ECE137B notes set 8. Multistage analysis

Methods of analysis of multistage frequency response.

1) breaking into individual stages
- Does not work, because Z_{in} of most stages is very complex.
Only exception: break in center of common-base stage.
Even this is not realistic if $R_{bb} \neq 0$.

2) Nodal analysis
- Learn to do! valuable for simple circuits
- Very hard for complex circuits
- Answers can be hard or impossible to interpret.
-> Limits understanding

3) Computer analysis (SPICE)
Good final check on design predictions
No understanding
No good for choosing a design.

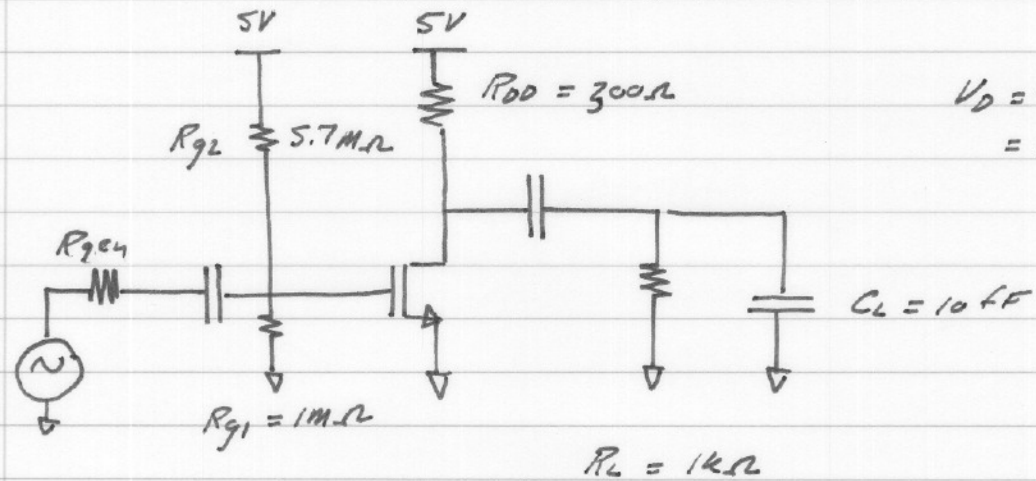
4) MOTC
Easy. Quick.
Get's 1^{st} pole right.
Gets 2^{nd} pole approximately.
Good insights.

<u>MOSFET:</u> $C_{ox} v_{sat} = 1\,mS/\mu m$ $\quad V_{th} = 0.25V$ $\quad \lambda = 0$ for simplicity

$W_g = 20\mu m$, $L_g = 200\,nm$, $v_{sat} = 10^7\,cm/sec$

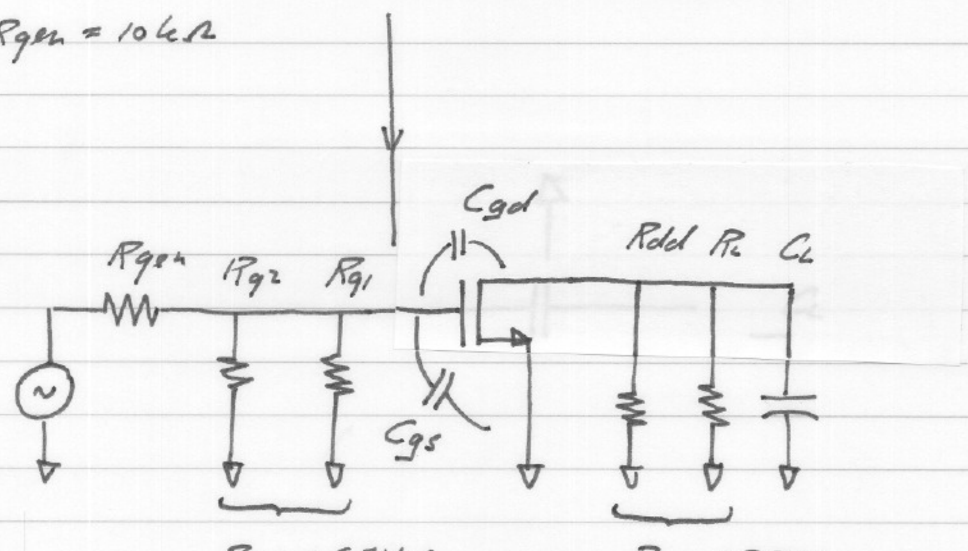
$\Rightarrow C_{gs} = 40\,fF$, $\quad C_{gd} = 10\,fF$, $\quad g_m = 20\,mS$ $\quad (f_T \cong 80\,GHz)$

