ECE ECE145A (undergrad) and ECE218A (graduate)

Final Exam. Tuesday, December 8, 12-3 p.m.

Do not open exam until instructed to.
Open notes, open books, etc. You have 3 hrs.
Use all reasonable approximations (5% accuracy is fine.),
AFTER STATING and justifying THEM.
Think before doing complex calculations. *Sometimes there is an easier way.*

Problem	Points Received	Points Possible
1a		5
1b		7
2a		7
2b		5
3a		5
3b		7
3c		8
3d		5
3e		5
3f		5
4a		5
4b		7
4c		5
4d		5
5a		6
5b		5
5c		8
total		100

Name: Solution.

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L|^2} \qquad G_P = \frac{1}{1-\|\Gamma_{in}\|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

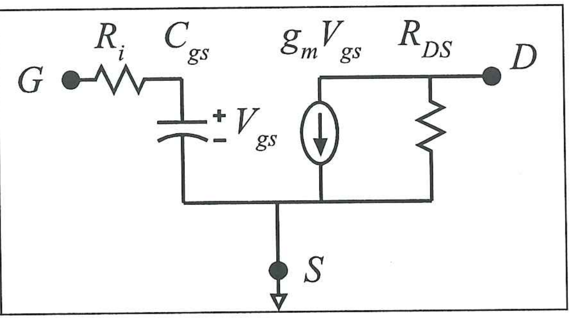
$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-\|\Gamma_{out}\|^2} \qquad G_{\max} = \frac{|S_{21}|}{|S_{12}|}\cdot\left[K-\sqrt{K^2-1}\right]\text{if } K>1$$

$$G_{MS} = \frac{|S_{21}|}{|S_{12}|}\cdot\text{if } K<1 \qquad K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{21}S_{12}|} \quad \text{where } \Delta=\det[S]$$

Unconditionally stable if : (1) K>1 **and** (2) $\|\det[S]\|<1$

Problem 1, 12 points
Two-port properties, Gain relationships

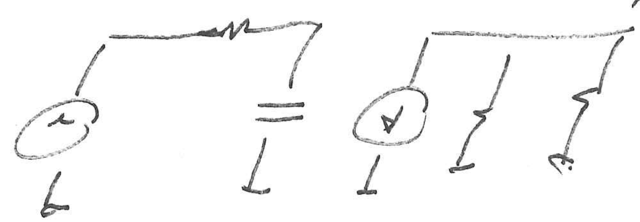
part a, 5 points

Transistor cutoff frequencies C_{gs}=100 fF, g_m=100 mS. R_{ds}=100 Ohms, R_i=10 Ohms, Find f_τ and $f_{\max}$.	

can work either by formula or by derivation.

by derivation:

$R_L = R_{DS}$.



2.5

compute/use Pout/Pin →

$$f_{max} = \frac{f_\tau}{2\sqrt{G_{ds}R_i}} = 251\,GHz$$

similarly:

2.5

$$f_\tau = \frac{g_m}{2\pi C_{gs}} = 159\,GHz$$

part b, 7 points

Find the short-circuit current gain and the maximum available power gain at 60 GHz

3.5 $h_{21} = \frac{159\,GHz}{60\,GHz} = \underline{\underline{2.65}} \rightarrow 8.5\,dB.$ ($20\log_{10}$)

3.5. un.lateral - so mag = u = $\left(\frac{\cancel{275}\,251\,GHz}{60\,GHz}\right)^2 = \underline{\underline{17.5}}$

