Mid-Term Exam, ECE-137B
May 14, 2002

Closed-Book Exam

There are 2 problems on this exam, and you have 75 minutes.

1) show all work. Full credit will not be given for correct answers if supporting work is not shown.
2) please write answers in provided blanks
3) Don’t Panic!
4) 137a, 137b crib sheets, and 2 pages personal sheets permitted.

Do not turn over the cover page until requested to do so.

Name:

Solution

Use any and all reasonable approximations. 5% accuracy is OK if the method is correct.

<table>
<thead>
<tr>
<th>Time function</th>
<th>LaPlace Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ(t)</td>
<td>$1$</td>
</tr>
<tr>
<td>U(t)</td>
<td>$1/s$</td>
</tr>
<tr>
<td>$e^{-at}U(t)$</td>
<td>$\frac{1}{s+\alpha}$</td>
</tr>
<tr>
<td>$e^{-at}\cos(\omega_dt)U(t)$</td>
<td>$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$</td>
</tr>
<tr>
<td>$e^{-at}\sin(\omega_dt)U(t)$</td>
<td>$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$</td>
</tr>
</tbody>
</table>
Problem 1, 62 points

A very broadband DC-coupled amplifier.

Transistor parameters:
- $C_{cb} = 2$ pF, $\beta = \infty$
- $V_a = \infty$
- $\tau_f = 0.2$ ns
- $C_{be,depl} = 1$ pF

- $RL = 100$ Ohm
- $R_{gen} = 100$ Ohms = $R_b$
- $CL = 5$ pF

$V_{cc} = 7.5$ Volts,
$-V_{ee} = -7.5$ volts
a) 5 points
Q1 and Q2 are to each operate at 7.5 mA emitter current. Q3 is to operate at 10 mA. The emitter of Q3 is to be biased at +5 volts. The current through Rb3b is 3 mA. The DC output voltage is zero volts.

Find I1, I2, Rb3a, Rb3b, Rc3

\[ I_1 = \frac{15\text{mA}}{1} \quad I_2 = \frac{17.5\text{mA}}{1} \]

\[ R_{b3a} = \frac{1.06\text{kΩ}}{1} \quad R_{b3b} = \frac{1.43\text{kΩ}}{1} \quad R_c = \frac{750\text{Ω}}{1} \]
b) 5 points
On the circuit diagram below indicate the currents through all resistors, all collector currents, and the voltages on each circuit node.

\[ V_{A} = \infty \]
\[ \beta = \infty \]
\[ \tau_{F} = 0.2 \text{ns} \]
\[ C_{\text{feedb.}} = 1 \text{pF} \]
\[ C_{bb} = 2 \text{pF} \]
b) 15 points
Find the mid-band amplifier parameters below

\[ V_{\text{out}}/V_{\text{in}} = \frac{12.73}{1}, \quad V_{\text{out}}/V_{\text{gen}} = 6.36 \]
\[ R_{\text{in, amplifier}} = 100 \Omega, \quad R_{\text{out, amplifier}} = 750 \Omega \]

Voltage gain of Q1 = \[\frac{1}{2}\]
Voltage gain of Q2 = 0.75
Voltage gain of Q3 = 3.47

Input impedance of Q2 = \[3.47 \Omega\]
Input impedance of Q3 = \[2.6 \Omega\]

\[ R_{\text{Q3}} = 100 \Omega / 750 \Omega = 88.24 \Omega \]
\[ R_{23} = 26 / 10 = 2.6 \Omega \]
\[ A_{\text{R3}} = \frac{R_{\text{Q3}}}{R_{23}} = 33.74 \]

\[ R_{\text{Q2}} = R_{23} = 2.6 \Omega \]
\[ R_{22} = 26 / 7.5 = 3.47 \Omega \]
\[ A_{\text{R2}} = \frac{3.47 \Omega}{2.6 / 3.47 \Omega} = 0.75 \]

\[ R_{\text{Q1}} = R_{22} = 3.47 \Omega \]
\[ R_{21} = 2.47 \Omega \]
\[ A_{\text{R1}} = \frac{1}{2} \]

\[ V_{\text{o1}}/V_{\text{i1}} = 12.73 \]
\[ V_{\text{o2}}/V_{\text{Q2}} = 12.73/6 \]
c) 5 points

