ECE137B Notes set 6: Method of time constants

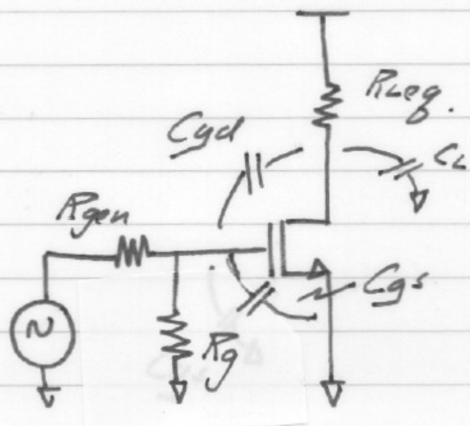
General circuit transfer function:

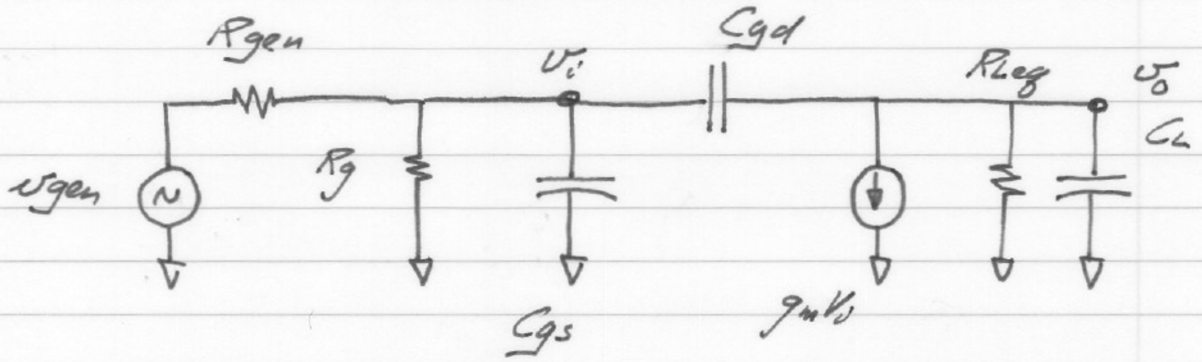
$$\frac{v_o}{v_{gen}} = \left.\frac{v_o}{v_{gen}}\right|_{MB} \frac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}$$

Methods of Solution

- Nodal analysis: hard to do for complex ckts
  answers often hard to interpret
- Miller Approximations: intuitive
  often very wrong
- SPICE: equivalent to randomly building and testing.
  zero understanding.
  SPICE is for design verification, not synthesis.
- Time constant method.
  exact - for the information it gives
  easy and fast
  answers often physically interpretable

let us start with a specific circuit:



