Power Amplifiers; Part 1  Class A

Device Limitations
Large signal output match
Define efficiency, power-added efficiency
Class A operating conditions
Thermal resistance

We have studied the design of small-signal amplifiers
- The designs were based on small-signal S-parameters.
- The output was often conjugately matched to increase gain.

conjugate match: SS AC Load Line
\[ \text{Re}\{Z_L\} = \text{Re}\{Z_{OUT}\} \]
Power Amplifier Design 1

The small signal conjugate match leads to limitations on voltage and current swing. Not important for SS amps, but crucial for power amps.

\[ \Delta V \ll V_{DQ} \]
\[ \Delta I \ll I_{DQ} \]

Power amps require a large-signal design methodology: \( \Delta V \) and \( \Delta I \) are significant compared with \( V_{DQ} \) and \( I_{DQ} \).

Power Amp Objective: Get the largest \( \Delta V \) and \( \Delta I \) without:

1. Clipping – large saturation of gain;
   - distortion generated

2. Destroying the device.
   - avoid breakdown
   - \( I_{MAX} \) must be within device specs
   - \( P_{DISS} \) must not overheat the device
Device limitations and clipping.
Every device has maximum voltage and current limits.
Breakdown voltage:
- Electric field large enough to generate electron-hole pair $qV > E_{\text{gap}}$ of semiconductor.
- Electrons injected into channel from source or into collector from emitter are accelerated in high field.

- Collisions transfer energy to Si atoms. Electron is released, accelerates
- creates hole.
- Electron-hole pairs are then accelerated further generating more electrons, holes.
- This leads to rapid increase in current for voltages beyond breakdown

**AVALANCHE BREAKDOWN!!!**
Maximum Current

GaAs FET:
• $I_{DSS}$ for FET = $I_D@V_{GS} = 0$
• $I_D \propto qn_{\text{sat}}$
• $n \propto (V_{GS} - V_T)^m$
• also must avoid forward gate conduction on MESFET or PHEMT

Si MOSFET:
• Imax specified by foundry or manufacturer
• Also must avoid gate oxide breakdown

BJT: $I_{MAX}$ is limited by collector electric field profile

when mobile charge $\approx$ fixed charge,

$$E = \frac{q(N_D - n)x}{\varepsilon}$$

electric field $\rightarrow 0$
transit delay ↑
$f_T \downarrow$

We will discuss thermal limitations later. Heat generation will limit the operation of all device types
Power Amplifier Design 1

Conjugate Match Revisited.

We have learned that max. power transfer occurs when $R_L = R_{ds}$.
- true for small signal condition
  - no device limitations.

But what happens when we have limitations on voltage and current?

Suppose $V_{max} = 10V$, $I_{max} = 1A$, $R_{ds} = 100\Omega$

1. **Conjugate Match.**
   
   $R_L \parallel R_{ds} = 50\Omega = R_L'$
   
   $V_{OUT} = I_{max} \cdot R_L' = 50V$!
   
   Clearly, $I_{max}$ can’t be reached since $V_{max} = 10V$
   
   $P_{OUT} = \frac{(V_{max} / 2)^2}{2 R_L'} = \frac{25}{100} = \sqrt{4} W$
2. Load Line Match. \[ R_L = \frac{V_{\text{max}}}{I_{\text{max}}} = 10\,\Omega \]

\[ P_{\text{OUT}} = \frac{25}{20} = 1.25\,W \]

This uses the maximum capability of the device more realistically, improvement of 2-3 dB is typical.

Here you can see the large signal load line with slope $1/R_L$.

**Device I-V Curves**
We have now shown that different criteria are used for output matching a power amp than a small signal amp.

You may have noticed that the large signal load line doesn’t extend to $V_{DS} = 0$. To avoid excessive distortion, we must also take into account the “knee” voltage ($V_{Dsat}$ or $V_{CEsat}$). Clipping will occur if the drain voltage swing extends into the ohmic region of the device characteristic.

Thus, our definition of the large signal load line resistance must take this into account:

$$R_L = \frac{V_{BR} - V_{knee}}{I_{max}} \text{ for maximum voltage swing}$$

**Some additional PA concepts**

**Efficiency**

$$\eta = \frac{P_{OUT}}{P_{DC}} \times 100\%$$

**Power Dissipation**

$$P_D = P_{DC} - P_{OUT}$$

- Must be removed as heat.
- $P_D$ can also limit the maximum $P_{OUT}$
- $T_{max} = 150^\circ C$
Power Amplifier Design 1

Power-Added-Efficiency

\[ PAE = \frac{P_{OUT} - P_{IN}}{P_{DC}} \]

Gain should be at least 10dB to avoid significant reduction in PAE.

