ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. October 26, 2015

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

Problem	Points Received	Points Possible
1		15
2a		10
2b		15
3a		10
3b		10
3c		10
4		15
5		15
total		100

Name: Solution.

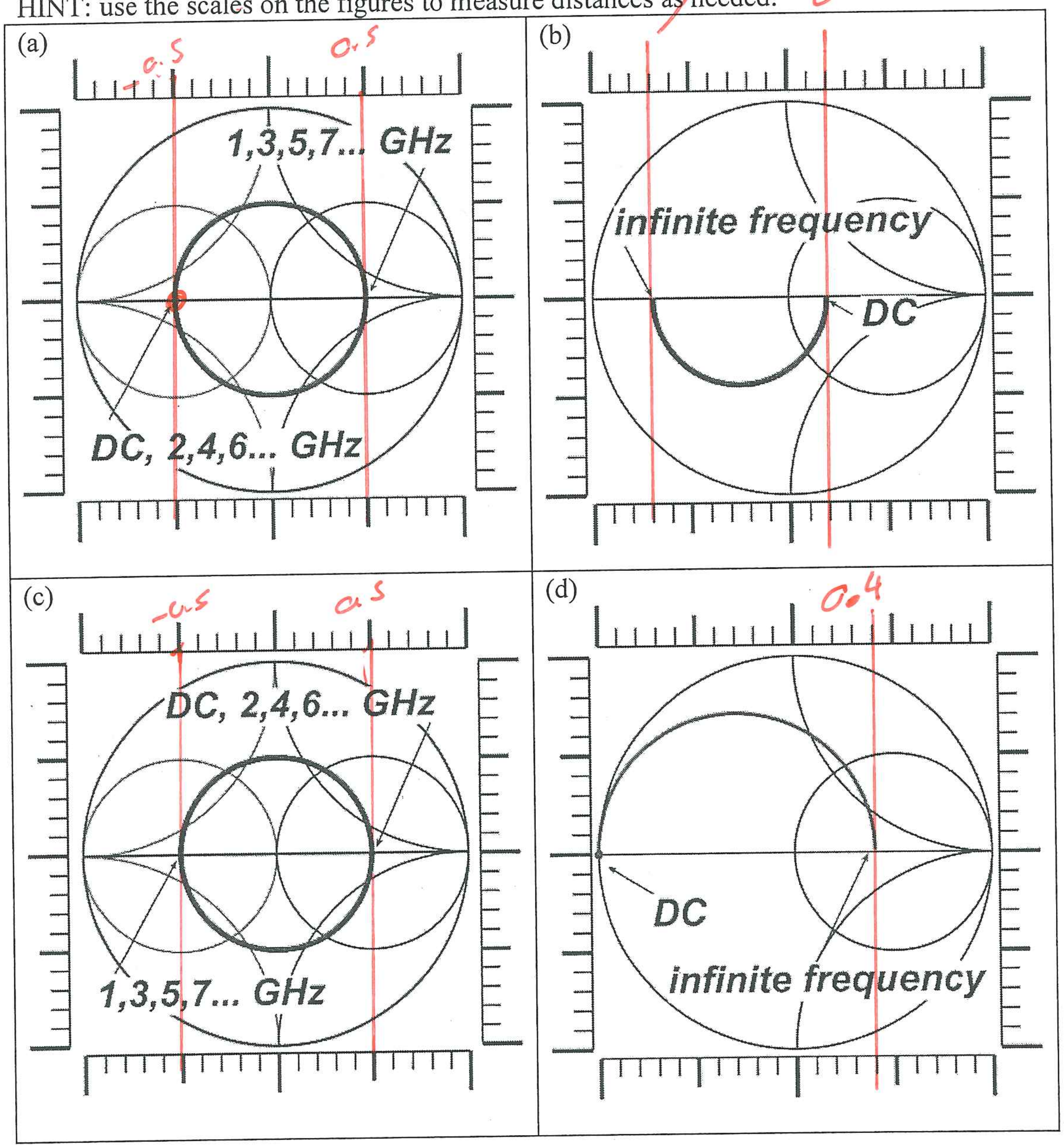
Problem 1, 15 points

The Smith Chart and Frequency-Dependent Impedances.