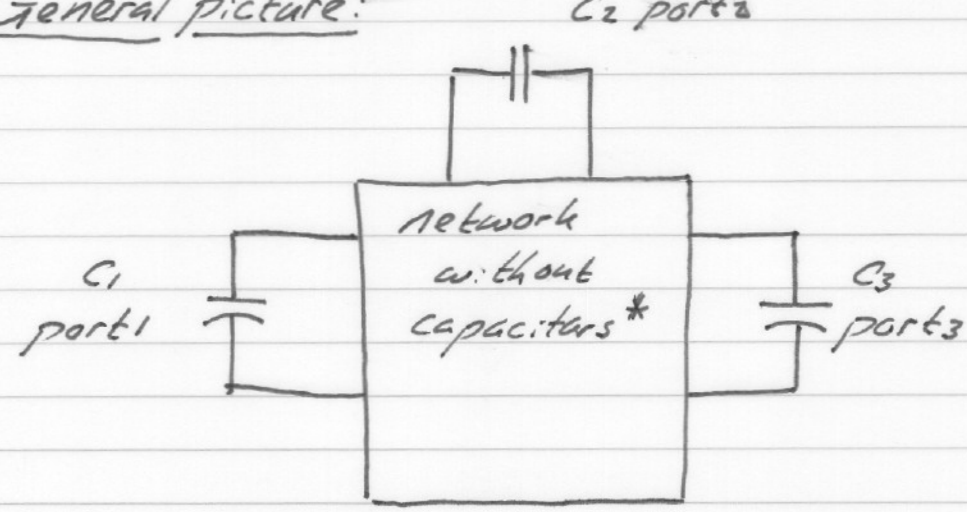


turn off independent voltage sources → short circuits

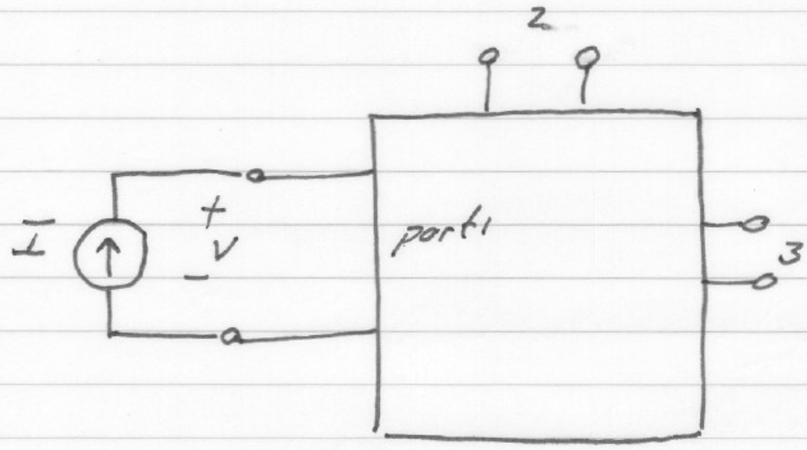
turn off independent current sources → open circuits

dependent sources <u>must</u> be kept.

<u>General</u> <u>picture:</u>



Define R_{11}^{o}: resistance at port1 with all capacitors replaced with open circuits



$R_{11}^{o} \triangleq V/I$

(one can also force a voltage & measure I)

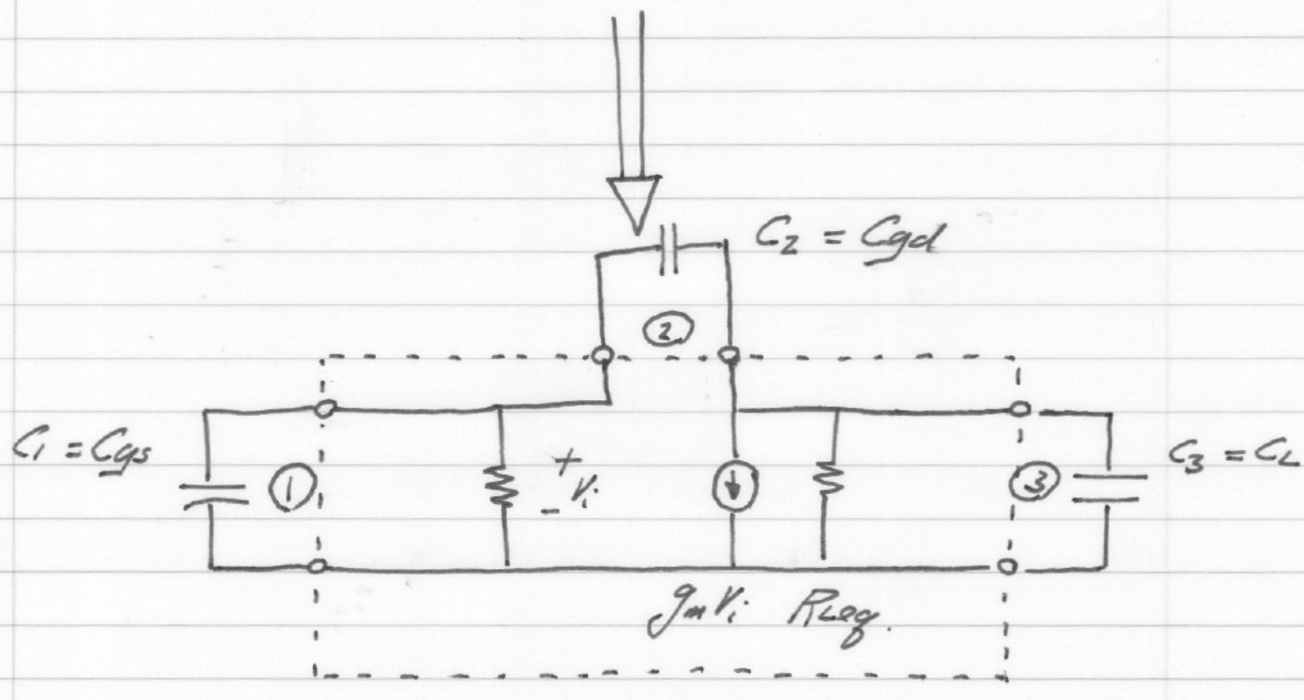
*inductors, transmission lines, time delay elements, or anything with frequency dependence is <u>not</u> allowed.

