% Example of AUTOMATED GENERATION OF LAGRANGIAN EOM
%
% Note: This file requires the shareware function "fulldiff.m"
% to do chain rule for time derivatives.
%
% Example system:
%                   ____----|                               | g (gravity)
%                           |---| b (damper)               \|/
%                           |----                        |
%                          | [m] (mass) - - - - -
%                          | <
%                          | (constant > k (spring) dm (rel displ of mass)
%                          | arm length) <
%                          | La ___---- - - - - -
%                          | ___---- alpha, tau (acting wrt inertial frame on alpha)
%                          |---- - - - - - -

% Process
% ========

% 0. Clear workspace (desirable):
clear all
format compact % more readable when single-spaced

% 1. Declare constant and variable symbolically.
syms m k b alpha dm g La tau

% 2. Define INDEPENDENT and COMPLETE set of Generalized Coordinates.
GC = {alpha, dm} % note CURLY BRACKET notation here and elsewhere!!

% 3. Define any DEPENDENT variables that are convenient. (For example,
% we may wish to define potential energy in terms of absolute height.)
xm = La*cos(alpha) % absolute x position of center of mass (COM)
ym = La*sin(alpha) + dm % absolute height of COM

% 4. Define TIME DERIVATIVES of the dependent variables:
dxm = fulldiff(xm,GC) % input arg GC are the independent vars of time
dym = fulldiff(ym,GC)
ddm = fulldiff(dm,GC)
for n=1:length(GC)
    dGC(n) = fulldiff(GC(n),GC)
end

% 5. Write expressions for...
%    a. T* (kinetic energy):
Tstar = (1/2)*m*(dxm^2 + dym^2)
%    b. V (potential energy):
V = m*g*ym + (1/2)*k*dm^2
%    c. L = Tstar - V, except "L" might already be defined previously
Lagran = Tstar - V
6. For each GC, calculate LHS of EOM:
   \[ \frac{d}{dt}(\text{partial } L \text{ wrt } d\xi_n) - (\text{partial } L \text{ wrt } \xi_n) \]
   for \( n=1:\text{length}(\text{GC}) \)
   
   ```matlab
   LHS{n} = fulldiff(diff(Lagran,dGC{n}),GC) - diff(Lagran,GC{n})
   end
   ```

7. Define NON-CONSERVATIVE forces (or torque) for RHS of each EOM:
   \( \text{RHS}(1) = \tau; \) % absolute torque on alpha DOF
   \( \text{RHS}(2) = -b*\ddm; \) % damping force; \( \ddm \) is a relative coordinate

8. Print equations nicely on screen!
   for \( n=1:\text{length}(\text{GC}) \)
   
   ```matlab
   fprintf('
\textbf{Equation }\%d:\n\textbf{\n}
   pretty(simplify(LHS{n})) % simplify, then print in "pretty" way...
   fprintf(' \textbf{\text{equals\n}}
   pretty(simplify(RHS{n}))
   latex_output{n} = [latex(simplify(LHS{n})), ' = ',...
   latex(simplify(RHS{n}))]; % to print in LATEX
   end
   ```

Now, the contents of \texttt{latex\_output{1}} and \texttt{latex\_output{2}} can be
% copied into your favorite LaTeX editor. To display nicely will
% still require some "hand editing" without latex, however!
Auto-Generation of Equations of Motion
Example, in $\text{L}_{\text{TEX}}$

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Abstract

This example equation comes from the output to m-file eom_4bar_example.

1 MATLAB auto-formatting for EOMs

By default, MATLAB’s \texttt{latex} function tends to display many variables in a non-\textit{italic} Roman font which is not pleasing to the eye, nor easy to read. Below are our two equations of motion for the 4-bar spring-mass-damper system, for example.

The first EOM MATLAB generates is:

$$\La\ m\ (\La\ d2alpha + d2dm\ \cos(\alpha) + g\ \cos(\alpha)) = \tau$$

Similarly, the second equation output from MATLAB is:

$$-\La\ m\ \sin(\alpha)\ \left\{ dalpha \right\}^2 + dm\ k + d2dm\ m + g\ m + \La\ d2alpha\ m\ \cos(\alpha) = -b\ ddm$$

In latex formatting, this becomes:

$$La m \ (La \ d2alpha + d2dm \ cos(alpha) + g \ cos(alpha)) = tau \quad (1)$$

Similarly, the second equation output from MATLAB is:

$$- La m \ \sin(alpha) \ dalpha^2 + dm \ k + d2dm \ m + g \ m + La \ d2alpha \ m \ cos(alpha) = -b \ ddm \quad (2)$$
2 Hand-editing within \LaTeX

Making sense of the equations requires some hand editing. The equations are more easily understood when re-formatted by hand as shown below.

An improved \LaTeX{} expression for the first EOM is given below:

\[
L_a m \left( L_a \ddot{\alpha} + (\ddot{d}_m + g) \cos(\alpha) \right) = \tau
\]

This becomes:

\[
L_a m \left( \ddot{\alpha} + \ddot{d}_m + g \right) \cos(\alpha) = \tau \tag{3}
\]

Similarly, the second equation can be written nicely as:

\[
-L_a m \sin(\alpha) \dot{\alpha}^2 + d_m k + \left( \ddot{d}_m + g \right) m + L_a \ddot{\alpha} m \cos(\alpha) = -b \dot{d}_m
\]

And this becomes:

\[
-L_a m \sin(\alpha) \dot{\alpha}^2 + d_m k + (\ddot{d}_m + g)m + L_a \ddot{\alpha} m \cos(\alpha) = -b \dot{d}_m \tag{4}
\]

Appendix: Example System

![Figure 1: 4-bar mechanism with spring-mass-damper](image)