ECE137B Final Exam

There are 5 problems on this exam and you have 3 hours.

There are pages 1-18 in the exam: please make sure all are there.

Do not open this exam until told to do so.

Show all work.

Credit will not be given for correct answers if supporting work is not shown.

Class Crib sheets and 4 pages (front and back → 8 surfaces) of your own notes permitted.

Don’t panic.

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<table>
<thead>
<tr>
<th>Time function</th>
<th>LaPlace Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta(t) )</td>
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</tr>
<tr>
<td>( U(t) )</td>
<td>( 1/s )</td>
</tr>
<tr>
<td>( e^{-\alpha t} \cdot U(t) )</td>
<td>( \frac{1}{s + \alpha} ) or ( \frac{1}{1 + s / \alpha} )</td>
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<tr>
<td>( e^{-\alpha t} \cos(\omega_d t) \cdot U(t) )</td>
<td>( \frac{s + \alpha}{(s + \alpha)^2 + \omega_d^2} )</td>
</tr>
<tr>
<td>( e^{-\alpha t} \sin(\omega_d t) \cdot U(t) )</td>
<td>( \frac{\omega_d}{(s + \alpha)^2 + \omega_d^2} )</td>
</tr>
</tbody>
</table>

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Name: _________________________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>points</th>
<th>possible</th>
<th>Problem</th>
<th>points</th>
<th>possible</th>
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<tbody>
<tr>
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<td>3</td>
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Problem 1, 33 points
method of first-order and second-order time constants. Some negative feedback

Above is a high-speed op-amp with, by design, a low input impedance to the negative input. It is connected, as the inset image suggests, as a positive voltage-gain stage.

All FETs are short-channel devices:
\[ I_d \approx v_{sat} c_{ox} W_g (V_{gs} - V_{th} - \Delta V) \] where \( v_{sat} c_{ox} = 1 \text{mS/micrometer and } (V_{th} + \Delta V) = 0.3 \text{ Volts.} \)

All FETs have \( \lambda = 0 \text{ V}^{-1} \), all have \( W_g = 10 \text{ micrometers.} \)

Q4a and Q4b have \( C_{gs} = 10 \text{fF and } C_{gd} = 1 \text{ fF}, \)
Q6a and Q6b have \( C_{gs} = 10 \text{fF and } C_{gd} = 0 \text{ fF}, \)
Q1a, Q1b, Q2a, Q2b, Q3a, Q3b, Q5a, Q5b, have \( C_{gs} = 0 \text{fF and } C_{gd} = 0 \text{ fF.} \)

\( I_o = 2 \text{ mA}, \ R_{sa} = R_{sb} = 2 \text{M} \Omega, \ 2R_L = \text{infinity (just to keep the exam simple).} \)
\( C_{ca} = C_{cb} = 99 \text{fF}. \ R_1 = 1000 \Omega, \ R_2 = 50 \Omega \)
The supplies are +1.5V and -1.5V.
Part a, 5 points

Draw all DC node voltages and all DC bias currents on the diagram below.
Part b, 3 points

Symmetry allows us to analyze bandwidth and gain with the half-circuit below:

To compute the loop transmission you must (1) set $V_{\text{gen}}$ to zero, (2) cut the feedback loop as shown (3) restore the stage loading which has been removed by making the cut, (4) insert an AC voltage generator at the cut point, and (5) compute the voltage gain once around the loop.

Indicate on the drawing above what circuit element must be placed in the box labeled with a "?", and give the value of this element.
Part c, 10 points

Working with the circuit diagram of the previous page, determine the DC value of the loop transmission.

\[ T_{DC} = \text{__________} \]
Part d, 15 points
Using MOTC, you will find the frequency, in Hz (not rad/sec), of the two major poles in the loop transmission $T$.

Find all the following.

\[
\begin{array}{|c|c|c|}
\hline
C_1 &= C_{gd4a} + C_{ca} & C_2 = C_{gs4a} : & C_3 = C_{gs6a} \\
R_{11}^0 = & R_{22}^0 = & R_{33}^0 = \\
R_{22}^1 = & R_{33}^1 = & R_{33}^2 = \\
\tilde{f}_{p1} = & \tilde{f}_{p2} = & \\
\hline
\end{array}
\]
The amplifier has a differential gain of $10^7$.

The op-amp has infinite differential input impedance and zero differential output impedance.

The differential amplifier has pole in its open-loop transfer function at 1 Hz, 10 MHz, and 50 MHz.

$$R_1 = 4 \text{ kOhm}, \quad R_2 = 1 \text{ kOhm},$$

Using the Bode plots on the next page, plot the open-loop gain ($A_d$ or $A_{of}$), the inverse of the feedback factor ($1/\beta$), closed loop gain ($A_{CL}$), and determine the following:

Loop bandwidth = _______________  
phase margin = _______________

$V_{out}/V_{gen}$ at DC = _______________

Be SURE to **label** and **dimension** all **axes** clearly, and to make clear and **accurate asymptotic** plots.
Draw open loop gain (Ad) and 1/\beta on this plot

draw closed loop gain on this bode plot
Problem 3, 10 points

negative feedback

The amplifier has a differential gain of $10^7$.