Give the following transistor parameters

\[ r_{e1} = \frac{3.47}{\Omega} \quad r_{e2} = \frac{3.47}{\Omega} \quad r_{e3} = \frac{2.6}{\Omega} \]

\[ C_{be1} = \frac{58.64}{\text{nF}} \quad C_{be2} = \frac{58.64}{\text{nF}} \quad C_{be3} = \frac{77.92}{\text{nF}} \]

\[ C_{be} = 9 \mu\text{F} + C_{be\text{dep}}/2 \]

\[ \sqrt{0.2\mu\text{s}} \]
d) 20 points
High-frequency response. Calculate the gain-frequency characteristics of the amplifier: give an expression for $V_{out}(s)/V_{gen}(s)$.

**Give all pole frequencies and zero frequencies (in Hz).**

Do NOT use the node-by-node Miller Approximations. Use either MOTC or the results we have derived from Nodal analysis of single-stage amplifiers *This part should not take an hour, so choose your analytic method sensibly. Please note that the circuit has separation points; this simplifies analysis considerably.*
First Part

The emitter follower.

\[ Q_1 = C_{be1} \left[ (R_{em} / R_{b1}) (1 - A_{R1}) + \Re_{11 / R_{e2}} \right] \]

\[ + C_{es1} \cdot R_{em} / R_{b1} + C_{be2} \left[ \Re_{e2} / R_{e1} \right] \]

\[ = 59 \text{pF} \left( 25 \Omega + 1.73 \Omega \right) + 2 \text{pF} (50 \Omega) \]

\[ + 59 \text{pF} (1.73 \Omega) \]

\[ = 1.78 \text{ns} \]

\[ Q_2 = 50 \Omega \cdot C_{es1} C_{be2} \cdot 1.73 \Omega + 50 \Omega \cdot C_{es1} C_{be1} \left( \Re_{11 / R_{e2}} \right) \]

\[ + 1.73 \Omega \cdot C_{be2} C_{be1} \cdot 50 \Omega \]

\[ = 50 \Omega \cdot 1.73 \Omega \left[ C_{be1} + C_{es1} + C_{be1} C_{be2} + C_{be2} C_{es1} \right] \]

\[ = 3.17 \times 10^{-19} \text{ sec}^2 \]

Use SPA:

\[ f_{p1} = \frac{0.159}{Q_1} = 89.3 \text{ MHz} \]

\[ f_{p2} = \frac{0.159}{Q_2/Q_1} = 892 \text{ MHz} \quad \text{OK!} \]

\[ f_3 = \frac{0.159}{C_{be1} R_{e1}} = 782 \text{ MHz} \]

Part 2

\[ f_p = \frac{0.159}{2.65 \text{a} (80 \text{pF})} = 764 \text{ MHz} \]

Part 3

\[ f_p = \frac{0.159}{8.9 \text{a} (77 \text{pF})} = 258 \text{ MHz} \]
Draw a Bode Plot (Straight-line asymptotes) of the amplifier transfer function $V_{\text{out}}/V_{\text{gen}}$, labeling all pole and zero frequencies and labeling the slopes of all asymptotes. *To keep the plot manageable, include only the 2 lowest-frequency poles in the transfer function*.

