A NOVEL APPROACH TO PLANAR CAMERA CALIBRATION

Ashutosh Morde, Mourad Bouzit, Lawrence Rabiner
CAIP, Rutgers University
96 Frelinghuysen Road, Piscataway, NJ 08855
amorde@caip.rutgers.edu, bouzit@caip.rutgers.edu, lrr@caip.rutgers.edu

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Abstract: Camera calibration is an important step in 3D reconstruction of scenes. Many natural and man made objects are circular and form good candidates as calibration objects. We present a linear calibration algorithm to estimate the intrinsic camera parameters using at least three images of concentric circles of unknown radii. Novel methods to determine the projected center of concentric circles of unknown radii using the projective invariant, cross ratio, and calculating the vanishing line of the circle are proposed.
The circular calibration pattern can be easily and accurately created. The calibration algorithm does not require any measurements of the scene or the homography between the images. Once the camera is fully calibrated the focal length of zooming cameras can be estimated from a single image. The algorithm was tested with real and synthetic images with different noise levels.

1 INTRODUCTION

Camera calibration is an essential step in many computer vision and photogrammetric applications. It consists of recovering the metric properties which are encoded as a set of so-called internal parameters. It has been a subject of active research with numerous methods (Tsai, 1987; Strum and Maybank, 1999; Zhang, 2000). Once the cameras are calibrated, the projective relationship from 3D space to 2D image can be established.

The existing camera calibration techniques can be broadly classified as linear, (Grosky and Tamburino, 1990; Strum and Maybank, 1999), and non-linear (Heikkila, 2000; Meng and Hu, 2003). The non linear techniques have the disadvantage of requiring good initial estimates of the intrinsic parameters and being computationally intensive. If the starting point of the algorithm is not well chosen the solution can diverge or can get trapped in a local minimum. A linear approach is not plagued with these problems.

Camera calibration can also be broadly classified based on the type of calibration object, viz. 3D calibration object, and 2D calibration object. Most commonly used calibration procedures described in the computer vision literature rely on a calibration object with control points whose 3D coordinates are known with a high degree of accuracy to obtain accurate results (Tsai, 1987; Heikkila, 2000). As compared to a 3D calibration object, 2D calibration patterns offer the advantage of easily creating an accurate calibration object; the calibration pattern can be printed on a laser printer and mounted on a flat surface. Techniques utilizing planar patterns require multiple views of the calibration object (Strum and Maybank, 1999; Zhang, 2000). The camera motion between the images need not be known.

Conics can be used instead of control points as it is easy to match correspondences. They project onto the image plane as ellipses from any view and have been widely used before to estimate the camera pose (Kanatani and Liu, 1993; Chen and Huang, 1999). The use of circles and ellipses as 2D calibration objects have been increasing. There are various non-linear planar calibration algorithms which use circles and ellipses of known dimension as calibration object (Yang et al., 2000; Kim and Kweon, 2001; Kim et al., 2002; Abad et al., 2004).

We propose a novel linear method, which exploits the Thales theorem for circles, to calculate the vanishing line of the circle and the corresponding Image of the Circular Points (ICP's). The camera intrinsic parameters are then determined, from at least 3 images, using the Image of Absolute Conic (IAC) as the calibration object. The projected center of concentric circles required by the calibration algorithm is accurately determined using the cross ratio without any knowledge of the circle radii. The calibration method has the advantage of not requiring any metric measurements nor any correspondences between the images.

The paper is organized as follows. In section 2 we describe the theory used; in section 3 we present novel algorithms to find the projected center of concentric circles and the vanishing line of the circle. In section 4 we discuss the camera calibration algorithm and the results, which is followed by the conclusion.

2 THEORY

We adopt a perspective camera model with intrinsic matrix K,

$$K = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (1)$$

where f_u and f_v are the effective camera focal lengths along the camera axes u and v, s is the skew of the CCD plane and (u_0,v_0) is the coordinate of the image center. A homogeneous point $\tilde{X} = [X, Y, Z, 1]$ gets projected to $\tilde{x} = [u, v, w]$ as

$$\begin{array}{rcl} \lambda\tilde{x} & = & K\,[R\,T]\,\tilde{X} \\ \text{i.e. } \lambda\tilde{x} & = & KM_{ext}\tilde{X} = M\tilde{X} \end{array}. \quad (2)$$

where M_{ext} is the camera extrinsic parameter matrix with R and T, the rotation matrix and the translation vector from the world to the camera system respectively and λ is a non zero scale factor.