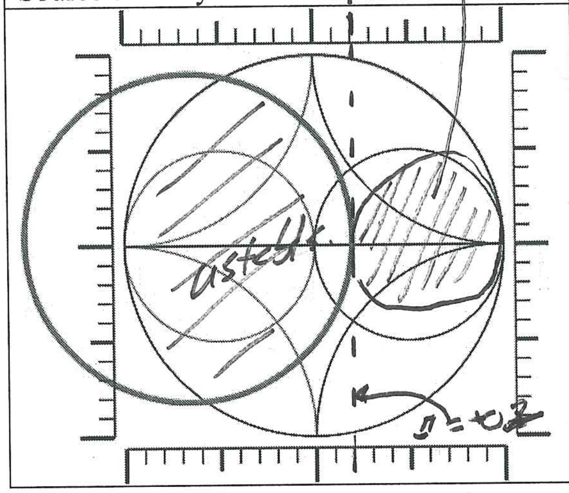
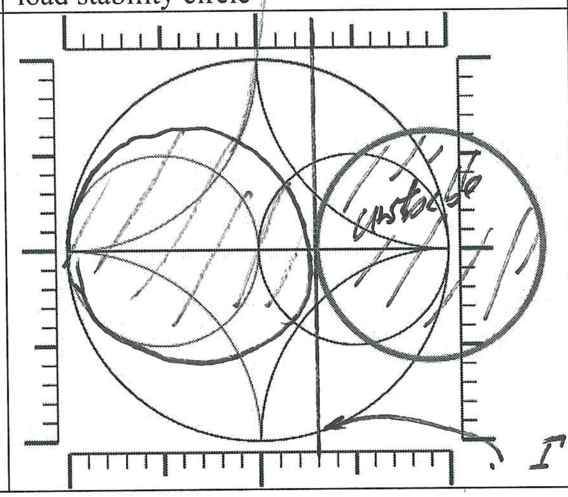
$\rightarrow$ 12.4 dB. ($10\log_{10}$).

restrict to this region!
restrict to this region.

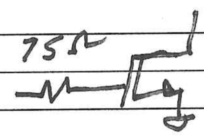
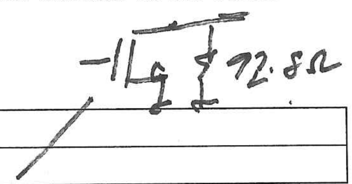
Problem 2, 12 points
Potentially unstable amplifier design

part a, 7 points

At a design frequency of 1 GHz, a common-source FET has source and load stability circles as below

Source stability circle	load stability circle
	

Given that S11=0.5 and S22=1.1 at 1GHz, draw two stabilization circuits in the boxes below, giving element values

Solution 1	Solution 2
	

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \quad ; \quad \Gamma_{out} = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{22}\Gamma_S}.$$

source stab. ckt: $\Gamma_S = 0$ gives $\Gamma_{out} = S_{22} = 1.1 > 1$, so center of smith chart is unstable] 2

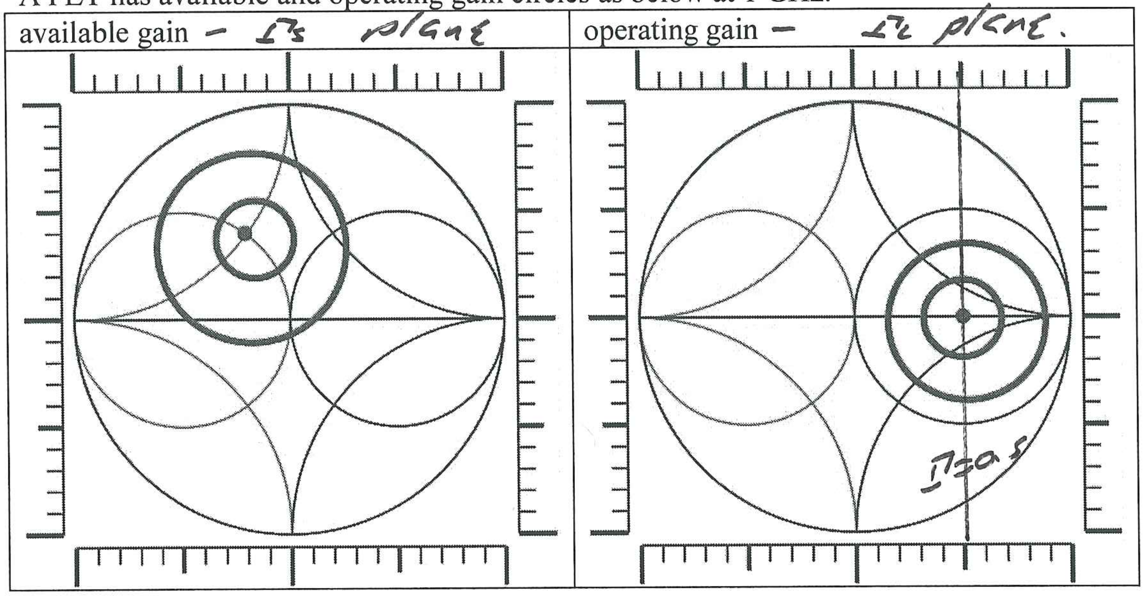
must add series resistance on input of $50\Omega \cdot \frac{1+0.2}{1-0.2} = 75\Omega$] 1.5