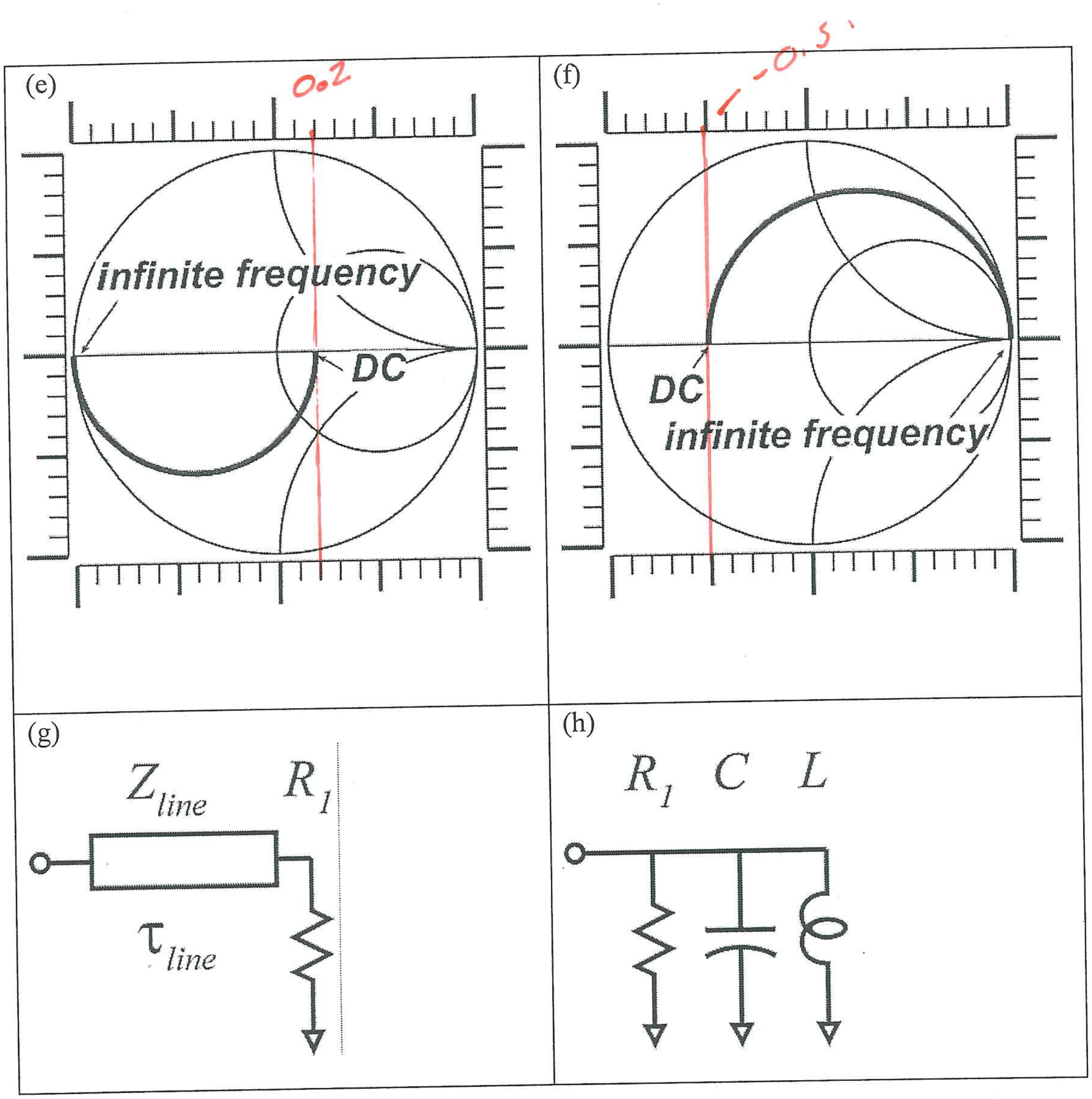
HINT: use the scales on the figures to measure distances as needed.

-0.7

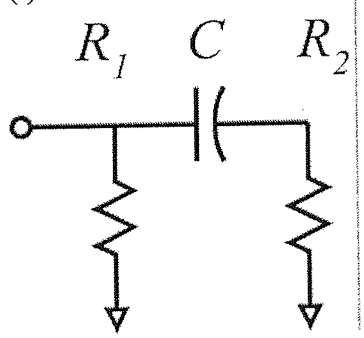
0.2



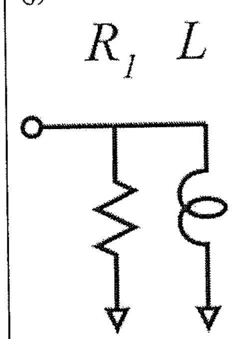
a)


(e)
0.2
infinite frequency
DC
(f)
-0.5
DC
infinite frequency
(g)
Z_{line}
R_1
τ_{line}
(h)
R_1 C L

