1. Short review for midterm (Q&A)
2. Derivation of Lagrangian equations of motion (EOMs):
   • Review of holonomic / nonholonomic definitions
   • The “Lagrangian”: \[ \mathcal{L} = T^* - V \]
     - Kinetic co-energy
     - Potential energy
   • Lagrange’s equations
     \[ \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = \mathbf{\Xi}_i \]

Notices:
• Midterm date: Thursday, May 10 (in class)
• Office hours today in 3120A (lab), 3:30-4:30pm
• One-sided single sheet of notes (8.5 x 11 inches) for midterm
• HW 3 due tonight; solutions on web after 7pm
Topics for particular focus:
1. **Terminology**: manipulator, end effector, redundancy, singularity, ...
2. **Workspace**: Reachable vs Dexterous
3. **Kinematics**: Forward and velocity kinematics
4. **The Jacobian matrix**: Geometric calculation of $J$
5. **Mechanical impedance**: Definition of; inertia; reflected inertia
6. **Work balance**: gear ratios; motor coefficients
7. **Motor dynamics model**: standard equations and block diagram
8. **Block diagrams**: understanding, creating and solving algebraically
9. **First- and second-order time response**
10. **Friction vs damping**: effect of; estimating parameters
11. **SISO control**: P, PD, PID; PID tuning methods; feedforward
12. **Control law partitioning**: “cancel out” all dynamics except mass (or inertia)
13. **Wheeled vehicles**: mobility, steerability, maneuv.; wheel types; ICR; ...
14. **Holonomic vs nonholonomic**: instantaneous DOFs vs long-term GC’s
15. **Rotation matrices**: body frame vs inertial reference frame velocities

Notices:
- HW 2 due MONDAY (5pm).
- HW 3 (practice quiz) due: ?
- Midterm date: Thursday, May 10 (in class)
Holonomic / Nonholonomic

- **Nonholonomic** when:
  - Accessible configuration space has higher dimension than the accessible velocity space.
  i.e.,
  - More generalized variables required to describe the long-term configurations that are achievable…
  - …than Degrees of Freedom (DOFs) for local motion, instantaneously

\[
\# GC = \xi_j > \delta \xi_k = \# DOF \quad \leftrightarrow \quad \text{Nonholonomic}
\]

\[
\# GC = \xi_j = \delta \xi_k = \# DOF \quad \leftrightarrow \quad \text{Holonomic}
\]
Wheeled Robots

Common terminology differs from pure definition:
• “kinematic posture”: x, y, φ for body.
  – If we produce appropriate control laws, we can create an “apparent” holonomic system
    • The internal variables (wheel rotations) have not gone away
    • BUT, they are effectively “hidden” within a set of feedback laws…
  – 3 GC’s (longterm);
  – 3 “DDOF” (instantaneous DOF, which we will just call “DOF”)

If we care about actual wheel positions, then:
• “configuration posture”: x, y, φ, q₁, q₂, q₃ (6 GC’s)
  – Omnibot from lab has an integrable constraint:
    \[ \phi_b = \frac{-r_w}{3L} (q_1 + q_2 + q_3) \]
  – 5 GC’s (longterm), since φ is a known function (above) of q₁, q₂, and q₃
  – 3 DOF instantaneous (dq₁/dt, dq₂/dt, dq₃/dt)
Lagrangian Approach

1. Define generalized coordinates that are:
   – Complete
   – Independent
   \( \xi_j \) (\( j = 1, 2, 3, \ldots \))
   *(For now, we will also assume system is holonomic…)*

2. Force-dynamics relationship requires:

\[
\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i
\]

where:

\[
\mathcal{L} = T^* - V
\]
Lagrangian Approach

ξₗ (j = 1, 2, 3, …)

\[
\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i
\]

\[
\mathcal{L} = T^* - V
\]
Lagrangian Approach

Little “$\xi_i$” is the set of DOF. Since we assume system is holonomic, there are the same number of GCs and DOFs.

$\xi_j$ (j = 1, 2, 3, …)

Big “$X_i$” represents all non-conservative forces, for each DOF. This includes both the ACTUATION and LOSSES (due to damping, most typically).

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i$$

Fancy “$\mathcal{L}$” is the “Lagrangian”

$$\mathcal{L} = T^* - V$$

$T^*$ is kinetic CO-ENERGY

$V$ is potential energy
Kinetic Co-energy

Usually, kinetic energy is identical to kinetic “co-energy”.

\[ T = \sum_i \frac{1}{2} m_i v_i^2 \]

For a rigid rotating body:

\[ T = \frac{1}{2} J \dot{\theta}^2 \]
Note: That said, just assume $T^* = T$ in ECE/ME 179d...
Relativistic (and other unusual) Physics

Note: That said, just assume $T^*=T$ in ECE/ME 179d...
Simple, Passive Pendulum Example

\[ T^* = \frac{1}{2} m u \cdot u = \frac{1}{2} m (L \dot{\theta})^2 \]

\[ \begin{align*}
V_1 &= mgh_1 = -mgL \cos \theta \\
V_2 &= mgh_2 = mg(L-L \cos \theta) 
\end{align*} \]

Does it matter where we define our fixed (inertial) reference frame?
Simple, Passive Pendulum Example
Spring-Damper Pendulum Example

\[ m L_m \ddot{\theta} + mg L_m \sin \theta + k L_k \dot{\theta} = -L_b b \dot{\theta} \]

or, write as:

\[ m L_m \ddot{\theta} + b L_b \dot{\theta} + k L_k \dot{\theta} + mg L_m \sin \theta = 0 \]
Spring-Damper Pendulum Example
Two-Link Pendulum ("Walker") Example

\[ L = L_a + L_b \]
\[ L_c = L_b \]
Two-Link Pendulum ("Walker") Example