4.3 The Ar ion laser

The argon-ion laser can provide powerful CW visible coherent radiation of several watts. The laser operation is achieved as follows: The Ar atoms are ionized by electron collisions in a high current electrical discharge. Further multiple collisions with electrons excite the argon ion, Ar$^+$, to a group of 4$p$ energy levels $\sim$35 eV above the atomic ground state as shown in Figure 4Q3. Thus a population inversion forms between the 4$p$ levels and the 4$s$ level which is about 33.5 eV above the Ar atom ground level. Consequently, the stimulated radiation from the 4$p$ levels down to the 4$s$ level contains a series of wavelengths ranging from 351.1 nm to 528.7 nm. Most of the power however is concentrated, approximately equally, in the 488 and 514.5 nm emissions. The Ar$^+$ ion at the lower laser level (4$s$) returns to its neutral atomic ground state via a radiative decay to the Ar$^+$ ion ground state, followed by recombination with an electron to form the neutral atom. The Ar atom is then ready for "pumping" again.

a Calculate the energy drop involved in the excited Ar$^+$ ion when it is stimulated to emit the radiation at 514.5 nm.

b The Doppler broadened linewidth of the 514.5 nm radiation is about 3500 MHz ($\Delta \nu$) and is between the half-intensity points.

(i) Calculate the Doppler broadened width in the wavelength; $\Delta \lambda$

(ii) Estimate the operation temperature of the argon ion gas; give the temperature in °C

c In a particular argon-ion laser the discharge tube, made of Beryllia (Beryllium Oxide), is 30 cm long and has a bore of 3 mm in diameter. When the laser is operated with a current of 40 A at 200 V dc, the total output power in the emitted radiation is 3 W. What is the efficiency of the laser?

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**Figure 4Q3**

Given wavelength $\lambda_o = 514.5$ nm or $514.5 \times 10^{-9}$ m, the change in the energy is

$$\Delta E = h c / \lambda_o = (6.626 \times 10^{-34} \text{ J s})(3.0 \times 10^8 \text{ m s}^{-1})/(514.5 \times 10^{-9} \text{ m})$$
\[ i.e. \quad \Delta E = 3.86 \times 10^{-19} \text{ J or } 2.41 \text{ eV} \]

b The center frequency of the lasing emission is
\[ v_o = \frac{c}{\lambda_o} = \frac{(3.0 \times 10^8 \text{ m s}^{-1})}{(514.5 \times 10^{-9} \text{ m})} = 5.831 \times 10^{14} \text{ s}^{-1} \]

The relationship between wavelength and frequency linewidth is
\[ \Delta \lambda = \frac{\lambda_o}{v_o} \Delta v = \frac{(514.5 \times 10^{-9} \text{ m})}{(5.831 \times 10^{14} \text{ s}^{-1})} (3500 \times 10^6 \text{ s}^{-1}) \]

\[ \therefore \quad \Delta \lambda = 3.09 \times 10^{-12} \text{ m or } 0.00309 \text{ nm} \]

The frequency linewidth \( \Delta v = 3500 \times 10^6 \text{ s}^{-1} \) is due to Doppler broadening. From \( \Delta v \) we can find the operating temperature by using,
\[ \Delta v = 2v_o \sqrt{\frac{2k_BT \ln(2)}{Mc^2}} \]

in which \( M \) is the mass of the Ar ion, that is, \( M = 39.95 \text{ g mol}^{-1} \) or \( 6.636 \times 10^{-26} \text{ kg} \).

\[ \therefore \quad T = \frac{Mc^2}{8k_B \ln(2)} \left( \frac{\Delta v}{v_o} \right)^2 = 2810 \text{ K or } 2537 \text{ °C} \]

which is the gas temperature. The tube gets very hot. Indeed, beryllia (BeO) is used because the operating temperature of the argon ion laser is high and beryllia has a high melting temperature and good thermal conductivity.

c The output power from the laser is 3 W. The input power is \( IV \) or \( (40 \text{ A})(200 \text{ V}) \).

\[ \text{Efficiency (\%)} = \frac{\text{Output Power from Laser}}{\text{Input Power into Laser}} \times 100\% = \frac{3 \text{ W}}{(40 \text{ A})(200 \text{ V})} \times 100\% = 0.0375\% \]

4.4 Einstein coefficients and critical photon concentration \( \rho(h \nu) \) is the energy of the electromagnetic radiation per unit volume per unit frequency due to photons with energy \( h \nu = E_2 - E_1 \). Suppose that there are \( n_{ph} \) photons per unit volume. Each has an energy \( h \nu \). The frequency range of emission is \( \Delta \nu \). Then,
\[ \rho(h \nu) = \frac{n_{ph} h \nu}{\Delta \nu} \]

Consider the Ar ion laser system. Given that the emission wavelength is at 488 nm and the linewidth in the output spectrum is \( 5 \times 10^9 \text{ Hz} \) (between half intensity points, that is \( \Delta \nu = 2 \times 5 \times 10^9 \text{ Hz} \)), \textit{estimate} the photon concentration necessary to achieve more stimulated emission than spontaneous emission.