load stability circle. $\Gamma_L = 0$ gives $\Gamma_{in} = S_{11} = 0.5 < 1$ so center of smith chart is stable] 2

must add parallel resistance on output of $50\Omega \cdot \frac{1+0.3}{1-0.3} = 92.8\Omega$] 1.5.

part b, 5 points

A FET has available and operating gain circles as below at 1 GHz.



Assuming a 50Ohm impedance normalization, what are the optimum generator and load impedances ?

$Z_{gen,opt}$ = $\cancel{}\,^{35}$ $(1+j)\ \Omega$. $Z_{l,opt}$ = $150 + j0\ \Omega$.

$$G_A = \frac{P_{AVA}}{P_{AVG}} \overset{\text{iff o/p matched}}{=} \frac{P_L}{P_{AVG}} = G_T$$

so $G_A = G_A(\Gamma_s)$

optimum Y_s $y_s = 1 - j1$

$$Y_{s,opt} = \frac{1-j1}{50\Omega}$$

$$Z_{s,opt} = \frac{50\Omega}{1-j1} = \frac{50}{\sqrt{2}}\Omega(1+j)$$

2.5 · ~~Z~~ 3

$$G_P = \frac{P_L}{P_{in}} \overset{\text{iff I/P matched}}{=} \frac{P_L}{P_{AVG}} = G_T$$

2

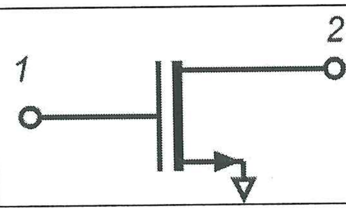
so $\underline{G_P} = G_P(\Gamma_L)$

2.5

optimum $\Gamma_L = 0.5$

3. optimum $Z_L = 50\Omega \dfrac{1+0.5}{1-0.5} = 50\Omega \dfrac{1.5}{0.5} = 150\Omega$

Problem 3, 35 points
Power gains and stability

The transistor has S11=0, S12=0.1, S21=8, S22=0.5	

part a, 5 points

If the load impedance is an open-circuit, what is the input reflection coefficient?

Γ_{in} = 1.6

3

5.

$$\Gamma_{in} = \underbrace{S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}}_{3} = S_{11} + \frac{S_{21} S_{12}}{1 - S_{22}}$$

2 for arithmetic

$$= 0 + \frac{8(0.1)}{1-0.5} = \frac{0.8}{0.5} = 1.6,$$ which is greater than 1

So, without having to compute K, B1, we can see that this network is potentially unstable

part b, 7 points

Is it necessary to stabilize the device before simultaneous input and output matching to it ? Assuming that you have stabilized, if necessary, or have not stabilized (if not necessary), what power gain will you obtain after matching on both input and output ?

Unconditionally Stable ? NO.

Power gain after simultaneous matching= 80:1.

3 from part a, with $|\Gamma_L| \le 1$, we have $|\Gamma_{in}| > 1$, so potentially unstable.

since it is potentially unstable,

2 the power gain after stabilizing & matching

is the MSG $= \frac{|S_{21}|}{|S_{12}|} = \frac{|8|}{|0.1|} = 80.$ (19.0 dB).