Let a circle with radius R_c and center at $[X_c, Y_c, 0]$ lie in the z plane of the world coordinate system. A homogeneous point $[X, Y, 1]$ on the circle satisfies the equation

$$\begin{array}{rl} [XY1] \begin{bmatrix} 1 & 0 & -X_c \\ 0 & 1 & -Y_c \\ -X_c & -Y_c & X_c^2 + Y_c^2 - R_c^2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} & = 0 \\ \text{i.e. } X^T C X & = 0 \end{array}. \quad (3)$$

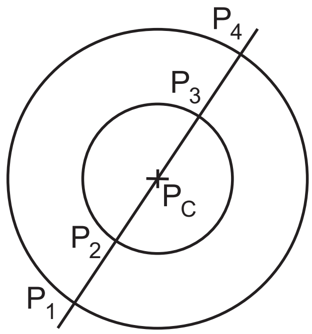
This circle gets projected as the ellipse $X^T A X = 0$, where A is the matrix defining the ellipse and is related to the circle up to a scale factor λ by

$$\lambda A = H^{-T} C H^{-1}. \quad (4)$$

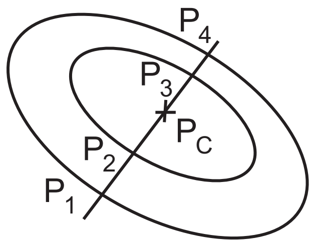
where $H = K\begin{bmatrix} R_1 & R_2 & T \end{bmatrix} = KM_h$ is the homography transforming the circle into an ellipse; R_1 and R_2 are the first and second columns of the rotation matrix R.

All circles pass through the circular points $I = (1, i, 0)^T$ and $J = (1, -i, 0)^T$ whose position is invariant to plane similarity transformation. A special circle is the absolute conic which is an imaginary circle located in the plane at infinity and is also invariant to similarity transformations. All circles intersect the absolute conic and the line at infinity at the circular points (Hartley and Zisserman, 2000). The absolute conic, $C_\infty = I$, forms a natural calibration object with the IAC, ω, being related to the intrinsic matrix under the homography $H_\infty = KM_{h_\infty}$ by

$$\begin{array}{rcl} \omega = H_\infty^{-T} C_\infty H_\infty^{-1} & = & (KM_{h_\infty})^{-T}\, I\, (KM_{h_\infty})^{-1} \\ = & & K^{-T} M_{h_\infty}^{-T} M_{h_\infty}^{-1} K^{-1} = K^{-T} K^{-1} \end{array}. \quad (5)$$



(a)



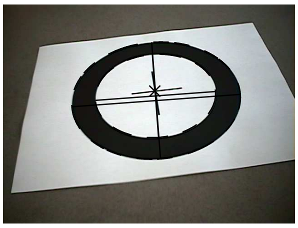
(b)

Figure 1: Projection of concentric circles

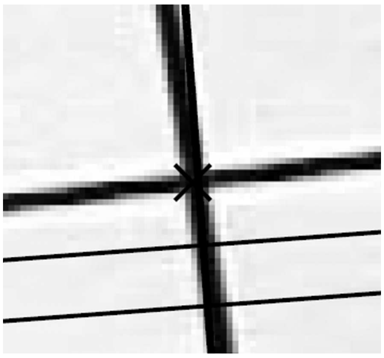
3 FROM CIRCLES TO RECTANGLES

3.1 Projected Center of Circle

A 3D circle gets projected as an ellipse from any view but the projected center of the 3D circle does not coincide with the center of the ellipse (J.L.Mundy and A.Zisserman, 1992). The projected center of concentric circles lies on the line joining the centers of the projected ellipses (Kim and Kweon, 2001) and it can be determined by using the projective invariant, cross ratio. The cross ratio of four points (P_1, P_2, P_3, P_4) is defined as