Efficiency is important because

1) PA’s are used for power
2) Wasted power must be removed as heat
3) Wasted power consumes batteries faster

Suppose Pout = 10 kW
(FM broadcast transmitter)

<table>
<thead>
<tr>
<th>( \eta ) (%)</th>
<th>( P_D ) (kW)</th>
<th>( P_{DC} ) (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1.1</td>
<td>11.1</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>100</td>
</tr>
</tbody>
</table>
Power Amplifier Design 1

First PA: Class A.  
- Most similar to small-signal amp.  
- $V_{DQ}$ and $I_{DQ}$ set so that amp is always on.  
- Conduction angle = \frac{"on" \ text term}{T} \cdot 2\pi = 2\pi

Case 1: resistive load. Bad idea for PA, but familiar.  
(we will refer later to this $V_{DC}$ as $V_{DC_1}$)

Neglecting $V_{knee}$, we find

$$R_L = \frac{V_{BR}}{I_{max}} = \frac{V_{DC}}{2ICQ}$$
**Power Amplifier Design 1**

\[ I_c(\theta) = I_{CQ} + I_m \sin \theta \quad (\theta = \omega t) \]

Maximum value of \( I_m = I_{CQ} \) (just begins to clip)

We can see that the average DC component is \( I_{CQ} \) and the fundamental component of current is \( I_m \), OR:

We can use Fourier integrals to determine \( P_{DC} \) and \( P_{OUT} \). The DC term \( (a_0) \) can be used to calculate power dissipation:

\[
P_{DC} = \frac{V_{DC}}{2\pi} \int_{0}^{2\pi} i_c(\theta) d\theta
\]

\[
= \frac{V_{DC}}{2\pi} \int_{0}^{2\pi} \left[ I_{CQ} + I_m \sin \theta \right] d\theta = V_{DC} I_{CQ}
\]

constant DC power, constant input current. You can use Fourier integrals to also find coefficients for fundamental and harmonics. \( T = 2\pi \)

\[
a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(x) dx
\]

\[
a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin(nx) dx
\]

\[
b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos(nx) dx
\]
Power Amplifier Design 1

Power at fundamental frequency $\omega$:  \( n = 1 \)

\[
a_1I_m = i_{OUT}(\omega) = \frac{1}{\pi} \int_{-\pi}^{\pi} I_m \sin^2 \theta \, d\theta = I_m
\]

\[
\left[ \int_{-\pi}^{\pi} \sin^2 \theta \, d\theta = \pi \right]
\]

\[
P_{OUT} = \frac{1}{2} \text{Re}\{V_m I_m^*\} = \frac{1}{2} V_m I_m
\]

\[
V_m = \frac{V_{DC}}{2}, \quad I_m = \frac{V_m}{R_L}
\]

Thus:

\[
P_{out} = \frac{1}{2} \frac{V_{DC1}}{2} \frac{I_{max}}{2} = \frac{V_{DC1} I_{max}}{8}
\]

and,

\[
P_{out} = \frac{I_m^2 R_L}{2} = \frac{I_{CQ}^2 R_L}{2} = \frac{V_{DC}^2}{8 R_L}
\]

since \( I_{max} = 2I_{CQ} \), \( V_{BR} = V_{DC} \), \( R_L = \frac{V_{BR}}{I_{max}} \)

In terms of device limitations, the maximum output power is

\[
P_{OUT} = \frac{1}{2} \frac{I_{max}^2}{4} \frac{V_{BR}}{I_{max}} = \frac{V_{BR} I_{max}}{8}
\]
what about harmonics? \( n > 1? \)

\[
I_m a_n = \frac{1}{n} \int_{-\pi}^{\pi} I_m \sin \theta \sin n \theta \, d\theta = 0
\]

with no nonlinearity in \( I_c(\theta) \), no harmonic currents.

**Efficiency:**

\[
P_{DC} = V_{DC} I_{CQ} = \frac{V_{DC}^2}{2R_L}
\]

\[
I_{CQ} = \frac{V_{DC}}{2R_L}
\]

\[
\eta = \frac{P_{OUT}}{P_{DC}} = \frac{1}{4} \quad (25\%) \quad \text{max!}
\]

If we AC couple the load resistor:

We have current divider.

Efficiency can be **much** worse.

so, **bad idea** for any power application
For RF applications we can do much better.