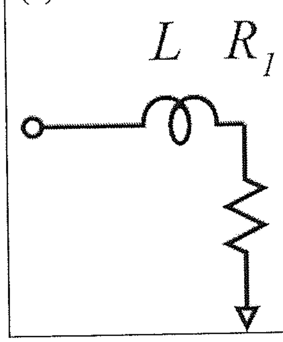
(i)



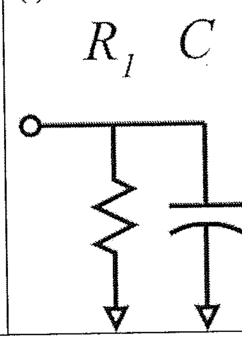
(j)



(k)



(l)



First match each Smith Chart with each circuit. ***Then determine as many component values as is possible*** (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given. The charts all use 50 Ohm normalization:

Smith chart (a). Circuit= g . Component values= $Z_{line} = 50\Omega$, $R_L = 17\Omega$, $T = 250ps$
Smith chart (b). Circuit= i . Component values= $R_1 = 75\Omega$, $R_2 = 10\Omega$.
Smith chart (c). Circuit= g . Component values= $Z_{line} = 50\Omega$, $R_L = 150\Omega$, $T = 250ps$
Smith chart (d). Circuit= j . Component values= $R = \cancel{133\Omega}$ 117
Smith chart (e). Circuit= (e). Component values= $R = 75\Omega$
Smith chart (f). Circuit= (h). Component values= $R = 16.7\Omega$

3

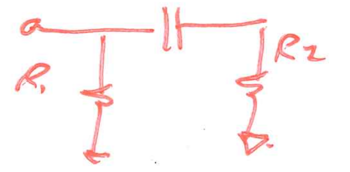
a) Is a 50Ω transmission line loaded. 1
with $\frac{Z_L}{Z_0} = \frac{1+\Gamma_{DC}}{1-\Gamma_{DC}} = \frac{1-0.5}{1+0.5} = 1/3 \rightarrow Z_L = 50/3 = 16.7\Omega$ 1

T: ~~at~~ the line is $\lambda/2$ long @ 2GHz, λ long @ 4GHz 1
4GHz → 250ps period ⇒ T = 250ps

3

b) $\Gamma = 0.2$ @ DC
$Z/Z_0 = \frac{1.2}{0.8} = 1.5 \rightarrow Z = 1.5 \cdot 50\Omega = 75\Omega = R_1$ 1

$\Gamma = -0.7$ @ $f \rightarrow \infty$.
$Z/Z_0 = \frac{1-0.7}{1+0.7} = \frac{0.3}{1.7} = 0.176 \rightarrow Z = 8.8\Omega = R_1 \| R_2$ 1

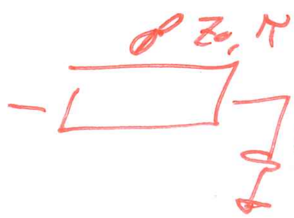


$\frac{1}{R_2} = \frac{1}{8.8} - \frac{1}{75} =$ 1
$R_2 = 9.97\Omega$

3

c) Similar to (a)

circuit is (g) 1



$Z_0 = 50\Omega$, $\tau = 250ps$ 1

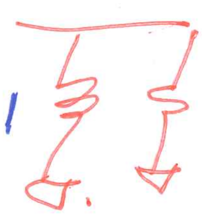
but $R_L = ?$

$\Gamma_{DC} = 0.5 \rightarrow R_L = Z_0 \frac{1+0.5}{1-0.5} = 3Z_0 = 150\Omega$ 1

2

d) this is network (j)

1



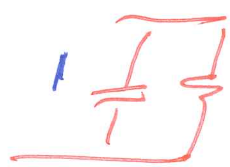
$R = Z_0 \frac{1+0.4}{1-0.4} = 50\Omega \frac{1.4}{0.6} = 1\cancel{33}\Omega$

117

2

e) Network (l)

1



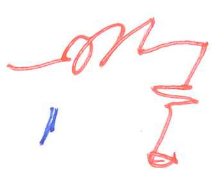
$R = 50\Omega \frac{1+0.2}{1-0.2} = 50\Omega \frac{1.2}{0.8}$ 1

$= 50\Omega \frac{3}{2} = 75\Omega$

2

(f) network (k)