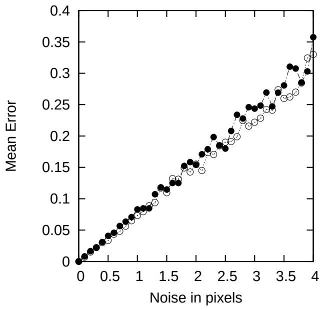


(a)

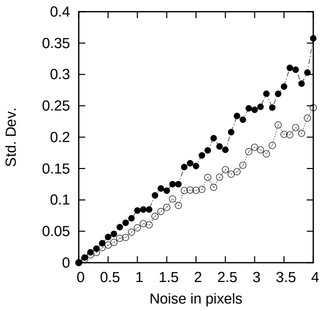


(b)

Figure 2: Projected center of circle

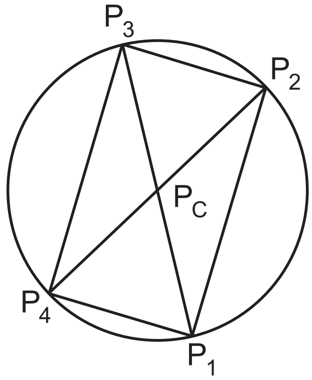


(a) Projected circle center error

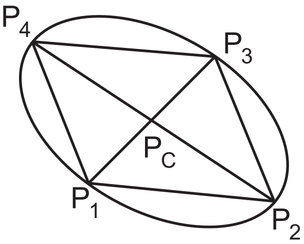


(b) Std. Dev. of projected circle center

Figure 3: Influence of noise on finding projected center of circle



(a) Rectangle inscribed by two diameters



(b) Its projection

Figure 4: Rectangle inscribed in a circle

$$CR(P_1, P_2, P_3, P_4) = \frac{d(P_1,P_2)*d(P_3,P_4)}{d(P_1,P_3)*d(P_2,P_4)} \; . \quad (6)$$

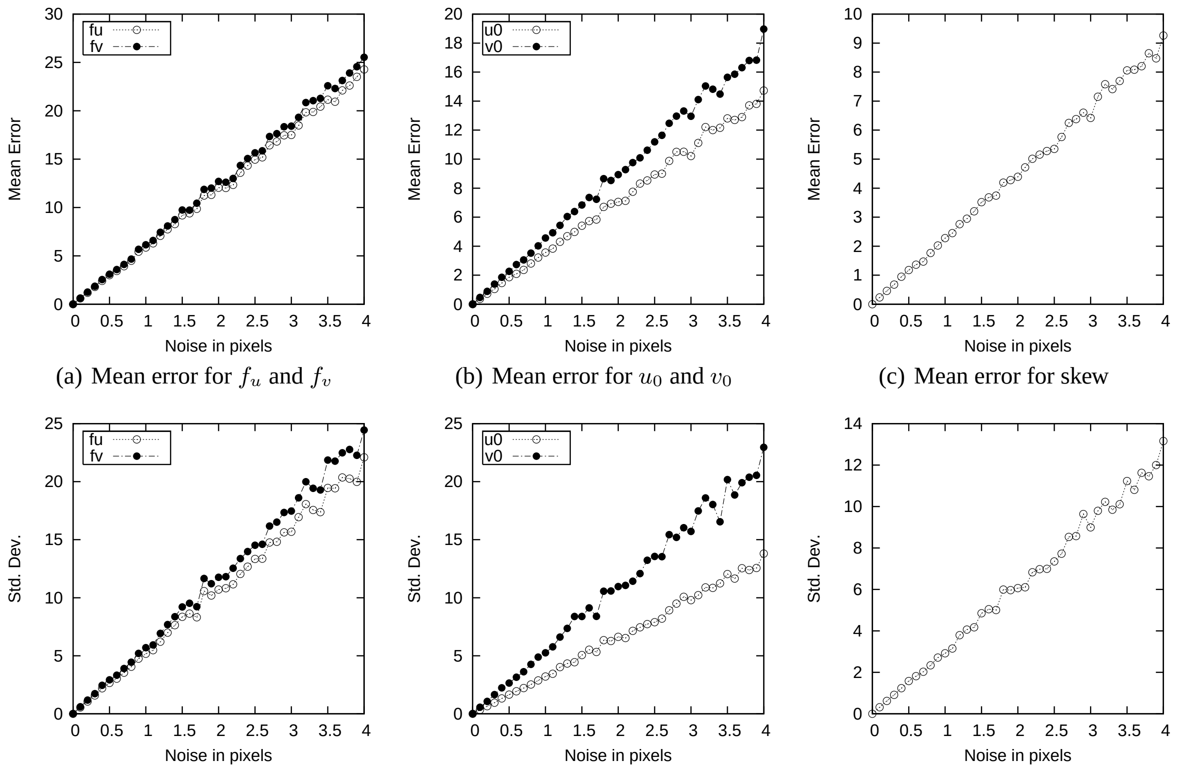
where $d(j,k) = \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2}$ is the distance between points j and k. With three fixed points the cross ratio is a monotonic function of the fourth point. As any one point is made to move along the line joining all the points the cross ratio increases/decreases. For example, in figure 1, moving point P_3 closer to point P_4 causes the cross ratio to decrease while moving point P_3 away from point P_4 causes the cross ratio to increase.