BIAS at $I_D = 10\,mA \rightarrow V_{gs} = 0.75V$



$V_D = 5V - 300\Omega \cdot 10mA$
$= 2V$

$R_{gen} = 10k\Omega$



$R_i = R_{gen} \,//\, R_{inA} = 9.9\,k\Omega$

Mid-band analysis

$$v_{in}/v_{gen} = 851/861 = 0.99$$

$$v_o/v_{in} = -g_m R_{Leq} = -4.62$$

$$v_o/v_{gen} = -4.57 \cong -4.6$$

High frequency analysis: dominant pole only:

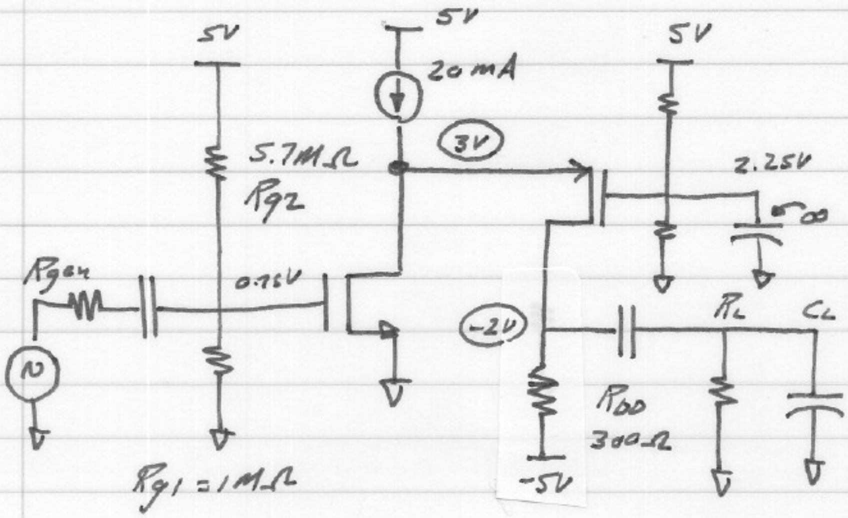
$$a_1 = R_i C_{gs} + C_{gd}\left[R_i(\overbrace{1-A_v}^{5.62}) + R_{Leq}\right] + C_L \cdot R_{Leq}$$

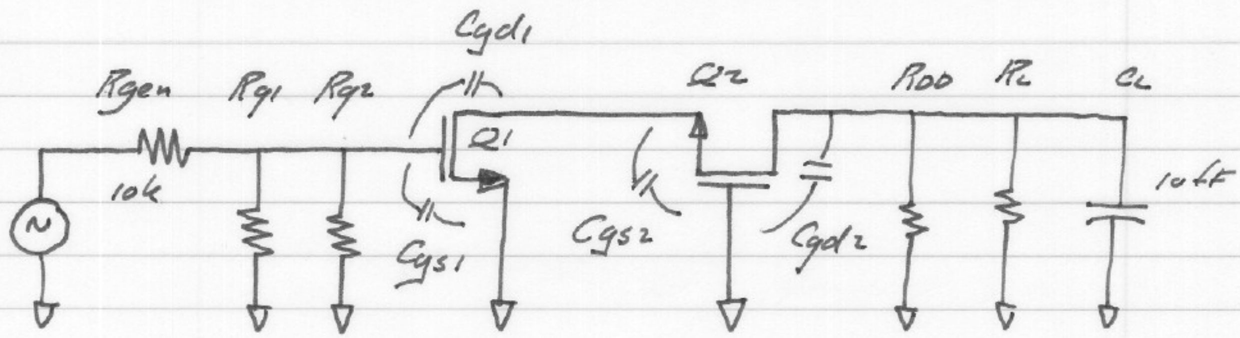
$$= 188\,ps + \underline{559\,ps} + 2.3\,ps = 749\,ps$$

$$f_{p1} \cong 1/2\pi a_1 = \underline{212\ MHz}$$

note that Miller Multiplication has made C_{gd} the dominant bandwidth limit

Compare to Cascode stage: (use same #'s for pMOSFET)





