windings of an ordinary 5-W mains transformer. Miniature neon indicator lamps are connected across the high-voltage windings. The lamps are situated below the strobe disk, the flashes pass through it, and the disk is observed from above. A large "reading glass" magnifier, mounted just above the disk, helps to make the divisions more easily visible when viewed from a distance.

The whole apparatus, including power pack, is mounted in a chassis measuring 20 by 30 by 12 cm, and weighs 10 lb.

RONALD ALPIAR[1]
Computation Cen.
Swiss Fed. Inst. Tech.
Zürich, Switzerland

References

[1] F. Maynard, "The electric strobotuner," *Electron. World*, Mar. 1970.
[2] Motorola Inc., Applications Note AN 435 (out of print).
[3] ——, Applications Information, MC1035 Dual Schmitt Trigger.
[4] C. G. Conn Ltd., Elkhart, Ind., "Strobotuner" and "Stroboconn" literature.
[5] "The Hohner visual tuning apparatus," Honer AG, Trossingen/Württemberg, Germany.
[6] "The Yamaha PT-3 oscilloscope tuner," Nippon Gakki Co. Ltd., Hammamatsu, Japan.
[7] Phillips Ltd., Eindhoven, The Netherlands, Motor 9904.111.05211.
[8] *The Transistor Manual*, 7th ed., Gen. Elec. Co., ch. 8.

[1] Now with the Computer Centre, University of London, London WC1, England.

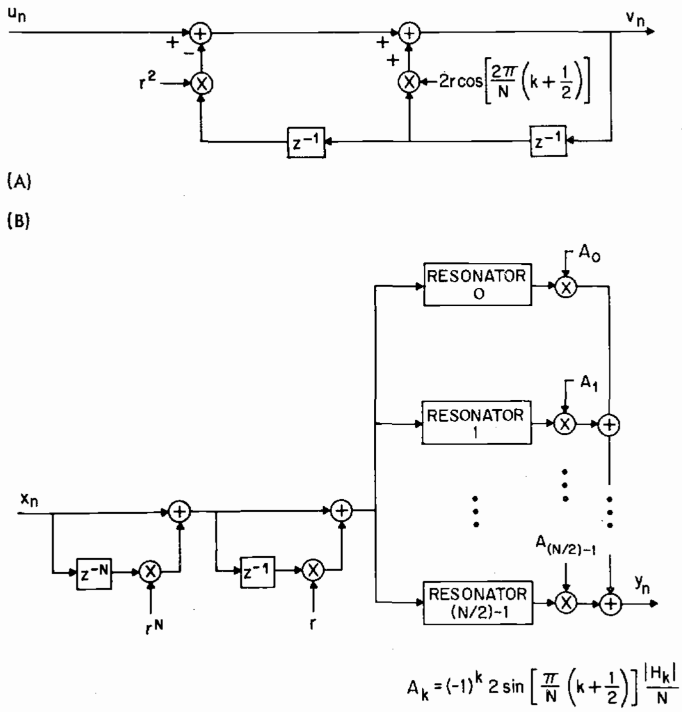


Fig. 10.

Correction to "Recursive and Nonrecursive Realization of Digital Filters Designed by Frequency Sampling Techniques"

Abstract

The purpose of this correspondence is to correct some inaccuracies in the above paper.[1] Specifically, we refer to the results in the Section "Linear Phase Type 2 Filters" (pp. 205–207). Although the results of that section are correct for the conditions stated, the constraints on phase delay and $H_{(N-1)/2}$ are more restrictive than necessary. Therefore, we offer the following as a correction to the original section.