Consider four points lying on the diameter of two concentric circles with center P_C as shown in figure 1(a). For these four points we have

$$CR(P_1, P_2, P_C, P_3) = CR(P_4, P_3, P_C, P_2) \, . \quad (7)$$

As the cross ratio is a projective invariant the same is true for figure 1(b) which is a projection of the concentric circles in figure 1(a). The algorithm for finding the circle center then involves :

1. Initializing the projected circle center as the inner ellipse center
2. A binary search along the line joining the two ellipse centers bounded by points P_2 and P_3



(a) Mean error for f_u and f_v

(b) Mean error for u_0 and v_0

(c) Mean error for skew

(d) Std. Dev. of f_u and f_v

(e) Std. Dev. of u_0 and v_0

(f) Std. Dev of skew

Figure 5: Influence of noise in calculating intrinsic parameters

(a) If $CR\,(P_1, P_2, P_C, P_3) > CR\,(P_4, P_3, P_C, P_2)$ then move P_C towards point P_3

(b) If $CR\,(P_1, P_2, P_C, P_3) < CR\,(P_4, P_3, P_C, P_2)$ then move P_C towards point P_2

Due to the monotonic nature of the cross ratio the algorithm always converges, usually in 2-3 iterations. In figure 2 the circle center was correctly calculated as (333.86, 190.76) for a real image. Figure 3 shows the result of a simulation of finding the projected center of circle; the simulation parameter details are given in section 4.2 along with the calibration results . The algorithm is observed to be very robust to noise perturbations and correctly calculates the projected circle center while the ellipse centers do not coincide with the projected circle center.

3.2 Vanishing Line of a Circle

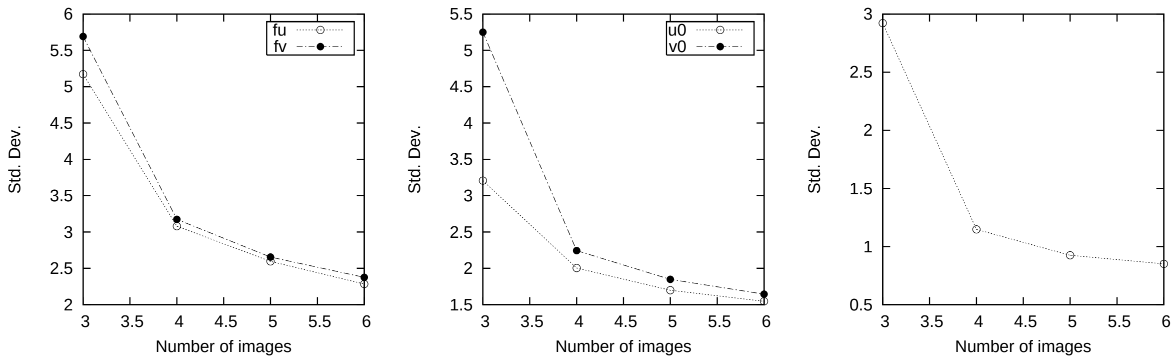
The vanishing line is the image of the line at infinity and its intersection with the image of a circle gives the image of circular points. Any two lines through a circle center form the diagonals of a rectangle. Figure 4 illustrates this with lines $P_1P_CP_3$ and $P_2P_CP_4$ being the diameters of the circle and inscribing a rectangle $P_1P_2P_3P_4$. The procedure for finding the vanishing line starts off with intersecting two lines passing through the projected center of circle with an ellipse, the image of the circle. This gives a quadrilateral $P_1P_2P_3P_4$, figure 4(b), which is the image of a rectangle. Thus intersecting lines P_1P_2 and P_4P_3 gives one vanishing point while intersecting lines P_1P_4 and P_2P_3 gives another vanishing point. The two vanishing points are then used to compute the vanishing line.