1



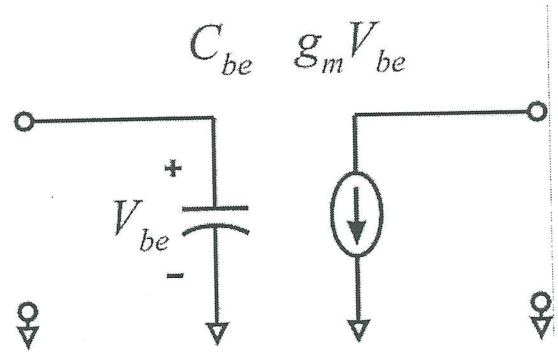
$R = 50\Omega \frac{1-0.5}{1+0.5} = 50\Omega \frac{0.5}{1.5}$

$= \frac{50\Omega}{3} = 16.7\Omega$

Problem 2, 25 points
2-port parameters and Transistor models

Part a, 10 points
For the network at the right, give algebraic expressions for the four S-parameters.

Assume a normalization to impedance Zo for the S parameters.



3

$$Z_{in}\Big|_{Z_L=Z_0} = 1/j\omega C_{be}.$$

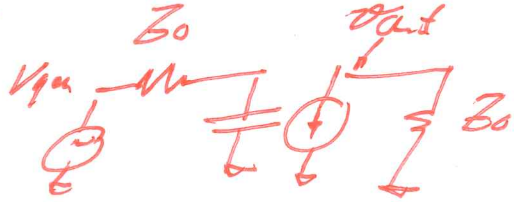
$$\Rightarrow S_{11} = \frac{1/j\omega C_{be}Z_0 - 1}{1/j\omega C_{be}Z_0 + 1} = \frac{1 - j\omega C_{be}Z_0}{1 + j\omega C_{be}Z_0}$$

2

$$Z_{out}\Big|_{Z_L=Z_0} = \infty.$$

$$\Rightarrow S_{22} = 1.0$$

2 by Inspection, $S_{12} = 0$.



3

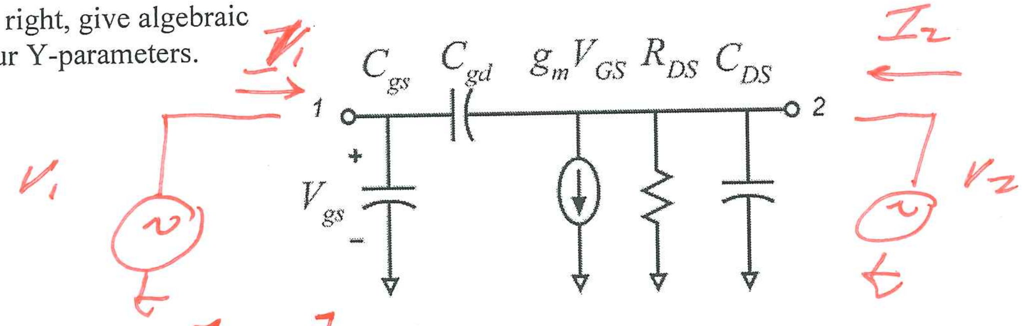
$$S_{21} = 2\, V_0/V_{gen}\Big|_{Z_{gen}=Z_L=Z_0}$$

$$= 2 \cdot \frac{1/j\omega C_{be}}{1/j\omega C_{be} + Z_0} \cdot (-g_m Z_0)$$

$$S_{21} = \frac{-2 g_m Z_0}{1 + j\omega C_{be} Z_0}.$$

Part b, 15 points

For the network at the right, give algebraic expressions for the four Y-parameters.



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

by inspection:

4 [$Y_{11} = j\omega(C_{gs} + C_{gd})$

3 [$Y_{12} = -j\omega C_{gd}$.

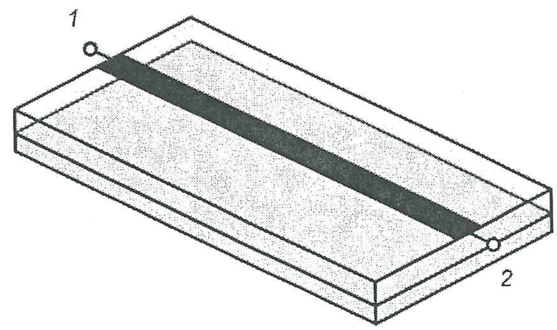
4 [$Y_{21} = g_m - j\omega C_{gd}$.

4 [$Y_{22} = \underbrace{1/R_{DS}}_{\text{or } G_{DS}} + j\omega C_{DS}$.