Case 2: inductive load.

\[ V_{DC} = V_{DC2} = V_{DC1}/2 \]

now, \( R_L = \frac{V_{BR}}{I_{max}} = \frac{V_{DC}}{I_{CQ}} \) (no change - assume same device limitations)

\[
P_{OUT} = \frac{V_{DC2}^2}{2R_L} = \frac{V_{DC2}I_{max}}{4} = \frac{V_{BR}I_{max}}{8} \quad \text{same as case 1}
\]

but: \( P_{DC2} = V_{DC2}I_{CQ} = \frac{V_{DC2}^2}{R_L} = \frac{1}{2} P_{DC1} \)

\[ \eta = \frac{1}{2} \times (50\%) \]
we have \( P_{\text{out}}(\text{case 2}) = P_{\text{out}}(\text{case 1}) \)

but \( P_{\text{DC}}(\text{case 2}) = \frac{1}{2} P_{\text{DC}}(\text{case 1}) \)

because: \( V_{\text{DC}}(\text{case 2}) = \frac{1}{2} V_{\text{DC}}(\text{case 1}) \)

Thus, the inductive feed allows the amplifier to produce the same output power with half the supply voltage. This also applies to tuned amplifiers.

So: 50\% of \( P_{\text{DC}} \) can be converted into useful output power if we swing rail to rail.

Alternatively, we can get 2 times more \( P_{\text{OUT}} \) for the same \( V_{\text{DC}} \) if the device has sufficient breakdown voltage. (twice the voltage; twice the \( R_L \))

\[
V_m = V_{\text{DC}2} \quad R_L = \frac{2V_{\text{DC}2}}{I_{\text{max}}} \\
P_{\text{out}} = \frac{V_{\text{DC}2}}{2R_L} = \frac{V_{\text{DC}2}I_{\text{max}}}{4} = \frac{V_{\text{DC}1}I_{\text{max}}}{2}
\]
But, what about a more typical situation where we have a large range of signal powers to be amplified?

How much power gets dissipated in the device?

\[ P_D = P_{DC} - P_{OUT} \]  
(heat in transistor)

We have doubled the efficiency (now 50%), but still have maximum power dissipated in device at zero input.

undesirable for power amp where high powers may be required.

ok for driver stage –low power; highly linear.
Thermal Limitations

\[ T_j \leq 150^\circ C \]

why?

reliability failure mechanisms are strongly temperature dependent

\[ MTTF \propto e^{-Ea/KT} \]

\[ Ea = \text{activation energy} \]

Thermal model.

\[ T_j < 150^\circ C \]

\[ T_C \]

\[ R_{TH_{j-c}} \]

package

heat sink

\[ R_{TH_{c-HS}} \]

\[ R_{TH_{HS-air}} \]

\[ T_{HS} \]

\[ T_{C} \]

\[ T_{Ambient} \]

\[ air \ T = 25^\circ C \]
Thermal Resistance relates $T$ to power dissipation (like ohm’s law for heat)

$$T_J = R_{TH,J-C} \cdot P_D + T_C$$

$$= (R_{TH,J-C} + R_{TH,C-HS} + R_{TH,HS-A}) P_D + T_A$$

$R_{TH}$ has units of °C/watt
Class A Power Amplifier Summary

1. Device limitations ($V_{BR}$ and $I_{MAX}$ and $T_{MAX}$) constrain the design for a PA.

$$R_{opt} = \frac{V_{BR} - V_{knee}}{I_{MAX}}$$

Large signal load line match

2. $T_{MAX} = 150^\circ C$ for reliable operation

3. Waste power, $P_D = P_{DC} - P_{OUT}$ is converted to heat. Must be removed

4. Efficiency, $\frac{P_{OUT}}{P_{DC}}$, or $PAE$, $\frac{P_{OUT} - P_{IN}}{P_{DC}}$ are critical for PAs.

For Class A with same output power, same $R_L$:

<table>
<thead>
<tr>
<th></th>
<th>Resistive DC feed (1)</th>
<th>Inductive feed (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{DC}$</td>
<td>$V_{DC1}$</td>
<td>$V_{DC2} = V_{DC1}/2$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>25% max</td>
<td>50% max</td>
</tr>
<tr>
<td>$P_{OUT}$</td>
<td>$\frac{V_{DC1}^2}{8R_L}$</td>
<td>$\frac{V_{DC2}^2}{2R_L} = \frac{V_{DC1}^2}{8R_L}$</td>
</tr>
<tr>
<td>$V_{DC\max}$</td>
<td>up to $V_{BR}$</td>
<td>up to $V_{BR}/2$</td>
</tr>
<tr>
<td>$R_L = \frac{V_{BR}}{I_{MAX}} = \frac{V_{DC1}}{2I_{CQ}}$</td>
<td>$\frac{V_{BR}}{I_{MAX}} = \frac{V_{DC2}}{I_{CQ}} = \frac{V_{DC1}}{2I_{CQ}}$</td>
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