2

part c, 8 points

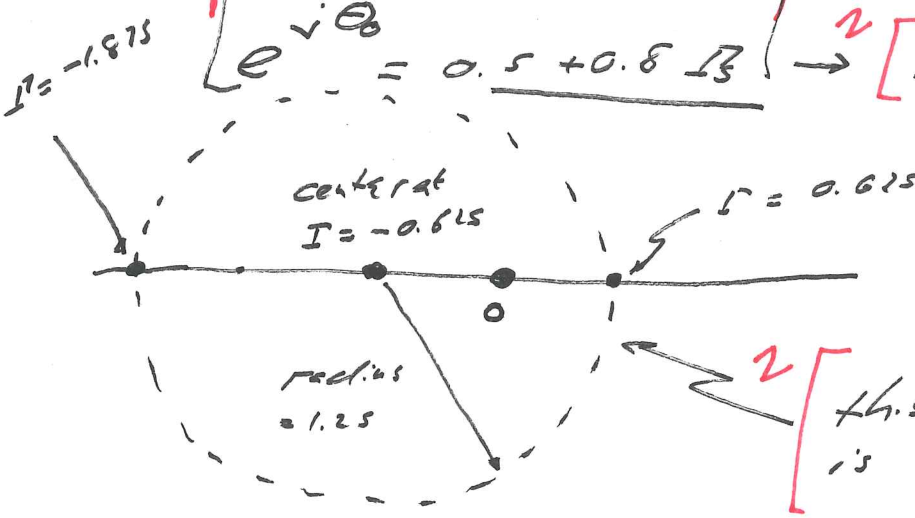
(hard thinking, ok math): Can you determine from the S-parameters above what values of source reflection coefficient would lead to potential instability ? Can you determine the necessary value of parallel input stabilization resistance ?

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S} = 0.5 + 0.8\,\Gamma_S$$

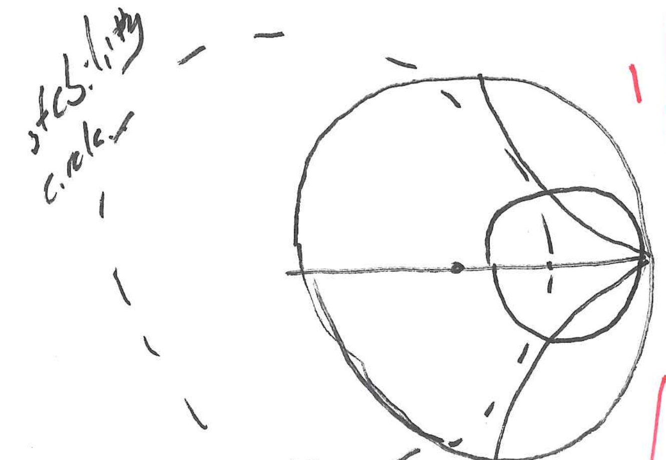
set $|\Gamma_{out}| = 1$, so $\left[\Gamma_{out} = e^{j\theta_0}\right]$ 1

$$\boxed{e^{j\theta_0} = 0.5 + 0.8\,\Gamma_S} \rightarrow \;2\; \left[\Gamma_S = 1.25\cdot e^{j\theta} - 0.625\right]$$

0.625, −1.875



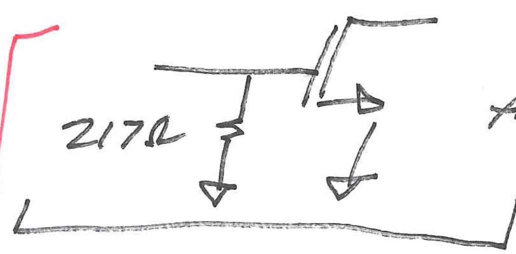
2 [this circle, in the Γ_S-plane, is the region of stability]



1 [so, we need to keep Γ_S below 0.625

$$50\,\Omega \; \frac{1+0.625}{1-0.625} = 217\,\Omega.]$$

1



this will stabilize the amplifier.

$S_{11} = 0, \quad S_{12} = 0.1, \quad S_{21} = 8, \quad S_{22} = 0.5$

part d, 5 points

Without stabilizing the FET, the FET is connected to a 100 Ohm generator, with 1mW available power, and a 100 Ohm load. Find the power in the load

$P_L =$ 91 mW.

2. we are being asked for the transducer gain, since $G_T = \frac{P_L}{P_{AVG}}$.