**Solution**

Suppose that there are \( n_{ph} \) photons per unit volume. Each has an energy \( h \nu \). The frequency range of emission is \( \Delta \nu \) as is the linewidth between half intensity points. Then,
\[
\rho(h\nu) = \frac{n_{ph} h\nu}{\Delta\nu}
\]

Now,
\[
\frac{R_2^\text{(stim)}}{R_2^\text{(spon)}} = \frac{c^3}{8\pi h}\rho(h\nu) > 1
\]
\[
\therefore \rho(h\nu) > \frac{8\pi h\nu^3}{c^3} = \frac{8\pi h}{\lambda^3} = \frac{8\pi(6.62 \times 10^{-34} \text{ J s})}{(488 \times 10^{-9} \text{ m})^3} = 1.43 \times 10^{-13} \text{ J s m}^{-3}
\]
\[
\therefore n_{ph} = \frac{\Delta\nu \rho(h\nu)}{h\nu} = \frac{(2 \times 5 \times 10^9 \text{ s}^{-1})(1.43 \times 10^{-13} \text{ J s m}^{-3})}{(6.62 \times 10^{-34} \text{ J s})(3 \times 10^5 \text{ m s}^{-1})}
\]
\[
\therefore n_{ph} = 3.5 \times 10^{15} \text{ Photons m}^{-3}
\]

Note that this represents the critical photon concentration for stimulated emission to just exceed spontaneous emission in the absence of any photon losses. It does not represent the photon concentration for laser operation. In practice, the photon concentration is much greater during laser operation.

### 4.6 Threshold gain and population inversion

**a** Consider a He-Ne gas laser operating at 632.8 nm. The tube length \( L = 40 \text{ cm} \), tube diameter is 1.5 mm and mirror reflectances are approximately 99.9\% and 98\%. The linewidth \( \Delta \nu = 1.5 \text{ GHz} \), the loss coefficient is \( \gamma = 0.05 \text{ m}^{-1} \), spontaneous decay time constant \( \tau_{sp} = 1/A_{21} \approx 300 \text{ ns} \), \( n \approx 1 \). What is the threshold gain and population inversion?

**b** Consider a 488 nm Ar-ion gas laser. The tube length \( L = 1 \text{ m} \), tube mirror reflectances are approximately 99.9\% and 95\%. The linewidth \( \Delta \nu = 3 \text{ GHz} \), the loss coefficient is \( \gamma = 0.1 \text{ m}^{-1} \), spontaneous decay time constant \( \tau_{sp} = 1/A_{21} \approx 10 \text{ ns} \), \( n = 1 \). What is the threshold population inversion?

**c** Consider a semiconductor laser operating at \( \lambda_o = 870 \text{ nm} \) with a GaAs laser cavity with cleaved facets. The cavity length is 50 \( \mu \text{m} \). The refractive index \( n \) of GaAs is 3.6. The loss coefficient \( \gamma \) at normal temperatures is of the order of \( \sim 10 \text{ cm}^{-1} \). Calculate the required threshold gain. What is your conclusion?

[Note: \( \gamma \) depends on a number of factors including the injected carrier concentration and at best the above calculation is an estimate. We cannot simply calculate the threshold population inversion, \( \Delta N_{th} = \Delta n_{th} \), from Eq. (9) in Section 4.9 which does not apply for a number of reasons; see Section 7.2 in P. Bhattacharya, *Semiconductor Optoelectronic Devices, Second Edition* (Prentice-Hall, New York, 1993).

**Solution**

**a**
\[
g_{th} = \gamma - \frac{1}{2L}\ln(R_1R_2) = 0.05 \text{ m}^{-1} - \frac{1}{2(0.4 \text{ m})} \ln(1 \times 0.99) = 0.077 \text{ m}^{-1}
\]

The emission frequency \( \nu_o = c/\lambda_o = (3 \times 10^8 \text{ m s}^{-1})/(632.8 \times 10^{-9} \text{ m}) = 4.74 \times 10^{15} \text{ s}^{-1} \). From laser characteristics,
\[ \Delta N_{th} = g_{th} \frac{8\pi \nu^2 \eta^2 \tau_{sp} \Delta \nu}{c^2} \]

and

\[ = (0.077 \text{ m}^{-1}) \frac{8\pi (4.74 \times 10^{14} \text{ s}^{-1})^2 (1)^2 (300 \times 10^{-9} \text{ s})(1.5 \times 10^9 \text{ s}^{-1})}{(3 \times 10^8 \text{ m s}^{-1})^2} \]

\[ = 2.1 \times 10^{15} \text{ m}^{-3}. \]