- Poles: $4.6 \, \frac{89.3}{258} \, \frac{892}{764} \, \frac{782}{782} \, \text{MHz}$
Problem 2, 38 points

Part a 15 points

Derive an expression for \( \frac{V_{out}}{V_{gen}} \) in the form below...

\[
\frac{V_{out}}{V_{in}} = \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots}
\]

You must use nodal analysis.

\[
\frac{V_{out}}{V_{gen}} = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{1 + SR_1 C}{1 + S \frac{R_1}{R_2} \frac{1}{C}} \right)
\]

\[\delta \]

\[
\frac{V_0}{R_2} + \frac{V_0 - V_g}{R_1} + \frac{V_0 - V_g}{\frac{1}{sC}} = 0 \quad \frac{V_{gen} - V_0}{R_1} + \frac{V_{gen} - V_0}{\frac{1}{sC}} = 0
\]

\[\beta\]

\[
V_0 R_1 \frac{1}{sC} + (V_0 - V_g)(R_2 \frac{1}{sC}) + (V_0 - V_g)(R_1 R_2) = 0
\]

\[\gamma\]

\[
V_0 R_1 \frac{1}{sC} + V_0 R_2 \frac{1}{sC} - V_g R_2 \frac{1}{sC} + V_0 R_1 R_2 - V_g R_1 R_2 = 0
\]

\[\delta\]

\[
V_0 \left( R_1 \frac{1}{sC} + R_2 \frac{1}{sC} + R_1 R_2 \right) = V_g \left( R_2 \frac{1}{sC} + R_1 R_2 \right)
\]

\[\epsilon\]

\[
\frac{R_2 \frac{1}{sC} + R_1 R_2}{R_1 \frac{1}{sC} + R_2 \frac{1}{sC} + R_1 R_2} = \frac{R_2 + R_1 R_2 sC}{R_1 + R_2 + sC R_1 R_2}
\]

\[\zeta\]

\[
\frac{R_2 \left( 1 + S R_1 C \right)}{(R_1 + R_2) \left( 1 + S \frac{R_1 R_2}{R_1 + R_2} C \right)}
\]
Part b) 10 points
We have the following values: $R_1 = R_2 = 7 \, \text{k}\Omega$, $C = 0.3 \, \mu\text{F}$
Give all the pole frequencies and all the zero frequencies, using units of Hz.

pole frequencies = \underline{151 \, \text{Hz}}

zero frequencies = \underline{75.7 \, \text{Hz}}

\[ \frac{V_o}{V_g} = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{1 + SR_1C}{1 + S[R_1 + R_2]C} \right) \]

\[ = \frac{1}{2} \left( \frac{1 + 0.00215}{1 + 0.0010554} \right) \]

$R_{11}R_2 = 3500 \, \Omega$

$T_{\text{pole}} = 0.01055 \Rightarrow \frac{1}{2\pi T_p} = \underline{151 \, \text{Hz}}$

$T_{\text{zero}} = 0.00215 \Rightarrow \frac{1}{2\pi T_z} = \underline{75.7 \, \text{Hz}}$
Part c) 13 points
Now calculate Vout(t) for
V_{gen}(t)=5 \text{ Volts}\cdot U(t),
where U(t) is the unit step-function.
Graph on the chart below, giving units and labeling axes. Clearly label initial and final values, and charging time-constants.

\[
\frac{V_0}{s} = \frac{5(s)}{s} \left( \frac{1 + 0.0021s}{1 + 0.00105s} \right) = \frac{2.5}{s} \left( \frac{1 + 0.0021s}{1 + 0.00105s} \right)
\]

\[
= \frac{952 \cdot 2.5 \cdot (476 + s)}{476 \cdot (952 + s)} = \frac{5}{s} \left( \frac{s + 476}{s + 952} \right)
\]

\[
= \frac{A}{s} + \frac{B}{s + 952}
\]

\[
A = 5 \left( \frac{s + 476}{s + 952} \right)
\]

\[
A = 2.5
\]

\[
B = \frac{5}{s} \left( s + 476 \right)
\]

\[
B = 2.5
\]

\[
\frac{V_0}{s} = \frac{2.5 \cdot 2.5}{s + 952} = \text{Inverse LaPlace} \Rightarrow
\]

\[
\begin{bmatrix}
2.5u(t) + 2.5e^{-952t}
\end{bmatrix}
\]

\[
\begin{bmatrix}
2.5 + 2.5e^{-t/1.05\text{ms}}
\end{bmatrix}u(t)
\]
Initial Value = 5 V

decreasing at $e^{-t/1.05 \text{ ms}}$

Final Value = 2.5 V

2 points