1. Review Lagrangian Method (for EOM):

\[ \mathcal{L} = T^* - V \]

\[ \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i \]

2. Examples

3. Use of Relative vs Absolute Coordinates

4. Non-conservative (damping) term “tricks”

Notices:
- Midterm was technically “Lecture 12”.
- No labs until:
  - FRIDAY, June 1 and MONDAY, June 4
  - Lab 6 (Segway-style car)

Lagrangian Approach: Recall

Little “\( \xi \)” is the set of DOF. Since we assume system is holonomic, there are the same number of GCs and DOFs.

\( \xi_j \) (\( j = 1, 2, 3, \ldots \))

Big “\( \Xi \)” represents all non-conservative forces, for each DOF. This includes both the ACTUATION and LOSSES (due to damping, most typically).

\[ \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i \]

Fancy “L” is the “Lagrangian”

\[ \mathcal{L} = T^* - V \]

\( T^* \) is kinetic CO-ENERGY

\( V \) is potential energy
Lagrangian Approach: Procedure

0. Assume system is **holonomic**:

\[ \# GC = \xi_j = \delta \xi_k = \# DOF \leftrightarrow \text{Holonomic} \]

1. Pick **generalized coordinates** (DOFs) that are:
   - **Complete**
   - **Independent**
     \( (j = 1, 2, 3, \ldots) \)

   - Possible GC set(s) **might not be unique**!
   - **Any** independent and complete set is OK.
   - Either **absolute or relative** coords is OK.

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Lagrangian Approach: Procedure

2. Define kinetic (co-)energy, \( T \). (Technically, \( T^* \))

**Kinetic energy** includes the sum of “one-half mass time velocity-squared” for every particle:

\[
T = \sum_{i} \frac{1}{2} m_i v_i^2
\]

For a **rigid rotating body**, we can use moment of inertia, \( J \):

\[
T = \frac{1}{2} J \dot{\theta}^2
\]

(Note: We are ignoring electrical kinetic or potential energy here, for now and focusing on **MECHANICAL energy**.)
Lagrangian Approach: Procedure

3. Define potential energy, V.

**Potential energy** can be stored in a spring:

\[ V_{spring} = \frac{1}{2} k (x - x_o)^2 \]

...and also includes any “mgh” of particles in a gravitational field:

\[ V_{gravity} = mgh \]

Note: “h” is relative to any arbitrary (but fixed) inertial reference frame.

4. Lagrangian is **kinetic minus potential** energy:

\[ \mathcal{L} = T^* - V \]

5. For each GC (DOF), solve for an equation of motion:

\[ \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\xi}_i} \right) - \frac{\partial \mathcal{L}}{\partial \xi_i} = \Xi_i \]

Note: A system with **N generalized coordinates** (GCs) will have **N equations of motion**.
Spring-Damper Pendulum Example

One DOF. One EOM. Final answer is below:

\[ m l_m \ddot{\theta} + m g l \sin \theta + k l_b \dot{\theta}^2 = -l_b^2 \dot{\theta} \]

or, write as:

\[ m l_m \ddot{\theta} + b l_b \dot{\theta}^2 + k l_b l_b \dot{\theta}^2 + m g l \sin \theta = 0 \]

Spring-Damper Pendulum: Derivation

- GC(s) ?
- T ?
- V ?
- L ?
Spring-Damper Pendulum: Derivation

• Now solve for EOM(s):

Two-Link Pendulum ("Walker") Example

\[ L = L_a + L_b \]
\[ L_c = L_b \]
Two-Link Pendulum (“Walker”) Example

• \textbf{GC(s) ?} Let’s use \textit{relative theta 2} here...

• \textbf{T ?}

• \textbf{V ?}

• \textbf{L ?}

Two-Link Pendulum (“Walker”) Example

• Set-up form to solve EOM:

• Be careful for non-conservative term(s): [One torque input, at “elbow”]
Relative vs Absolute Coordinates

• As mentioned earlier, valid GCs can be either absolute (wrt an inertial reference frame) or relative.

• The non-conservative terms must be written to be compatible with the choice of absolute vs relative coordinates!

• Example: For the acrobot (relative coords), tau appears ONLY in the “theta 2” EOM!
Example: Two-cart spring system

Choice of GCs is NOT UNIQUE

Example: Two-cart spring system
Example: Two-cart spring system

Example: Acrobot (Rel vs Abs)

Theta 2 can be relative (as shown) OR absolute.
Example: Acrobot (Rel vs Abs)
Example: “Segway” style inverted pendulum

Example: “Segway” style inverted pendulum
Example: “Segway” style inverted pendulum