4 CAMERA CALIBRATION

4.1 Calibration Algorithm

The procedure for calibrating the camera involves the following steps :

- Capture 3 views of the concentric circles and for each image
  – fit ellipses using the direct least squares method (Fitzgibbon et al., 1999)



(a) Std. Dev. of f_u and f_v (b) Std. Dev. of u_0 and v_0 (c) Std. Dev of skew

Figure 6: Effect of increasing the number of images on calculating intrinsic parameters

Table 1: Comparison of Calibration results

	f_u	f_v	s	u_0	v_0
Proposed Algorithm	812.03	810.31	-0.6	314.25	239.47
Zhang's Planar Calibration	788.83	789.27	0	318.73	241.13

- Calculate the projected center of the circles as described in section 3.1
- Estimate the vanishing line as explained in section 3.2
- Intersect the vanishing line with an ellipse to get the two ICP's.

• Fit an ellipse to the 6 ICPs to obtain the Image of IAC

• Perform Cholesky Decomposition of the IAC matrix to obtain the camera intrinsic parameters

Once the camera is calibrated the ICP's from a single image can be used to estimate the focal length of zooming cameras (Strum and Maybank, 1999).

4.2 Calibration Result

The accuracy of the proposed algorithm was tested using simulations. The simulations were performed with circles of radii 120mm and 60mm with 400 points per circle and a camera with intrinsic matrix

$$K = \begin{bmatrix} 845.79 & 0.1 & 315.24 \\ 0 & 875.46 & 226.13 \\ 0 & 0 & 1 \end{bmatrix}. \quad (8)$$

The camera was setup in 6 different locations with the projection of the larger circle varying as an ellipse with half major axis length of 130-190 pixels and the half minor axis length of 80-150 pixels. For trials with less than 6 images, all possible camera pose combinations were used. Thus when calibrating using just 3 images, the 6 camera poses provide $\binom{6}{3} = 120$ sets to work with. The projected circle points were perturbed with zero mean gaussian noise while the standard deviation was varied from 0 to 4 pixels. Figure 5 shows the accuracy of the technique in estimating the intrinsic parameters at various noise levels for 200 independent trials. The estimation error is found to be small and its standard deviation increases with the noise levels. In figure 6 we observe that the standard deviation of the estimated intrinsic parameters decreases with the increase in the number of images used for calibration.

The algorithm was also tested with real images of dimension 640 by 480 taken by a Sony Lipstick camera. Table 1 gives the results of camera calibration with the proposed algorithm and Zhang's (Zhang, 2000) method. The results are comparable, the difference could be accounted by the use of non linear minimization and estimation of distortion coefficients in (Zhang, 2000).

5 CONCLUSION

We proposed a novel camera calibration method using concentric circles. An algorithm for estimating the projected center of concentric circles without using any radii information was developed. A method for finding the vanishing line of a circle and the corresponding ICP's was proposed. The calibration process does not require any measurements of the planar pattern; thus any natural pattern of concentric circles can also be used as a calibration object. Experiments with simulated data as well as real images showed the insensitivity of the algorithm to varying levels of noise. The estimated intrinsic parameters had low mean errors and standard deviation. Increasing the number of images beyond the minimum value of 3 resulted in a decrease in standard deviation of the errors.

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