Mid-band analysis

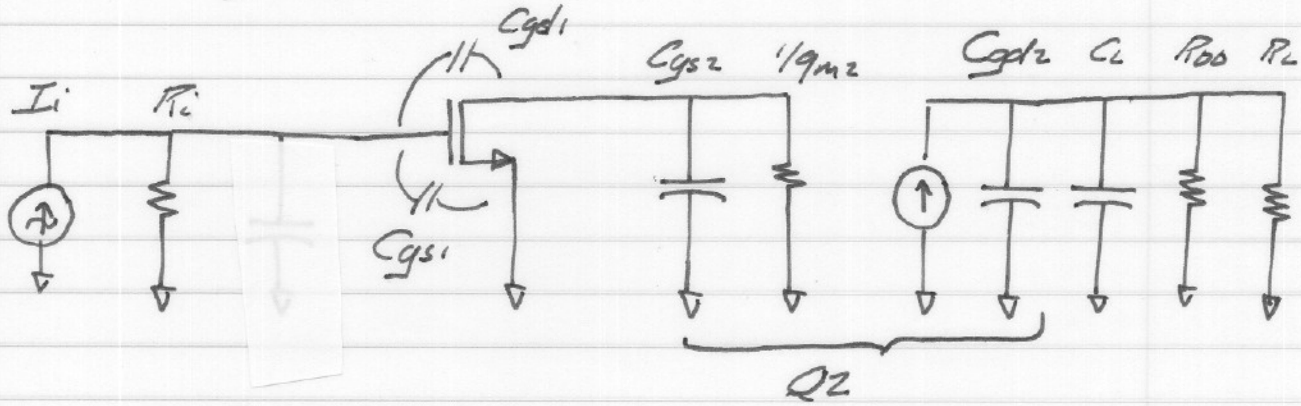
$C_{gs} = 40\ fF$
$C_{gd} = 10\ fF$

Q2: $R_{Leq} = R_{DD} \| R_L = 231\ \Omega$
$R_{in} = 1/g_m = 50\ \Omega$
$A_v = R_{Leq}/R_{in} = 4.62$

Q1: $R_{Leq} = R_{in2} = 50\ \Omega$
$A_v = -g_m R_{Leq} = -1$
$R_i = R_{gen} \| R_{g1} \| R_{g2} = 9.9\ k\Omega$
$v_{in}/v_{gen} = 0.99$

$\rightarrow v_o/v_{gen} = -4.6$

High frequency analysis:



part 1 of problem can be completely separated from ← | → part 2 of problem.

Part 1

$$a_1 = R_i C_{gs1} + C_{gd1}\left(R_i(1-A_{v1}) + 1/g_{m2}\right) + C_{gs2}\cdot 1/g_{m2}$$

$$= 396ps + 198ps + 2ps = 596ps$$

$$a_2 = R_i\cdot(1/g_{m2})\left[C_{gs1}C_{gd1} + C_{gs1}C_{gs2} + C_{gd1}C_{gs2}\right]$$
$$= 1.19(10^{-21})\,sec^2$$
$$= (34.5ps)^2$$