1 $$G_T = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)(1 - |\Gamma_S|^2)}{|(1 - \Gamma_S S_{11})(1 - \Gamma_L S_{22}) - S_{21} S_{12} \Gamma_S \Gamma_L|^2}$$

$$\Gamma_L = \Gamma_S = \frac{100 - 50}{100 + 50} = 1/3.$$

1 $$= \frac{8^2 [1 - 1/9][1 - 1/9]}{|(1)(1 - \frac{1}{3}\frac{1}{2}) - \frac{0.8}{9}|^2}$$

$$= \frac{50.56}{|0.741|^2} = 91.23 = \frac{P_L}{P_{AVG}}$$

1 $P_L = 91.23 \cdot 1\,mW = 91.23\,mW$

part e, 5 points

Without stabilizing the FET, the FET is connected to a 50 Ohm generator, with 1mW available power, and a 50 Ohm load. Find the power in the load

$P_L =$ 64 mW.

4. we are being asked for the insertion gain, which is $|S_{21}|^2$

1. $P_L = 1\,mW\,(8)^2 = 64\,mW$

$S_{11} = 0$ $S_{12} = 0.1$ $S_{21} = 8$ $S_{22} = 0.5$

part f, 5 points

Without stabilizing the device, the generator, with 1mW available power, is impedance-matched to the FET input, and is then connected directly to a 100 Ohm load. Find the power in the load

$P_L =$ 90.9 mW

$G_T = \frac{P_L}{P_{AVG}}$ $\frac{P_L}{P_i} = G_P$ / $\Gamma_L = \frac{100-50}{100+50} = 1/3$

1

Since the generator is (?) matched to the input, we have $G_P = G_T$ and $P_{load} = G_P \cdot P_{AVG}$.

1

$\Gamma_{in} = \cancel{S_{11}}^{0} + \frac{S_{21} S_{12} \Gamma_L}{1 - S_{22}\Gamma_L} = \frac{0.8 \cdot 1/3}{1 - 0.5 \cdot 1/3} = \frac{0.2666}{0.8333}$

$= 0.320$ i.e. with this particular Γ_L.

note that $|\Gamma_{in}| < 1$ with the FET is potentially unstable, In other words, though the load we have been given lies within the stable region on the Γ_L-plane. Had this not been true, the solution would not have existed (!).

1

$$G_P = \frac{1}{1 - ||\Gamma_{in}||^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L S_{22}|^2}$$

1

$$= \frac{1}{1 - (0.32)^2} \, 64 \, \frac{1 - 1/9}{(1 - 1/2 \cdot 1/3)^2}$$

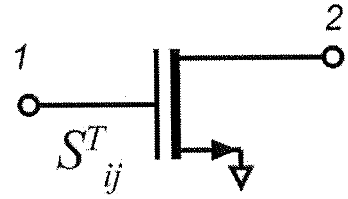
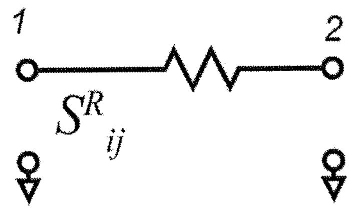
$$= 1.11 \cdot 64 \cdot \frac{8/9}{0.69} = 90.93$$

1

$P_L = 90.93 \cdot 1\text{mW} = 90.93\text{mW}$

Problem 4, 22 points

S parameters and Signal flow graphs

A transistor has the following s-parameters: S11=0.5 S22=0.25 S12=0.5 S21=5	
A second two-port consists of a 25 Ohm resistor between its input and output ports	

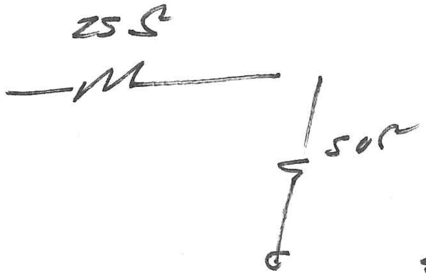
part a, 5 points

Using a 50 Ohm impedance standard, compute the four S-parameters of the resistor network.