It can be shown that

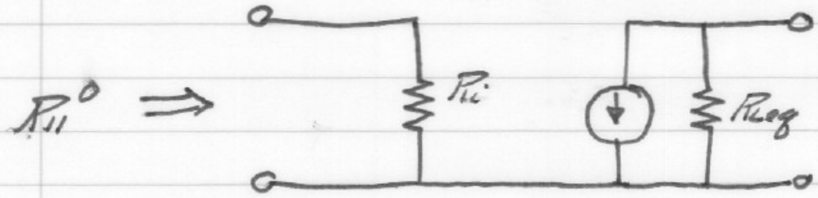
$$a_1 = R_{11}^{\,0} C_1 + R_{22}^{\,0} C_2 + R_{33}^{\,0} C_3 + \cdots$$

these are called the open circuit time constants.

Example:



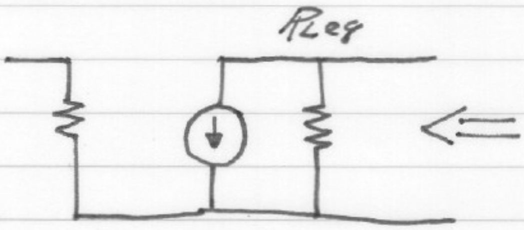
what is R_{11}^{0}? - Easy!



$R_{11}^{0} = R_i$

- so, first time constant is $C_{gs} R_i$

what is R_{33}^{0}? - Easy!

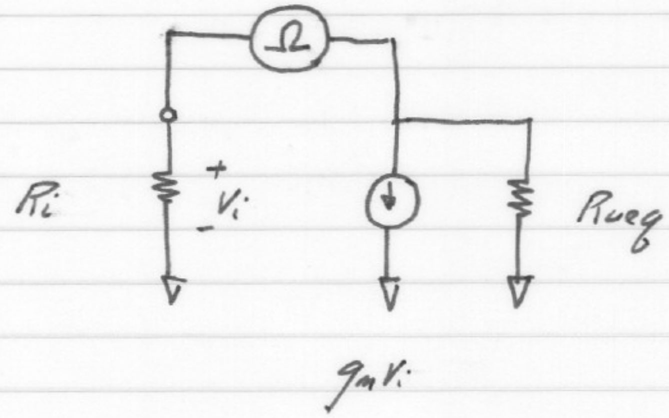


$R_{33}^{0} = R_{Leq}$

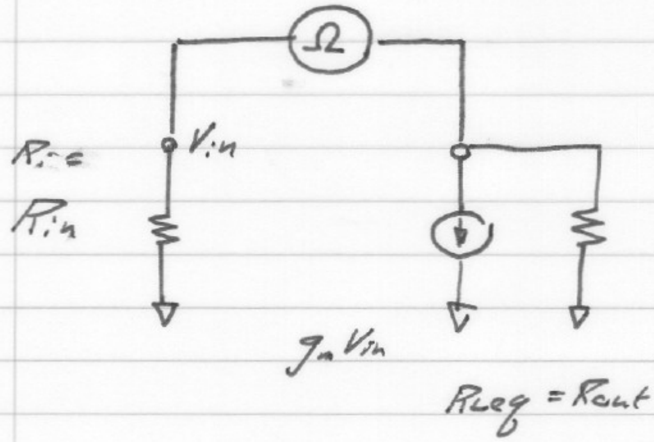
so, third time constant is $R_{Leq} C_L$