Note: The number of Ne atoms per unit volume \( n_{Ne} \) can be found from the Gas law using the partial pressure of Ne:

\[ P_{Ne}V = \frac{N_{Ne}}{N_A}RT \]

\[ \therefore n_{Ne} = \frac{N_{Ne}}{V} = \frac{N_A P_{Ne}}{RT} \left( \frac{6.02 \times 10^{23} \text{ mol}^{-1})(\frac{1}{6} \text{ 2.5 torr} \times 10^5 \frac{\text{N m}^2}{\text{torr}})}{(8.315 \times 10^{23} \text{ J K}^{-1} \text{mol}^{-1})(300 \text{ K})} \right) \]

\[ = 1.3 \times 10^{22} \text{ m}^{-3}. \]

b \[ g_{th} = \gamma - \frac{1}{2L} \ln(R_1 R_2) = 0.1 \text{ m}^{-1} - \frac{1}{2(1 \text{ m})} \ln(0.999 \times 0.95) = 0.126 \text{ m}^{-1}. \]

The emission frequency \( \nu_o = \frac{c}{\lambda_o} = (3 \times 10^8 \text{ ms}^{-1})/(488 \times 10^{-9} \text{ m}) = 6.14 \times 10^{15} \text{ s}^{-1}. \) From laser characteristics,

\[ \Delta N_{th} = g_{th} \frac{8\pi \nu_o^2 \eta^2 \tau_{sp} \Delta \nu}{c^2} \]

and

\[ = (0.126 \text{ m}^{-1}) \frac{8\pi (6.14 \times 10^{14} \text{ s}^{-1})^2 (1)^2 (10 \times 10^{-9} \text{ s})(3 \times 10^9 \text{ s}^{-1})}{(3 \times 10^8 \text{ m s}^{-1})^2} \]

\[ = 4.0 \times 10^{14} \text{ m}^{-3}. \]

c \[ g_{th} = \gamma - \frac{1}{2L} \ln(R_1 R_2) = 1000 \text{ m}^{-1} - \frac{1}{2(50 \times 10^{-6} \text{ m})} \ln(0.32 \times 0.32) \]

\[ = 2.4 \times 10^{4} \text{ m}^{-1}. \]

Substantially larger than a gas laser!

4.11 InGaAsP-InP Laser Consider a InGaAsP-InP laser diode which has an optical cavity of length 250 microns. The peak radiation is at 1550 nm and the refractive index of InGaAsP is 4.

The optical gain bandwidth (as measured between half intensity points) will normally depend on the pumping current (diode current) but for this problem assume that it is 2 nm.

a What is the mode integer \( m \) of the peak radiation?

b What is the separation between the modes of the cavity?

c How many modes are there in the cavity?

d What is the reflection coefficient and reflectance at the ends of the optical cavity (faces of the InGaAsP crystal)?
What determines the angular divergence of the laser beam emerging from the optical cavity?

**Solution**

a. The wavelength $\lambda$ of a cavity mode and length $L$ are related by

$$m \frac{\lambda}{2n} = L$$

so that

$$m = \frac{2nL}{\lambda} = \frac{2(4)(250 \times 10^{-6})}{(1550 \times 10^{-9})} = 1290.3.$$  

When $m = 1290$, $\lambda = 2nL/m = 1550.39$ nm so that the peak radiation has $m = 1290$.

b. Mode separation is given by,

$$\Delta \lambda_m = \frac{\lambda^2}{2nL} = \frac{(1550 \times 10^{-9})^2}{2(4)(250 \times 10^{-6})} = 1.20 \text{ nm}$$

c. The linewidth is 2 nm

Let the optical linewidth $\Delta \lambda$ be between $\lambda_1$ and $\lambda_2$. Then $\lambda_1 = \lambda - (\frac{1}{2})\Delta \lambda = 1549$ nm and $\lambda_2 = \lambda + (\frac{1}{2})\Delta \lambda = 1551$ nm and the mode numbers corresponding to these are

$$m_1 = \frac{2nL}{\lambda_1} = \frac{2(4)(250 \times 10^{-6})}{(1549 \times 10^{-9})} = 1291.15$$

$$m_2 = \frac{2nL}{\lambda_2} = \frac{2(4)(250 \times 10^{-6})}{(1551 \times 10^{-9})} = 1289.49$$

Now $m$ must be an integer and the corresponding wavelength must fit into the optical gain curve.

Taking $m = 1290$, gives $\lambda = 2nL/m = 1550.39$ nm; within optical gain 1549 – 1551 nm

Taking $m = 1291$, gives $\lambda = 2nL/m = 1549.18$ nm; within optical gain 1549 – 1551 nm

Taking $m = 1289$, gives $\lambda = 2nL/m = 1551.59$ nm; just outside optical gain 1549 – 1551 nm

There are 2 modes.

d. $r = (n - 1)/(n + 1) = (4 - 1)/(4 + 1) = 0.6$

$R = r^2 = 0.36$ or 36%.

e. Diffraction at the active region cavity end.