Linear Phase Type 2 Filters

The basic difficulty with the original discussion lies in the interpretation of the implications of the linear phase constraint. The basic implication of course is that the impulse response must be symmetric. This symmetry in turn implies that *when N is even*

$$H(z)\,|_{z=-1} = 0$$

for *any* linear phase FIR filter. This can be easily shown by using the constraint $h_n = h_{N-1-n}$ in the definition of the z transform. For the case of Type 1 frequency sampling designs, the frequency sample corresponding to $k=N/2$ falls on the point $z=-1$. Therefore the constraint $H_{N/2}=0$ arises. For Type 2 designs however, there is no frequency sample at $z=-1$ and the required zero is obtained in a way that will become clear in the discussion that follows. When *N is odd* the impulse response symmetry does not imply that $H(-1)=0$. Therefore, although the frequency sample corresponding to $k=(N-1)/2$ occurs at the point $z=-1$, we do not require $H_{(N-1)/2}=0$ as in the corresponding case of Type 1 designs with N even. Thus the statement to the contrary in the original paper is incorrect. It is true however, that if we assume:

a) $H_{(N-1)/2} = 0$, for N odd

b) delay $\tau = \frac{N}{2} - 1$, for N even or odd,

as in the original paper, then we do indeed get an impulse response of duration $N-1$ samples, even though N-frequency samples are specified in the design. Thus *for these conditions* the original section is correct except for (44) which should be

$$h_n = \frac{1}{N}\sum_{k=0}^{N/2-1} 2\,|H_k|\,(-1)^k \cdot \sin\left[\frac{2\pi}{N}(n+1)(k+1/2)\right]. \qquad (44')$$

Type 2 linear phase filters of the form shown in Fig. 3 of the original paper can in fact be obtained if we do not impose the constraints a) and b), but instead require the delay to be $(N-1)/2$ samples. If N is even, substitution of the linear phase

$$\theta_k = \begin{cases} \dfrac{-2\pi}{N}\left(\dfrac{N-1}{2}\right)(k+1/2) & k = 0, 1, \cdots, N/2-1 \\ \dfrac{2\pi}{N}\left(\dfrac{N-1}{2}\right)(N-(k+1/2)) & k = N/2, \cdots, N-1 \end{cases} \qquad (45)$$

into (33) of the original paper results in

$$H(z) = \frac{(1+z^{-N})}{N}\sum_{k=0}^{N/2-1} \frac{2\,|H_k|\,(-1)^k(1+z^{-1})\sin\left[\frac{\pi}{N}(k+1/2)\right]}{1-2z^{-1}\cos\left[\frac{2\pi}{N}(k+1/2)\right]+z^{-2}}. \qquad (46)$$

The recursive realization corresponding to the above equation is shown in Fig. 10 included in this letter. The upper part of the figure shows how the kth resonator

Manuscript received December 2, 1971.

[1] L. R. Rabiner and R. W. Schafer, *IEEE Trans. Audio Electroacoust.*, vol. AU-19, pp. 200–207, Sept. 1971.

section is realized. The difference equation of the resonator is of the form

$$v_n = u_n + 2r \cos\left[\frac{2\pi}{N}(k+1/2)\right]v_{n-1} - r^2 v_{n-2} \quad (47)$$

and requires only two multiplications per iteration. The full realization of the filter is shown at the bottom of the figure.

We have noted that for *N even*, $H(-1)=0$ to achieve linear phase. For Type 1 designs, this implies the constraint $H_{N/2}=0$. For Type 2 designs, (46) shows that the term $(1+z^{-1})$ occurs explicitly in *each* term of the sum, thus providing the required zero at $z=-1$.

The direct convolution realization of Type 1 and Type 2 filters are identical. Thus, (22) and (23) of the original paper hold for Type 2 filters with the impulse response being determined by the equation

$$h_n = \frac{1}{N}\sum_{k=0}^{N/2-1}(-1)^k 2\,|H_k| \cdot \sin\left(\frac{2\pi}{N}(k+1/2)(n+1/2)\right). \quad (48)$$

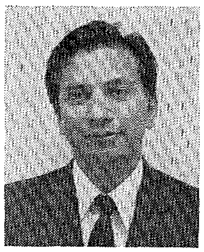
When N is odd the linear phase Type 2 filter has a z transform of the form

$$H(z) = \frac{(1+z^{-N})}{N}\left[\sum_{k=0}^{(N-3)/2}\frac{2\,|H_k|\,(-1)^k(1+z^{-1})\sin\left[\frac{\pi}{N}(k+1/2)\right]}{1-2z^{-1}\cos\left[\frac{2\pi}{N}(k+1/2)\right]+Z^{-2}} + \frac{H_{(N-1)/2}}{1+z^{-1}}\right] \quad (49)$$