what is R_{22}°? – harder

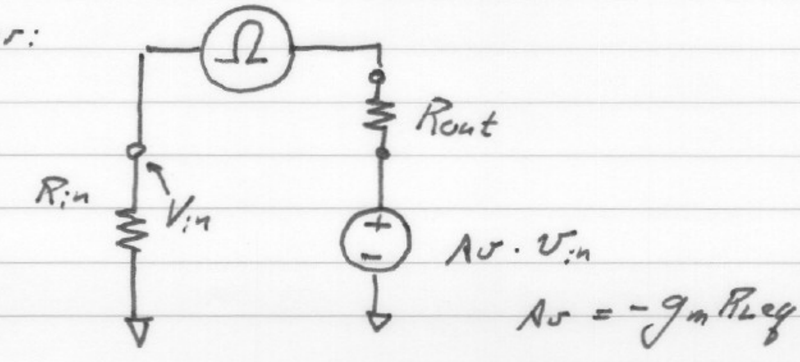
we are faced with analyzing this problem:



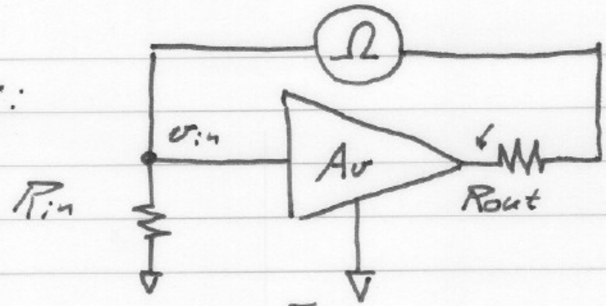
- this situation is going to show up with common emitter/source and common collector/drain.
- Let us analyze the problem once in a general way.



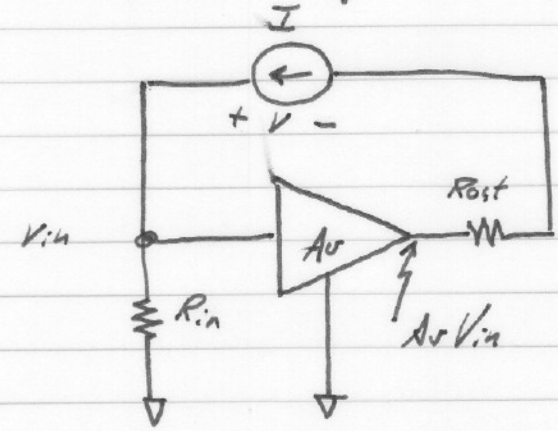
or:



or:



the 3 diagrams are equivalent.



$V_{in} = R_{in} \cdot I$,
voltage at amplifier output = $+ A_v R_{in} \cdot I$
voltage drop across R_{out}: = $I \cdot R_{out}$

$\Rightarrow \quad V = I \cdot R_{in} - I \cdot R_{in} \cdot A_v + I \cdot R_{out}$

$\Rightarrow \quad \boxed{R_{22}^{\circ} = R_{in}(1 - A_v) + R_{out}}$ general answer. similar to Miller Effect

For our specific common-source example:

$$\boxed{R_{22}^{\circ} = R_i(1 + g_m R_{Leq}) + R_{Leq}}$$

we will use the general form several times.

So the second time constant is $C_{gd} \cdot R_{22}^{o}$

e.g. $C_{gd} \cdot \left[R_i(1 + g_m R_{Leq}) + R_{Leq} \right]$

The sum of time constants is

$$a_1 = R_i C_{gs} + R_{Leq} \cdot C_L + C_{gd}\left[R_i(1 + g_m R_{Leq}) + R_{Leq} \right]$$

Recognize that this is the same as that found by nodal analysis.

what about higher order poles?

$$v_o/v_{gen} = v_o/v_{gen}\Big|_{MB} \frac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}$$