Problem 3, 30 points
Transmission-line theory

Hint: we are testing here your understanding of transmission-lines and their relationships to lumped elements. If the calculation appears to be extremely difficult, you may possibly be missing some key insight.

Part a, 7.5 points
You have a microstrip line of 10 cm length, and 5mm width. The substrate is 2mm thick and has a dielectric constant of 2.0. Treat fringing fields approximately by assuming that the effective conductor width is the physical conductor width plus twice the substrate thickness



Find the characteristic impedance of the line, the velocity, the total line inductance, and the total line capacitance.

$H_{eff} = 2\,mm$, $W = 5\,mm$, $W_{eff} = 9\,mm$.

what follows is very approximate:

2 $\left[Z_0 \cong \frac{377\Omega}{\sqrt{\epsilon_r}} \cdot \frac{H}{W_{eff}} = \frac{377\Omega}{\sqrt{2}} \frac{2\,mm}{9\,mm} = 59.3\Omega \cong 60\Omega. \right.$

1.5 $\left[v \cong \frac{c}{\sqrt{\epsilon_r}} = 2.12 \cdot 10^8\, m/s. \right.$

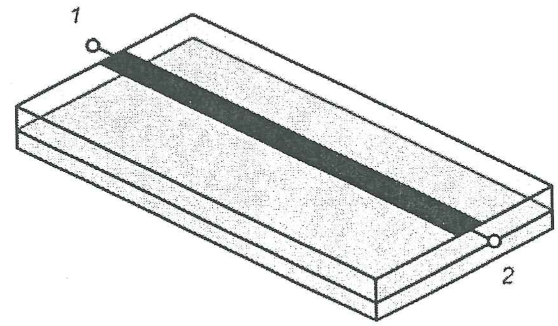
1 $\left[\tau = \text{line delay} = \frac{10\,cm}{v} = 4.7 \cdot 10^{-10} \text{ seconds} = 0.47\,ns \right.$

2 $\left[\text{total inductance} = L = \tau Z_0 = 0.47\,ns \cdot 60\Omega = 28.2\,nH. \right.$

2 $\left[\text{total capacitance} = C = \frac{\tau}{Z_0} = \frac{0.47\,ns}{60\Omega} = 7.8\,pF \right.$

Part b, 7.5 points

We now load port 2 with a resistance of 10 kOhm. Give an ***approximate*** expression for the frequency-dependent input impedance, measured at port 1. The approximation need be valid only over a DC-100MHz signal frequency range.



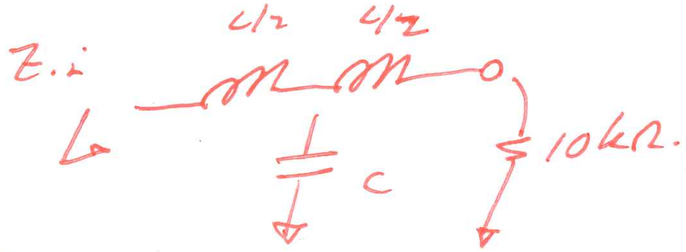
$L = 28\,nH$

$C = 7.8\,pF$

2.5

$100\,MHz \cdot 0.48\,ns = \underline{0.047\lambda}$ long

$\Rightarrow$ use T or Π.



2.5

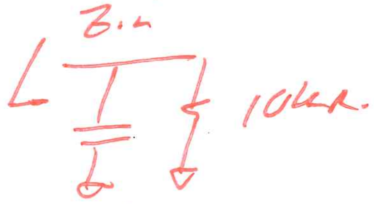
~~Ac~~ now $R_L = 10k\Omega \gg \cancel{Z_0} = 50\Omega$.

so $L/R_L \ll R_L C$

$\Rightarrow$ Effect of inductance is negligible.

2.5

$\Rightarrow$

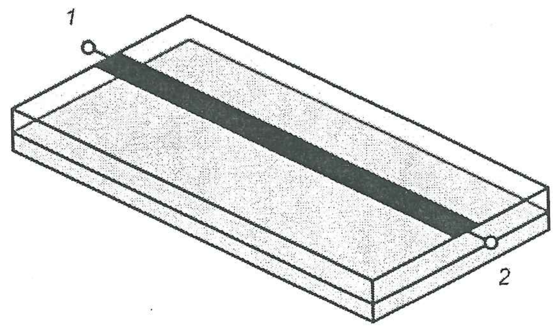


$$Z_{in} = \frac{1}{j\omega C} \| 10k\Omega = \frac{10k\Omega}{1 + j\omega C(10k\Omega)}$$

$$Z_{in} = \frac{10k\Omega}{1 + j\omega(7.8pF)10k\Omega} = \frac{10k\Omega}{1 + j\omega(780\,ns)}$$

Part c, 7.5 points

We now load port 2 with a resistance of 2 Ohms. Give an ***approximate*** expression for the frequency-dependent input impedance, measured at port 1. The approximation need be valid only over a DC-100MHz signal frequency range.