SPA: $f_{p1} \cong 1/2\pi a_1 = 266\ MHz.$

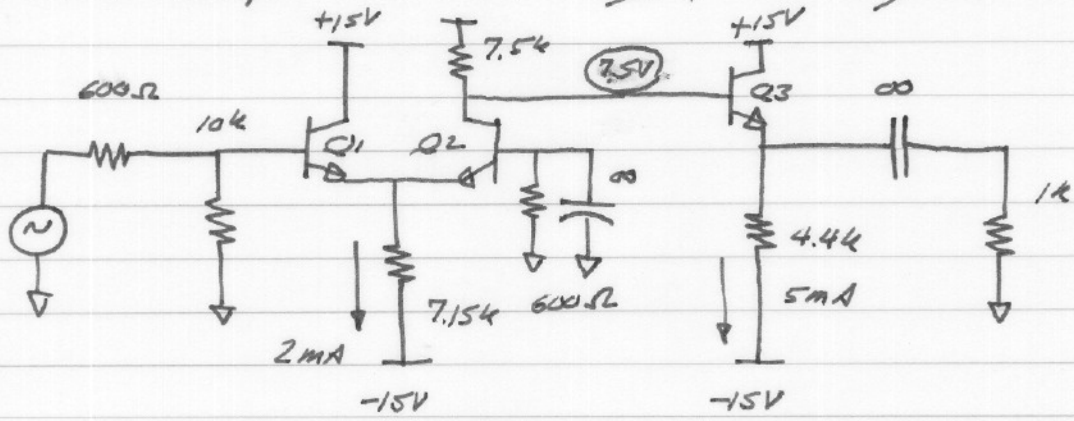
$f_{p2} \cong a_1/2\pi a_2 = 80\ GHz.$

Part 2 $\quad a_1 = (C_{gd2}+C_L)(R_{DD}\|R_L) = 4.6ps$

$f_p = 1/2\pi a_1 = 34\ GHz.$

Cascode stage has eliminated (no-reduced) Miller multiplication of C_{gd1} → increased bandwidth

Second example - this illustrating partitioning at C.B. stage



Transistors are typical of those used in the lab, ~1000:1 slower that state of art BJTs circa 2004.

$\tau_f = 0.5\,ns$, $\beta = 100$, $C_{cb} = 4pF$, $C_{je} = 4pF$, $V_A = \infty V$

use $C_{be} = g_m \tau_f + C_{je}$; $g_m = 1/r_e = I_E/V_t$

	I_c	r_e	g_m	f_T	C_{cb}	C_{be}
Q1	1mA	26Ω	38mS	223 MHz	4pF	23pF
Q2	1mA	26	38mS	223 MHz	4pF	23pF
Q3	5mA	5.2	192mS	293 MHz.	4pF	100pF

f_T from $g_m / 2\pi (C_{cb} + C_{be})$

Mid band analysis

Q3: $R_{Leq3} = 1k \,//\, 4.4k = 815\,\Omega$

$A_{v3} = R_{Leq3} / (R_{Leq3} + r_{e3}) = 0.994$ ← Don't round!!

$R_{in3} = \beta (R_{Leq3} + r_{e3}) = 82.3\,k\Omega$

Q2: $R_{Leq2} = 82.3k \,//\, 7.5k = 6.9\,k\Omega$

$R_{in2} = r_{e2} = 26\,\Omega$

$A_{v2} = 6.9k / 26\,\Omega = 264$

Q1: $R_{Leq1} = r_{e2} \,//\, 7.5k \cong r_{e2} = 26\,\Omega$

$A_{v1} = r_{e2} / (r_{e2} + r_{e1}) = 1/2$

$R_{in1} = (\beta + 1)(26 + 26\,\Omega) = 5.2\,k\Omega$

$R_{inA} = R_{in1} \,//\, 10k = 3.4\,K$

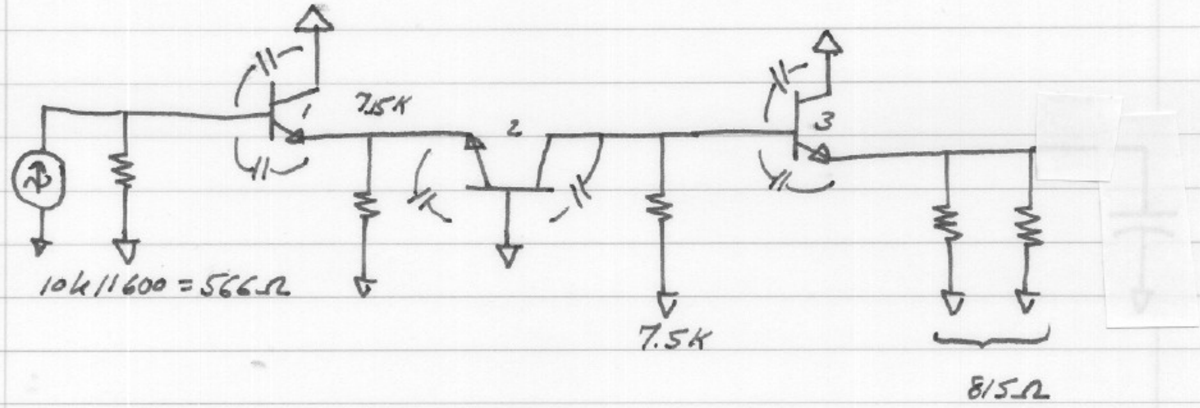
$R_i = R_{inA} \,//\, R_{gen} = 510\,\Omega$