S11= 1/5 S12= 4/5 S21= 4/5
S22= 1/5

2.5

S_{11} (and S_{22})



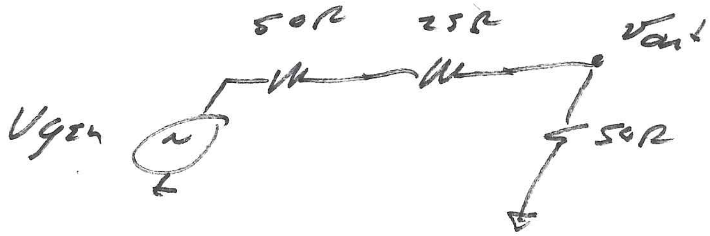
$Z_{in} = 75\Omega$

so $S_{11} = \dfrac{75\Omega - 50\Omega}{75\Omega + 50\Omega} = \dfrac{25}{125}$

$= 1/5.$

2.5

S_{21} (and S_{12})



$\dfrac{v_{out}}{v_{gen}} = \dfrac{50\Omega}{125\Omega}$

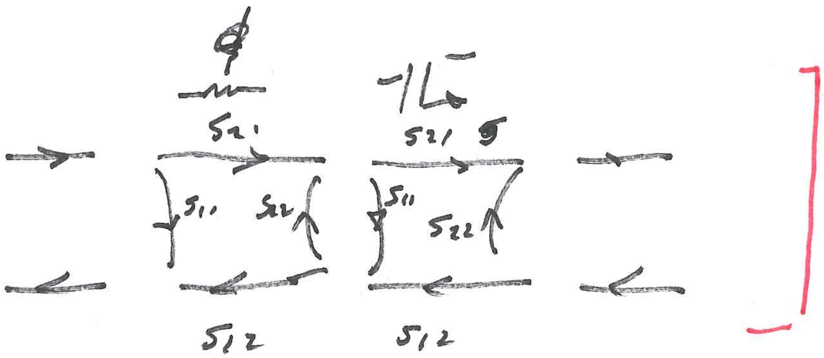
$= \dfrac{2}{5}$

so $S_{21} = 2\dfrac{V_o}{V_g}\Big|_{Z_L = Z_0 = Z_S} = 4/5$

part b, 7 points

The resistor network is connected between to the FET input. Compute the four S-parameters of the combined network.

S11= 0.555 S12= 0.444 S21= 4.44.
S22= 0.805.



2

$$S_{21}^{overall} = \frac{S_{21}^R S_{21}^T}{1 - S_{22}^R S_{11}^T} = \frac{(4/5)(5)}{1-(1/5)(0.5)}$$

$$= \frac{4}{1-0.1} = 4.44$$

half for formula
half for math

2

$$S_{12}^{overall} = \frac{S_{12}^R S_{12}^T}{1 - S_{21}^R S_{11}^T} = \frac{(4/5)(0.5)}{1 - 1/5\,(0.5)} = 0.444.$$

2

$$S_{11}^{overall} = \frac{S_{11}^R(1 - S_{22}^R S_{11}^T) + S_{21}^R S_{11}^T S_{12}^R}{1 - S_{22}^R S_{11}^T} = S_{11}^R + \frac{S_{21}^R S_{11}^T S_{12}^R}{1 - S_{22}^R S_{11}^T}$$

$$= 1/5 + \frac{(4/5)(0.5)(4/5)}{1 - 1/5\,(0.5)} = 1/5 + \frac{0.32}{0.9} = 0.555$$

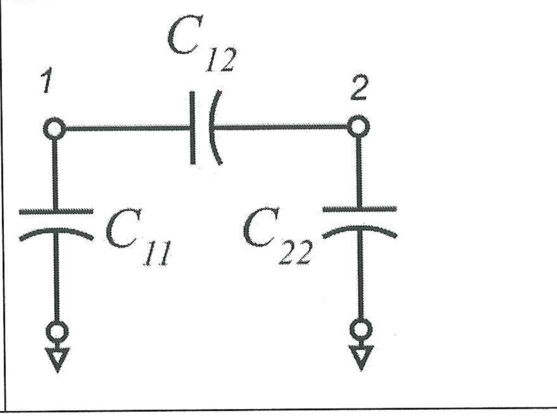
1

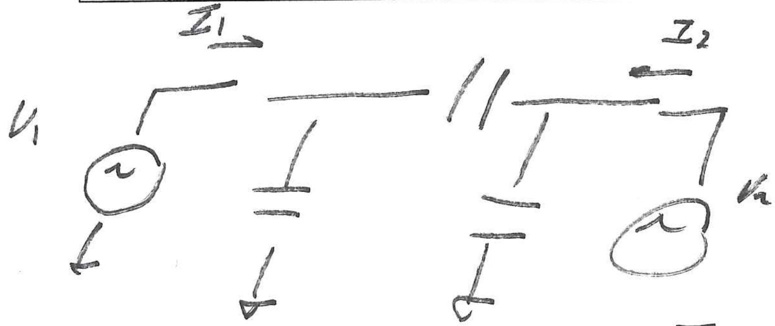
$$S_{22}^{overall} = S_{22}^T + \frac{S_{12}^T S_{21}^T S_{22}^R}{1 - S_{22}^R S_{11}^T} = 0.25 + \frac{5(1/2)(1/5)}{1 - 1/5\,(0.5)} = 0.25 + \frac{0.5}{0.9}$$