2.5

again $\omega T \ll 1$

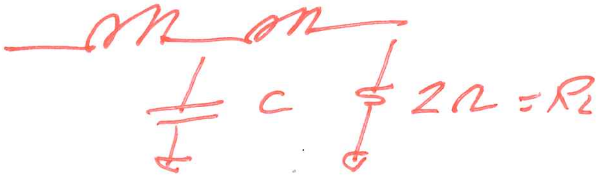
$\Rightarrow$ length $\ll \lambda$

so use π or T approximation.

L/2 L/2

$L = 28nH$

$C = 7.5pF$



2.5 now $R_L \ll Z_0$, so $RC \ll L/R$. $\Rightarrow$ ignore C.

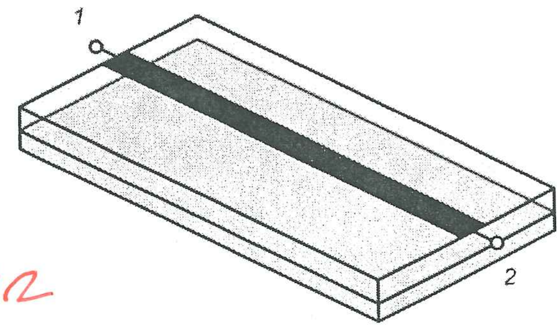
2.5.

Z_{in} 28nH.

$Z_{in} = 2\Omega + j\omega(28nH)$.

Part d, 7.5 points

At what frequency is the line one quarter-wavelength in length ? If we load port 2 with a 30 Ohm resistance, what would be the input impedance at port 1 at this frequency ?



