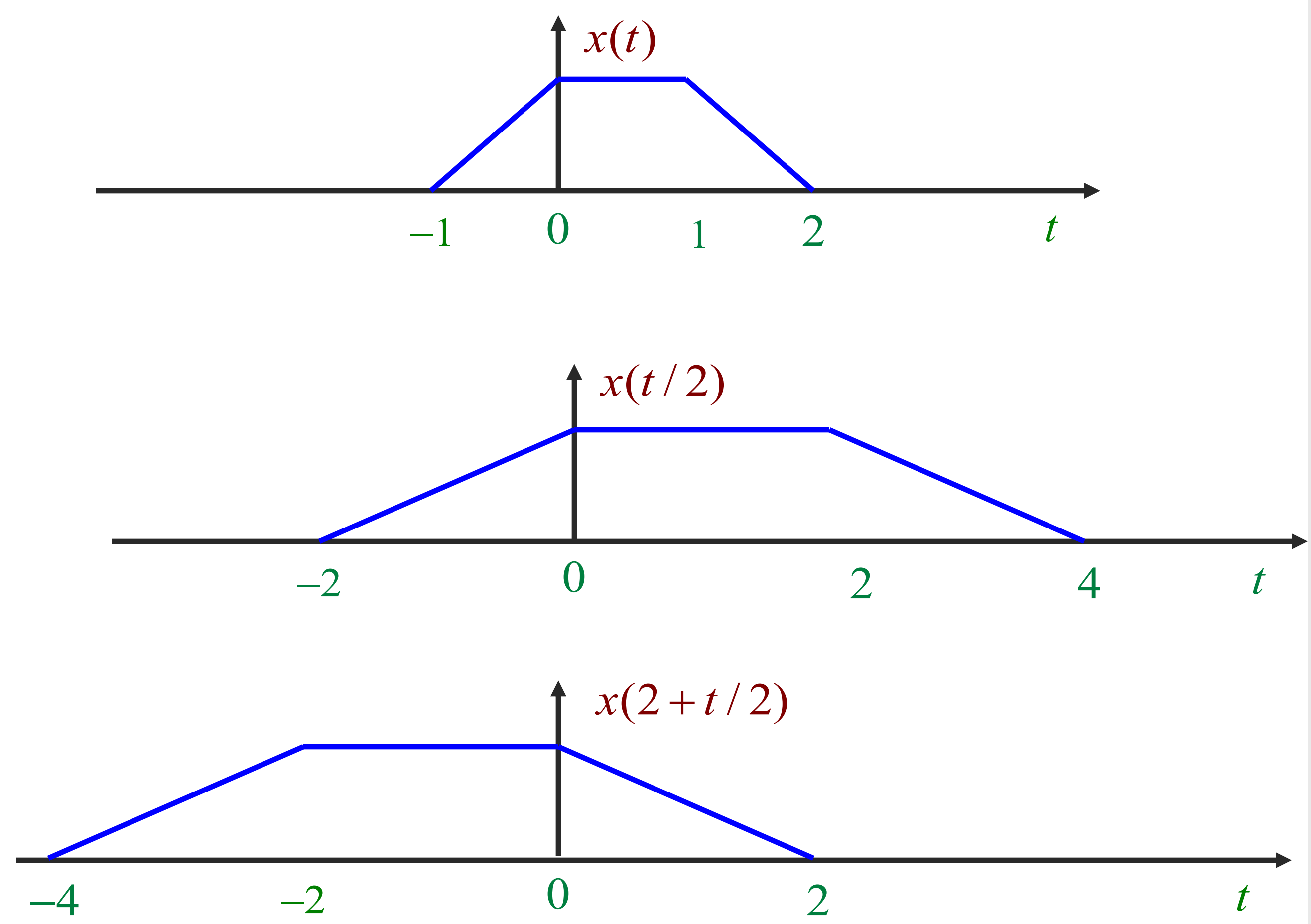


Fig 16.28

Reference

James H. McClellan, Ronald W. Schafer and Mark A. Yoder, “ 9.1, and 9.2 “Signal Processing First”, Prentice Hall, 2003