Input $v_{in} / v_{gen} = R_{inA} / (R_{inA} + R_{gen}) = 0.85$

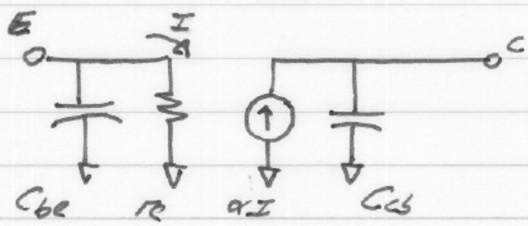
Overall gain

$v_o / v_{gen} = 0.85 \cdot 1/2 \cdot 264 \cdot 0.994 = 112$

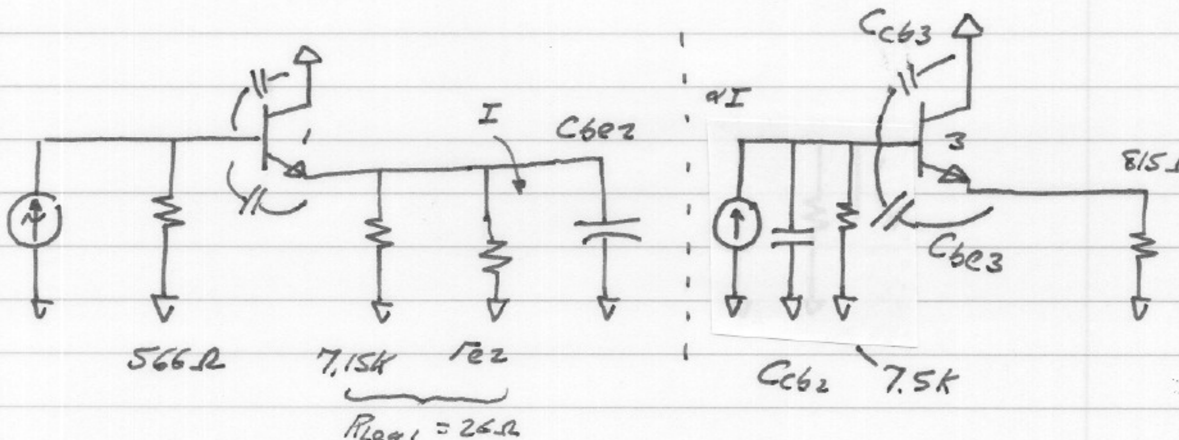
High frequency analysis



note Q2 common-base T model:



overall model: can be separated into 2 parts



part1

part 2

Part A: We have the choice of substituting into the E.F. gain expressions we have derived, or of doing MOTC step-by-step. Lets' do substitution:

$$a_1 = C_{cb1}\cdot(R_i \| R_{inT1}) + C_{be2}\left[R_{Leq1} \| (r_{e1} + R_i/\beta)\right]$$
$$+ C_{be1}\left[\beta r_{e1} \middle\| \left[R_i(1-A_{v1}) + R_{Leq1} \| \frac{1}{g_{m1}}\right]\right]$$

$$= 4pF \cdot 510\Omega + 23pF\left[26\Omega \| (26\Omega + \frac{566\Omega}{100})\right]$$
$$+ 23pF\left[2.6K \middle\| \left[566\Omega(1-1/2) + 26\Omega \| 26\Omega\right]\right]$$

$$= 2.04ns + 0.33ns + 6.11ns = 8.48ns$$

$$a_2 = (R_i \| R_{inT1})(R_{Leq1} \| \frac{1}{g_{m1}})\left[C_{be1}C_{cb1} + C_{be1}C_{be2} + C_{cb1}C_{be2}\right]$$

$$= (510\Omega)(13\Omega)\left[23pF\cdot 4pF + 23pF\cdot 23pF + 4pF\cdot 23pF\right]$$