$$= 0.805$$

part c, 5 points

Y-parameters

Compute the Y-parameters of this network	

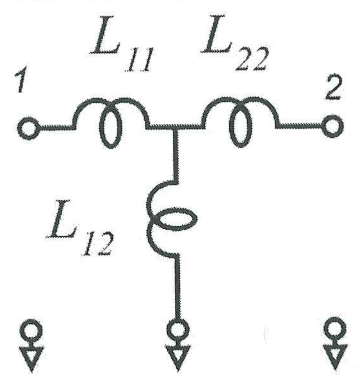


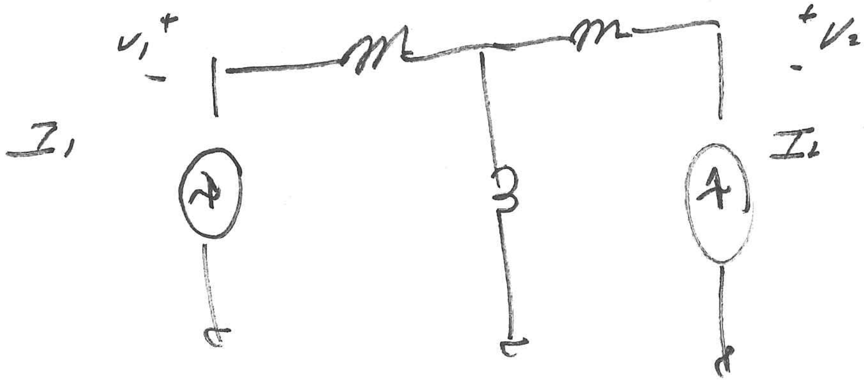
By inspection: $[Y_{ij}] = \begin{bmatrix} j\omega C_{11} + j\omega C_{12} & -j\omega C_{12} \\ -j\omega C_{12} & j\omega C_{12} + j\omega C_{22} \end{bmatrix}$

1.25 for each parameter

part d, 5 points

Z-parameters

Compute the Z-parameters of this network	

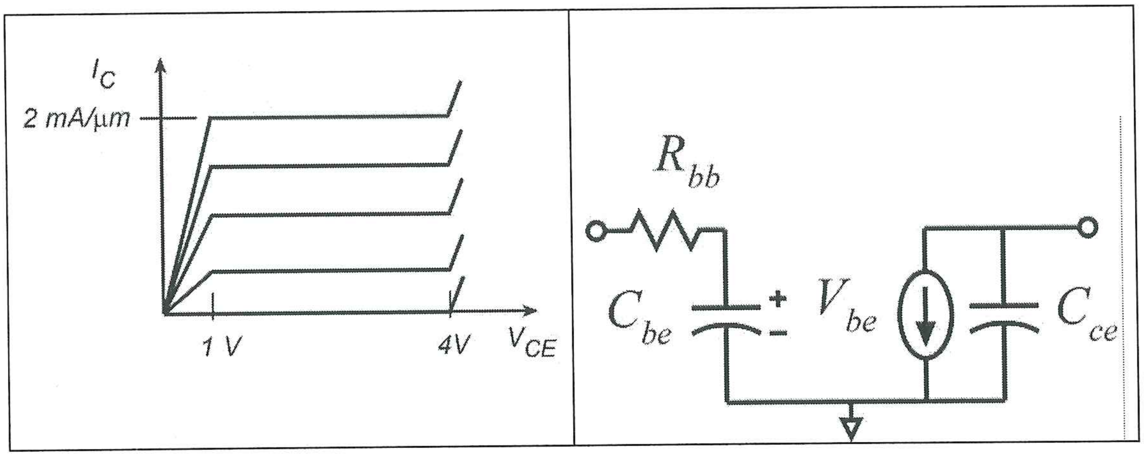


by inspection:

$$[Z_{ij}] = \begin{bmatrix} j\omega L_{11} + j\omega L_{12} & j\omega L_{12} \\ j\omega L_{12} & j\omega L_{22} + j\omega L_{12} \end{bmatrix}$$

1.25 for each parameter.