The op-amp has infinite differential input impedance and zero differential output impedance.

The differential amplifier has pole in its open-loop transfer function at 1 Hz.

$$R_1 = 9 \text{ kOhm}, \quad R_2 = 1 \text{ kOhm}, \quad C = 100 \text{nF}$$

Using the Bode plots on the next page, plot the open-loop gain ($A_d$ or $A_{ol}$), the inverse of the feedback factor ($1/\beta$), closed loop gain ($A_{CL}$), and determine the following:

Loop bandwidth = ____________

Be SURE to *label* and *dimension* all *axes* clearly, and to make clear and *accurate asymptotic* plots.
Draw open loop gain (Ad) and 1/beta on this plot.

Draw closed loop gain on this bode plot.
Problem 4, 10 points

negative feedback

The amplifier has a differential gain of $10^7$.

The op-amp has **100 Ohms differential input impedance** and zero differential output impedance.

The differential amplifier has one pole in its open-loop transfer function at 1 Hz.

$R_1 = 9 \text{kOhm}$, $R_2 = 1 \text{kOhm}$,

Using the Bode plots on the next page, plot the loop transmission ($T$), plot $A_s$, and plot the closed loop gain ($A_{CL} = \text{Vout/Vgen}$), and determine the following:

Loop bandwidth=________________ . Amplifier 3dB bandwidth=______________

Be SURE to **label and dimension all axes** clearly, and to make clear and **accurate asymptotic** plots.
Plot $T$, $A_{\infty}$, on this plot

draw closed loop gain on this bode plot
Problem 5: 12 points  
method of time constants analysis

part a, 10 points

Using MOTC, find the transfer function $V_{out}(s)/V_{gen}(s)$. Working with the transfer function in standard form, i.e. \[
\frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}}{V_{gen\ DC}} \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^3 + \ldots}
\] give algebraic answers in the blanks below

\[
\frac{V_{out}}{V_{gen\ DC}} = \quad a_1 = \quad a_2 = \\
\]

\[
b_1 = \quad b_2 = \quad
\]

We will give you that $b_1 = b_2 = 0$ seconds. This can be found without nodal analysis, just by looking at the form of the circuit.
part b, 2 points

Now, $R_1 = 1 \, k\Omega$, $R_2 = 2 \, k\Omega$, $R_3 = 3 \, k\Omega$, $C_1 = 1 \, \mu F$, $C_2 = 2 \, \mu F$. Again find $a_1$ and $a_2$ and $V_{out}/V_{gen}$ at DC.

$$\frac{V_{out}}{V_{gen,DC}} = a_1 = \text{________________________} \quad a_2 = \text{________________________}$$
Problem 6: 23 points
Nodal analysis and transistor circuit models

Part a, 10 points
Draw an accurate small-signal equivalent circuit model of the circuit above. Do not show components whose element values are zero or infinity (!).

Important hints:
(1) Use a hybrid-pi model, not a T-model, for the FET
(2) Use a Norton model (not a Thevenin model) for the generator.

Ignore DC bias; you don't need it.
FET: $C_{gs} = 0 \text{fF}$, $C_{gd} = 0 \text{ fF}$, $R_{ds} = \text{infinity}$.

** $g_m$ is nonzero**
Part b, 10 points

Using NODAL ANALYSIS, find the transfer function \( \frac{V_{\text{out}}(s)}{V_{\text{gen}}(s)} \).

The answer must be in standard form \( \frac{V_{\text{out}}(s)}{V_{\text{gen}}(s)} = \frac{V_{\text{out}}}{V_{\text{gen}}_{\text{DC}}} \left( \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots} \right) \).

\[
\frac{V_{\text{out}}}{V_{\text{gen}}_{\text{DC}}} = \quad a_1 = \quad a_2 = \\
b_1 = \quad b_2 =
\]
Part c, 3 points

Now set: $R_L = 1000 \, \Omega$, $g_m = 1 \, \text{mS}$, $C = 1 \, \text{pF}$. Find numeric values for $\frac{V_{out}}{V_{gen, DC}}$, $a_1$, $a_2$, $b_1$ and $b_2$.

Find the frequencies (in Hz) of all pole and zero frequencies

$$\frac{V_{out}}{V_{gen, DC}} = \text{__________________}$$

$$a_1 = \text{__________________}, \quad a_2 = \text{__________________}$$

$$b_1 = \text{__________________}, \quad b_2 = \text{__________________}$$

$$f_{p1} = \text{__________________} \quad f_{p2} = \text{__________________}$$

$$f_{z1} = \text{__________________} \quad f_{z2} = \text{__________________}$$
Problem 7, 10 points

*mental Fourier Transforms*

An amplifier has low-frequency and high frequency poles, with an upper -3 dB frequency of 10kHz and a lower -3dB frequency of 2kHz. It has a **MIDBAND** voltage gain of 10. Plot below an accurate Bode plot of Vout/Vgen and an accurate plot of its step response with a 1 V step-function input. Label and dimension axes.
output voltage with \( V_{\text{in}}(t) = 1 \text{ Volt step function} \)

\[
\begin{array}{c}
\text{Vout}(t) \\
\text{time}
\end{array}
\]