$$= 4.7(10^{-18})\,sec^2 = (2.17ns)^2$$

<u>use SPA:</u>

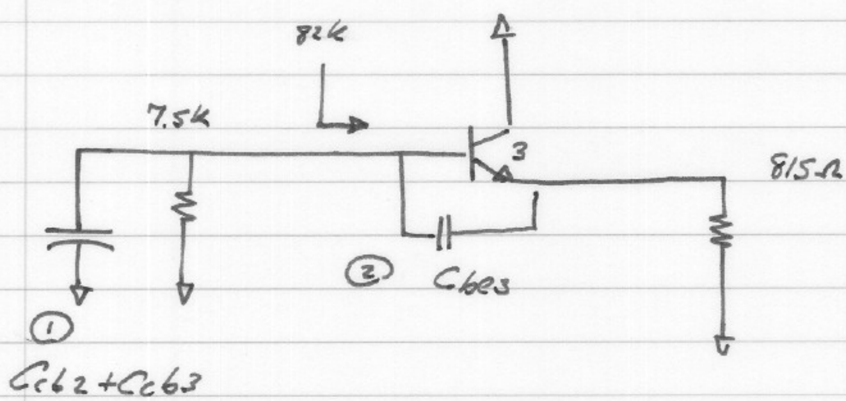
$$f_{p1} \cong 1/2\pi a_1 = 18.7\,MHz$$

$$f_{p2} \cong a_1/2\pi a_2 = 286\,MHz$$

use of SPA a little risky here: ok if 10:1 separation.

there is also a zero @ $f_z = g_m/2\pi C_{be} = 260\,MHz$.

Part B Lets' work this by MOTC:



$a_1 = R_{11}^{\circ} C_1 + R_{22}^{\circ} C_2$

Charging resistance for C_1: $7.5k \parallel 82k = R_{eq2} = 6.9K$

$R_{11}^{\circ} C_1 = 6.9k \cdot 8pF = 55.2\ ns$

Charging resistance for C_2: $\{7.5k[1 - A_{v3}] + r_{e3} \parallel 815\Omega\} \parallel \beta r_{e3}$

$= \{7.5k(1 - 0.994) + 5.2 \parallel 815\} \parallel 520\Omega$

$= 45.7\Omega$

$R_{22}^{\circ} C_2 = 45.7\Omega \cdot 100pF = 4.57ns$

→ $a_1 = 60\ ns$

$a_2 = R_{11}^{\circ} C_1 C_2 R_{22}' = 6.9k \cdot 8pF \cdot 100pF \cdot 5.1\Omega = 2.8(10^{-17})sec$
$= (5.3\ ns)^2$

$R_{22}' = 5.2 \parallel 815 \parallel 520\Omega = 5.1\Omega$

use SPA again

$f_{p1} \cong 1/2\pi a_1 = 2.65\ MHz$

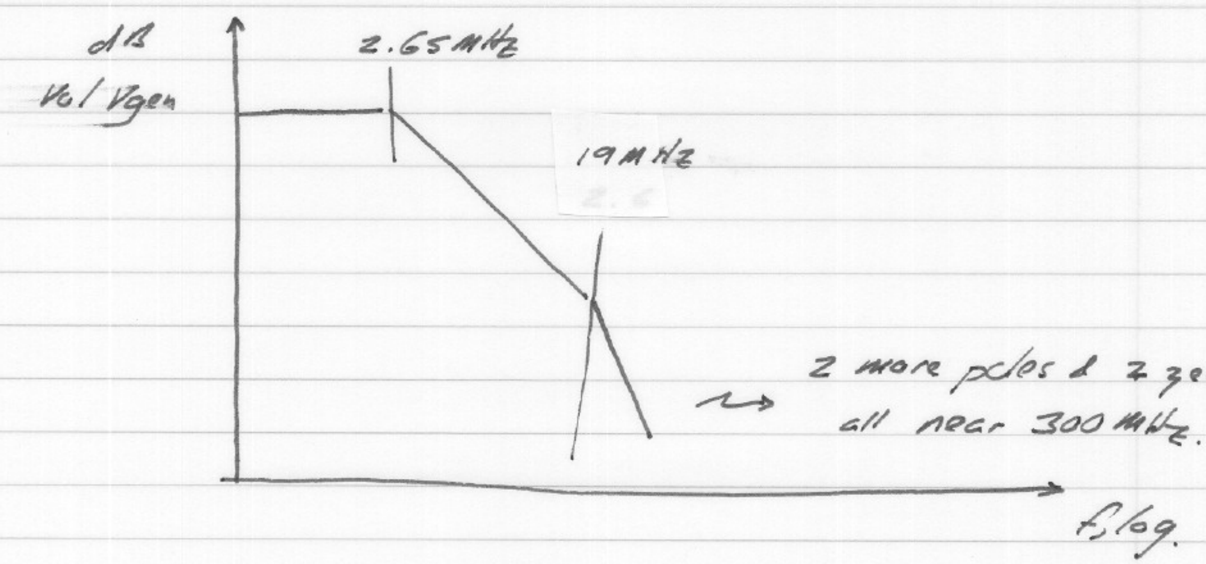
$f_{p2} \cong a_1/2\pi a_2 = 340 MHz$; also a zero @ $f = g_m/2\pi C_{be3} = 305 MHz$.