$Z_0 = 60\Omega$

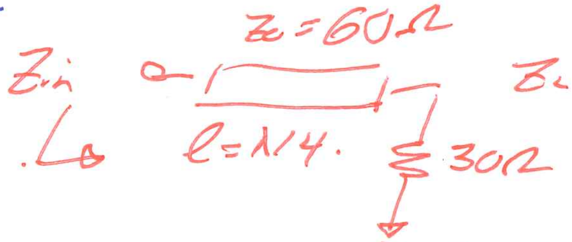
$\tau = 0.47ns \cong 0.5ns.$

3.5

quarter-wavelength ~~on~~ when

$f = 1/4\tau = 1/2ns = 500MHz.$

4



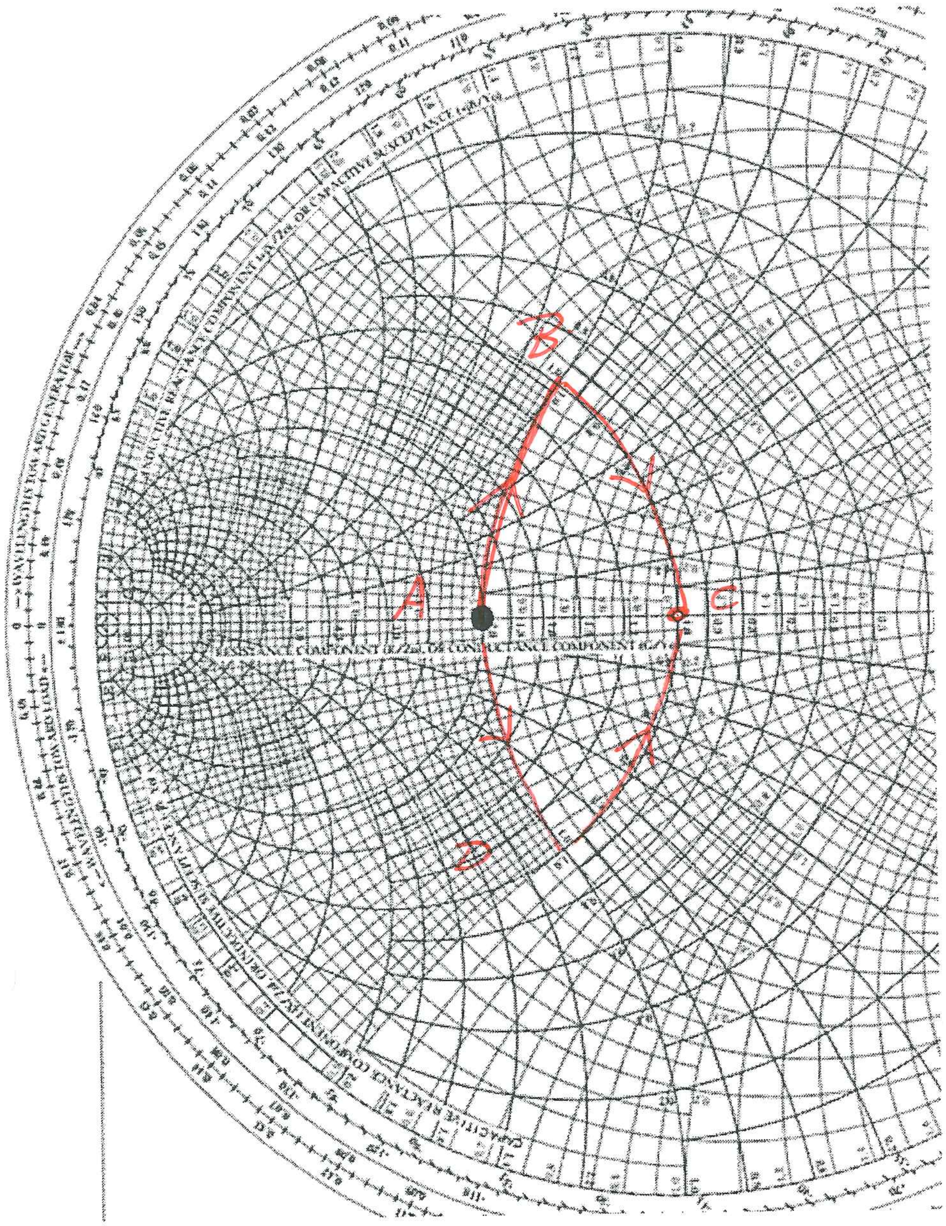
$Z_L/Z_0 = 30\Omega/60\Omega = 1/2$

at $\lambda/4 = L$, ~~Z_{in}~~ $Z_{in}/Z_0 = Z_0/Z_L = 2$

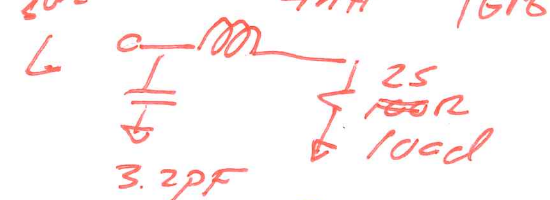
$Z_{in} = 2 \cdot 60\Omega = 120\Omega$

Problem 4, 15 points
Impedance-matching exercise.

The (50 Ohm normalization) Smith chart gives the input impedance of a circuit at 1 GHz signal frequency. Design a lumped-element matching network which converts this impedance to **50Ohms** at 1 GHz. Give all element values.



Solutions are A–B–C
and A–D–C.



Path ABC

point A: $z = 0.5 + j0$; $y = 2 + j0$ 3

point B: $z = 0.5 + j0.5$. 4

$\Delta z = j0.5 \Rightarrow \Delta Z = j0.5 \cdot Z_0 = j25\Omega = j\omega L$

$\omega L = 25\Omega \Rightarrow L = \dfrac{25\Omega}{2\pi(1GHz)} = \underline{\underline{4.0nH}}$

point B: $y = 1 - j1.0$.

point C $y = 1 + j0$. 4

$\Delta y = +j1.0 \Rightarrow \Delta Y = \dfrac{j1.0}{50\Omega} = j\omega C$ 4

$\Rightarrow C = \dfrac{1}{50\Omega \cdot 2\pi f} = 3.2pF$.

Solution 1

---

or Solution 2. Path ADC

point A: $y = 2 + j0$, $z = 0.5 + j0$ 4

Point D $z = 0.5 - j0.5$

$\Delta z = -j0.5 \Rightarrow \Delta Z = -j25\Omega = 1/j\omega C$ 4

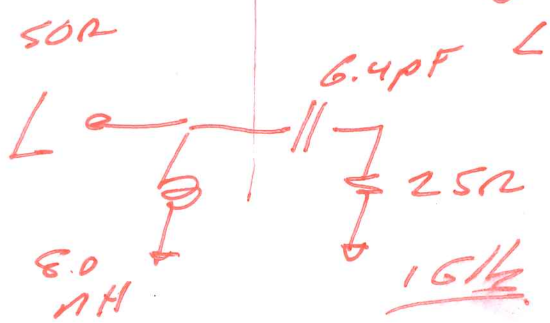
$C = \dfrac{1}{25\Omega \cdot 2\pi f} = 6.4pF$.