1) If a_3 and a_4 etc are small

-and-

2) If $a_2/a_1 << a_1$

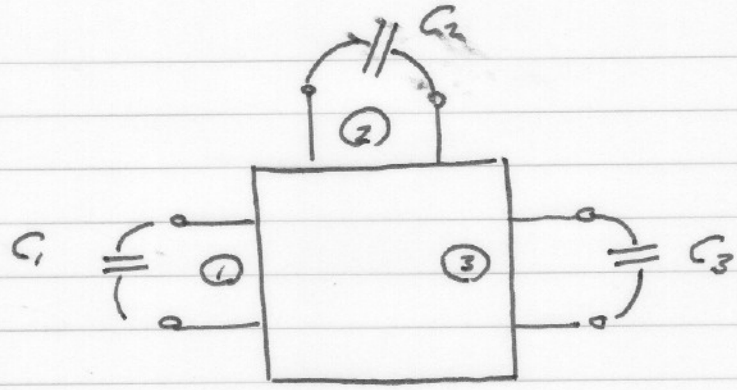
-then-

$$v_o/v_{gen} \cong \frac{v_o}{v_{gen}}\Big|_{MB} \frac{1 + b_1 s + b_2 s^2 + \cdots}{(1 + a_1 s)(1 + a_2/a_1 \cdot s)}$$

(separated pole approximation)

whether or not we use the SPA,

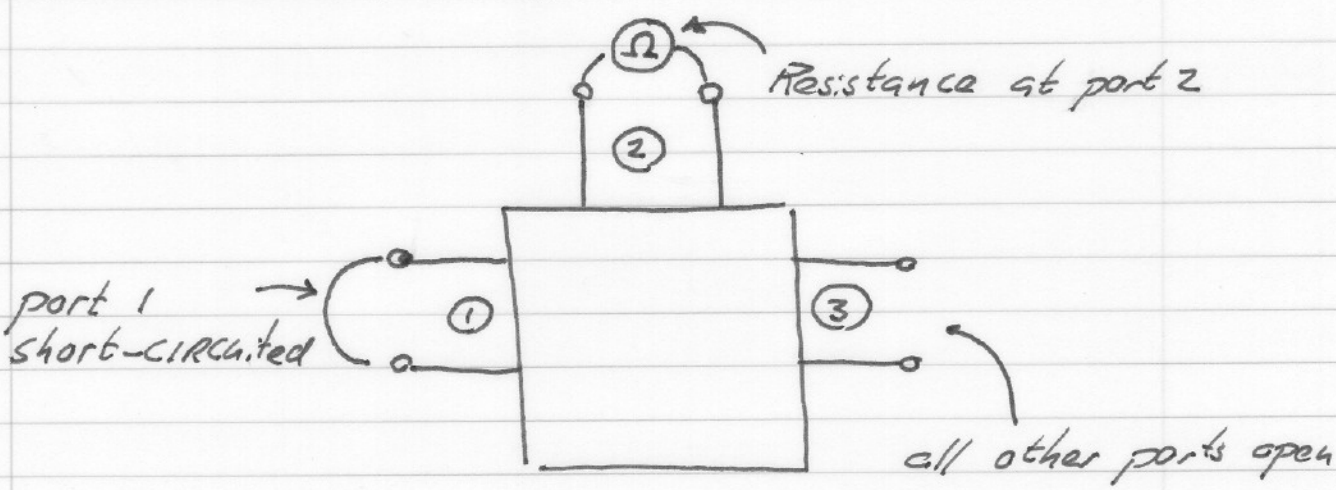
- How do we find a_2 ? -



$$a_2 = R_{11}^{0} C_1 C_2 R_{22}^{1} + R_{11}^{0} C_1 C_3 R_{33}^{1} + R_{22}^{0} C_2 C_3 R_{33}^{2}$$

(can be applied this way to more than 3 $\dashv\vdash$)

where we define R_{22}^{1} as below:

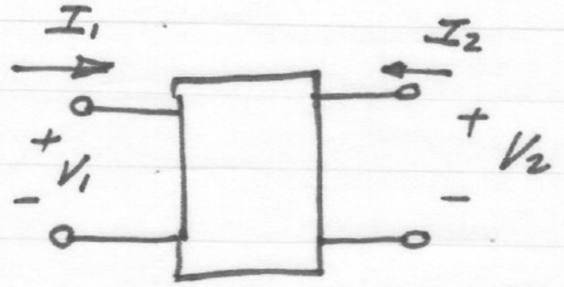


R_{22}^{1} = "resistance at port 2

with port 1 short-circuited

but all other ports open-circuited"