Part A poles: 18.7 MHz, 286 MHz

zero: 260 MHz

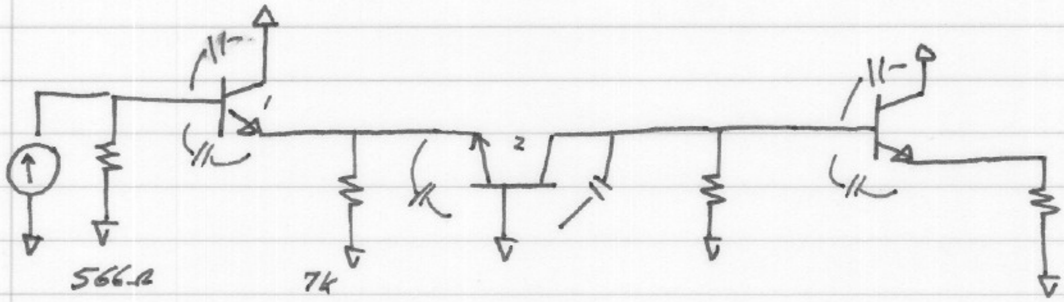
Part B: poles: 2.65 MHz, 340 MHz

zero: 305 MHz.

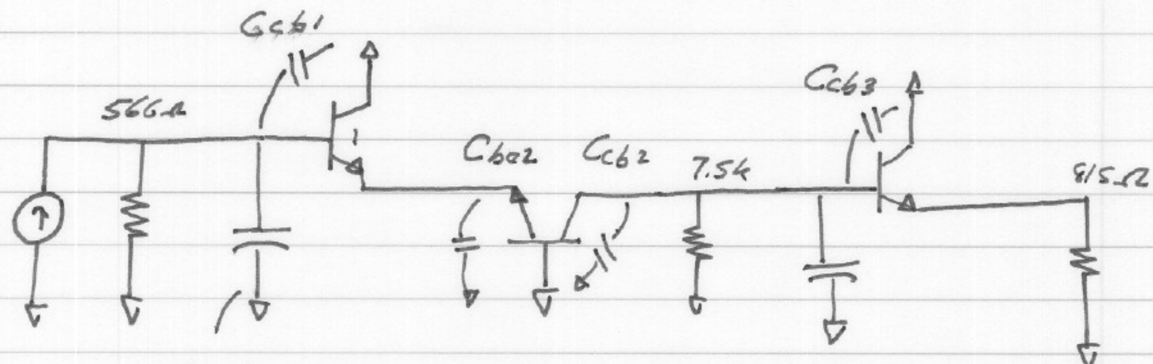


Now lets look at a very approximate way of estimating bandwidth very quickly

Use of Miller Approximations Node-by-Node.



Replace each capacitor not-to-ground with a Miller capacitor $C(1-A) = C_{miller}$.



$C_{miller1}$

$= C_{be1}(1-A_{v1}) = 11.5\,pF$

$C_{miller2}$

$= C_{be3}(1-A_{v3}) = 0.6\,pF$

We can now calculate time constants node by node

Node: base of Q1

$R = 566\Omega \parallel R_{in1} = 510\Omega$

$C = C_{cb1} + C_{miller1} = 4pF + 11.5\ pF = 15.5pF$

$\tau = RC = 7.9ns$

$f_p = 1/2\pi\tau = 20.1\ MHz \leftarrow$

Node: emitter of Q1

$R = r_{out1} \parallel R_{in2} = (566\Omega/\beta + r_{e1}) \parallel r_{e2} = 14.3\Omega$

$C = C_{be2} = 23pF$

$\tau = RC = 328ps$

$f_p = 1/2\pi\tau = 484\ MHz.$

Node: collector of Q2

$R = 7.5k \parallel R_{in3} = 6.9k$

$C = C_{cb3} + C_{miller2} = 4pF + 0.6pF = 4.6pF$

$\tau = RC = 31.7ns$

$f_p = 1/2\pi\tau = 5.0\ MHz.$

Node: emitter of Q3: no capacitances.

This method gave poles @ 5 & 20 MHz & at 484 MHz.

One pole & 2 zeros have been lost.

Method is good for quick checks when doing designs, but usually gets $a_2/2\pi a_1$ pole wrong by vast amount.