point D: $y = 1 + j$

point C: $y = 1 + j0$ 3

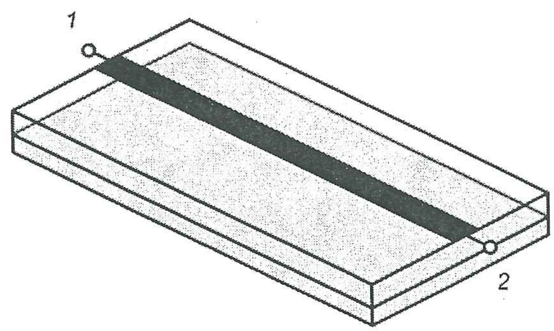
$\Rightarrow \Delta y = -j1.0 \Rightarrow \Delta Y = \dfrac{-j1.0}{50\Omega} = \dfrac{1}{j\omega L}$ 4

$L = \dfrac{50\Omega}{2\pi f} = 8.0nH$



Problem 5, 15 points
Transmission-line parasitics.

Part a, 7.5 points
You have a microstrip line of 10 cm length, and 5mm width. The substrate is 2mm thick and has a dielectric constant of 2.0.



Neglect the fringing fields.

The conductivity of gold is $44.2*10^6$ Siemens/meter and $\mu_0 = 4\pi * 10^{-7}\, H/m$. Find (i) the skin depth, (ii) the attenuation constant α, and (iii) the total line attenuation at 10GHz signal frequency.
Hint----the skin depth is $\delta = \sqrt{2/\omega\mu_0\sigma}$

1 [If we neglect fringing fields, then
$Z_0 = \frac{377\Omega}{\sqrt{\epsilon_r}} \frac{h}{W} = \frac{377\Omega}{\sqrt{2}} \frac{2mm}{5mm} = 106.6\Omega.$ (high, because we have neglected f.f.)

1 [$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 0.76\mu m$

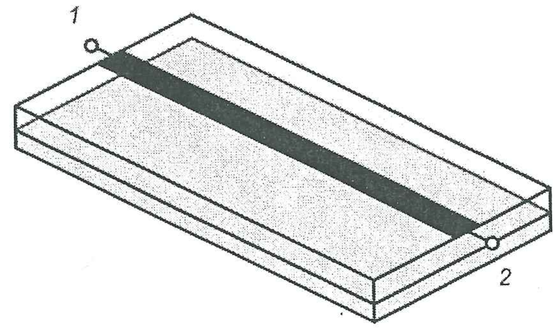
1 [series resistance per unit length:
$r_s = 1/\sigma\delta w = 6\,\text{ohms/meter}.$

1.5 [α = loss coefficient $= r_s/2Z_0 = \cancel{0.096}\ 0.028\ \text{meters}^{-1}.$

1.5 [voltage attenuation $= \exp[-\alpha l] = \exp[\cancel{0.096}\ 0.028\ \text{meters}^{-1} \cdot 0.1\,\text{meter}]$
$= \exp[\cancel{-0.0096}\ -0.0028] = 0.9972$

1.5 [Attenuation, dB $= -20\log_{10}[0.9972] = -0.024\,dB$
very small.

Part b, 7.5 points
We are not happy with the line attenuation we calculated above. So, we choose to make the circuit board thicker, while adjusting the conductor width to keep the same characteristic impedance. (i) If we increase the board thickness by 5:1, what is the total line attenuation now ? (ii) At what frequency might we expect to see lateral modes on the transmission-line ?



2.5

board 5:1 thicker
$\Rightarrow Z_0$ increased 5:1, α decreased 5:1
$\Rightarrow 5 \cdot 10^{-3}$ dB attenuation

2.5

~~board~~
board is 5:1 thicker,
so for constant Z_0, the
line is 5:1 wider, i.e. = 25 mm width.

2.5.

transverse modes when $\lambda_d/2 = 25$ mm

$\Rightarrow \lambda_d = 50$ mm $= 5$ cm.

$\lambda_0 = \lambda_d \cdot \sqrt{\epsilon_r} = 5\text{ cm} \cdot \sqrt{2} = 7.1$ cm.

$f\lambda_0 = c \Rightarrow f = c/\lambda_0 = \frac{3 \cdot 10^8 \text{ m/s}}{7.1 \text{ cm}}$

$= 4.2$ GHz

so, widening the line was a bad idea.