which has a realization similar to the one discussed above for N even. From (49) we see that at $z=-1$ one of two cases can occur. If $H_{(N-1)/2}$ is nonzero, then $H(-1)$ is nonzero and there is no phase discontinuity. This is consistent with an *integer* number of samples phase delay when N is odd. If $H_{(N-1)/2}$ is zero, then $H(-1)$ is zero, and $H(z)$ has a *double* zero at $z=-1$. The double zero comes from a simple zero in the term $(1+z^{-1})$ and a simple zero from the term $(1+z^{-N})$. In this case the phase discontinuity at $z=-1$ is an integer times 2π rad, which is again consistent with a phase delay of an integer number of samples.

L. R. Rabiner
R. W. Schafer
Bell Telephone Lab., Inc.
Murray Hill, N. J. 07974

Contributors



J. K. Aggarwal (S'62–M'65) was born in Amritsar, India, on November 19, 1936. He received the B.Sc. degree in mathematics and physics from the University of Bombay, Bombay, India, in 1956. He received the B.Eng. degree in electrical engineering from the University of Liverpool, Liverpool, England in 1960. He was a fellow at the University of Illinois, Urbana, in the year 1960–1961, where he received the M.S. degree in 1961. In 1964 he received the Ph.D. degree from the University of Illinois.

He joined the University of Texas at Austin as an Assistant Professor in the Department of Electrical Engineering in 1964. Since September 1968 he has been an Associate Professor at the University of Texas at Austin. He was a Visiting Assistant Professor at Brown University, Providence, R. I., during the spring semester of 1968 and a Visiting Associate Professor at the University of California, Berkeley during 1969–1970. His research interests are functional differential systems, computational methods, nonlinear systems, and digital filters.

Dr. Aggarwal is a member of Sigma Xi and Eta Kappa Nu. He is the editor of the IEEE Circuit Theory Group Newsletter.

❖

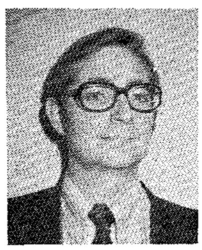
Robert R. Boorstyn (S'58–M'59) was born in New York City, N. Y., on May 28, 1937. He received the B.E.E. degree in 1958 from the City College of New York, New York, N. Y., and the M.S. and Ph.D degrees, both in electrical engineering, from the Polytechnic Institute of Brooklyn, Brooklyn, N. Y., in 1963 and 1966, respectively.

From 1958 to 1961 he worked as an Engineer for the Advanced Studies Department of the Sperry Gyroscope Company. In 1961 he joined the Department of Electrical Engineering of the Polytechnic Institute of Brooklyn, where he is currently an Associate Professor.

Jim D. Echard was born in Richmond, Va., on May 7, 1937. He received the B.S. and M.S. degrees in electrical engineering from Brigham Young University, Provo, Utah, in 1963 and 1964, respectively. He is presently working towards a Ph.D. degree in engineering (system science) from the Polytechnic Institute of Brooklyn, Brooklyn, N. Y.

Since 1964, he has worked for Heavy Military Electronics Systems, General Electric Company, Syracuse, N. Y. His work experience has been in military radar systems and radar signal and data processing.

❖



Aaron J. Goldberg (S'67–M'70) was born in New Haven, Conn., on November 4, 1943. He received the S.B. and S.M. degrees simultaneously in 1967, and the Ph.D degree in electrical engineering in 1970, all from the Massachusetts Institute of Technology, Cambridge.

From 1963 through 1965, he participated in the cooperative work plan in electrical engineering at Bell Telephone Laboratories, Inc., Murray Hill, N. J., Andover, Mass., and Holmdel, N. J. There he worked on semiconductor development, microwave transmission networks, and system engineering problems. From 1967 to 1968 he consulted on pattern recognition problems for Itek Corporation, Bedford, Mass., and on computer graphics for Control Data Corporation, also of Bedford, Mass. Since June 1970 he has been engaged in development and evaluation of speech communication terminals for the Defense Communications Agency, System Engineering Facility, Reston, Va.

Dr. Goldberg is a member of Tau Beta Pi, Sigma Xi, and the Acoustical Society of America.