Now Consider a 2-port for a moment:



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$R_{11}^{0} = Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \& \quad R_{11}^{2} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

If $V_2 = 0 \Rightarrow Z_{21} I_1 + Z_{22} I_2 = 0 \Rightarrow \dfrac{V_1}{I_1} = \boxed{Z_{11} + \dfrac{Z_{12}(-Z_{21})}{Z_{22}} = R_{11}^{2}}$

Similarly: $R_{22}^{1} = Z_{22} - \dfrac{Z_{12} Z_{21}}{Z_{11}}$

So: $R_{11}^{0} R_{22}^{1} = Z_{11}\left[Z_{22} - \dfrac{Z_{12} Z_{21}}{Z_{11}}\right] = Z_{11} Z_{22} - Z_{12} Z_{21} = \Delta Z$

and $R_{11}^{2} R_{22}^{0} = \left[Z_{11} - \dfrac{Z_{12} Z_{21}}{Z_{22}}\right] Z_{22} = \ldots = \Delta Z$

So: $\boxed{R_{11}^{0} R_{22}^{1} = R_{22}^{0} R_{11}^{2} = \Delta Z}$

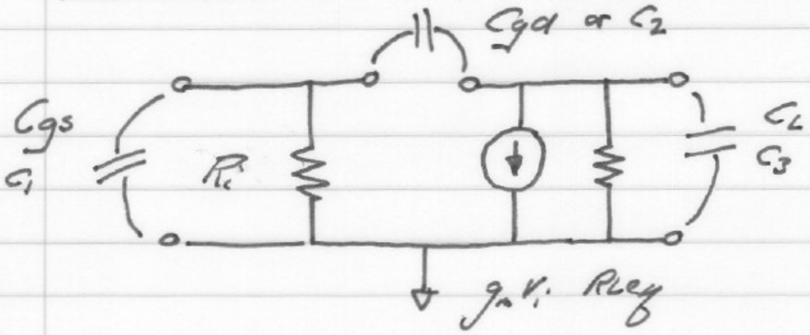
so:

$$a_2 = R_{11}^{0} C_1 C_2 R_{22}^{1} + R_{11}^{0} C_1 C_3 R_{33}^{1} + R_{22}^{0} C_2 C_3 R_{33}^{2}$$

$$= R_{11}^{2} C_1 C_2 R_{22}^{0} + R_{11}^{3} C_1 C_3 R_{33}^{0} + R_{22}^{3} C_2 C_3 R_{33}^{0}$$

- Each term can be calculated two ways.

<u>Returning to our example:</u>



<u>$C_1 R_{11}^{0} R_{22}^{1} C_2$</u> $\quad R_{11}^{0} = R_i$, $\quad R_{22}^{1} = R_{Leq}$ — DRAW sketches for each case

<u>$C_1 R_{11}^{0} R_{33}^{1} C_3$</u> $\quad R_{11}^{0} = R_i$, $\quad R_{33}^{1} = R_{Leq}$

<u>$C_2 R_{22}^{0} R_{33}^{2} C_3 = C_2 R_{22}^{3} R_{33}^{0} C_3$</u>

$R_{33}^{0} = R_{Leq} \quad R_{22}^{3} = R_i$

so: $R_{11}^{0} R_{22}^{1} = R_{11}^{0} R_{33}^{1} = R_{22}^{0} R_{33}^{2}$ for this case only

$= R_i R_{Leq}$

hence: $a_1 = R_i R_{Leq} \left[C_{gs} C_{gd} + C_{gs} C_L + C_{gd} C_L \right]$

For our example

$$v_o / v_{gen} = v_o / v_{gen}\Big|_{mbs} \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

$$a_1 = R_i C_{gs} + R_{Leq} C_L + \left[R_i (1 + g_m R_{Leq}) + R_{Leq} \right] C_{gd}$$

$$a_2 = R_i R_{Leq} \left[C_{gs} C_{gd} + C_{gs} C_L + C_{gd} C_L \right]$$

... as was found earlier by nodal analysis.