Problem 5, 19 points
Power amplifier design



An HBT has the output characteristics as shown, with a maximum 2mA/micron collector current. The (somewhat contrived) device model is to the right, with
$g_m = 20.0mS/\mu m \cdot L_E$ $\quad R_{bb} = 20\Omega - \mu m / L_E$ $\quad C_{be} = g_m \tau_f$, where $\tau_f = 0.5$ ps,
$C_{CE} = 2 fF/\mu m \cdot L_E$

<u>part a, 6 points</u>

The optimum load *admittance* is parallel combination of a conductance G and an inductive susceptance. Setting G to 40 milliSiemens, and setting the signal frequency to 100GHz, find (1) the appropriate HBT emitter length Le and (2) the required parallel load inductance L.

a.1 [optimum load impedance for $L_E = 1\mu m$ is
$Z_{L,opt} = \frac{3V}{2mA} = 1.5k\Omega$ with parallel inductive loading.

So... we have a load of $40mS = 1/25\Omega$

2 [$\rightarrow L_E = \frac{1500\Omega}{25\Omega} = \frac{6000\Omega}{100\Omega} = \underline{60\mu m}$

1 [$\Rightarrow C_{CE} = 2fF/\mu m \cdot 60\mu m = 120$ ~~p~~ fF

2 [need $2\pi f = 1/\sqrt{LC} \rightarrow$ $L = \frac{1}{C(2\pi f)^2} = \underline{21.1 pH}$.

2

part b, 5 points

What is the maximum saturated output power ? What is the correct collector bias voltage and collector bias current ?

3 $P_{max} = \frac{1}{8} \Delta V \cdot \Delta I = 1/8 \cdot 3V \cdot (2\frac{mA}{\mu m} \cdot 60 \mu m)$

$= \frac{1}{8} \cdot 3V \cdot 120 mA = \underline{\underline{45 mW}}$

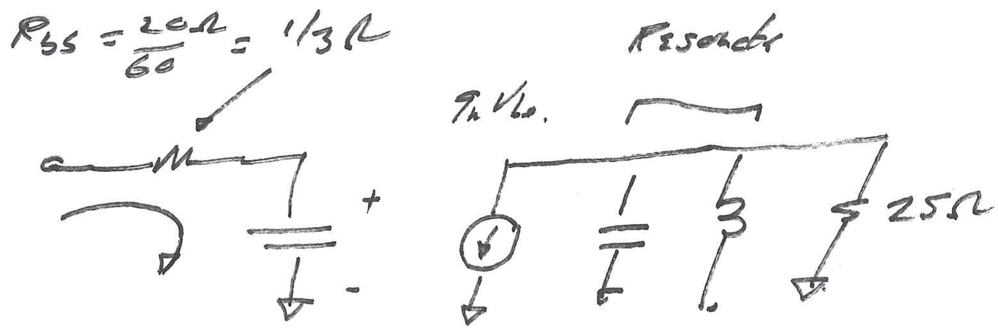
1 $V_{bias} = \frac{1V + 4V}{2} = 2.5V$

1 $I_{bias} = 60 mA.$

part c, 8 points

After impedance-matching on the amplifier input and output, what is the amplifier power gain ?

ckt and parameters. 2



$C_{be} = g_m \tau_f \qquad g_m = \frac{2\,\text{mS}}{\mu\text{m}} \cdot 60\,\mu\text{m} = 120\,\text{mS}.$

$= 60\,fF$

2 $P_{out} = I_{out}^2 \cdot 25\Omega. = (g_m V_{be})^2\, 25\Omega$

2 $P_{in} = I_{in}^2 \cdot (1/3\,\Omega) = (\omega C_{be} V_{be})^2 (1/3)$

1 $\frac{P_o}{P_{in}} = \left(\frac{g_m}{\omega C_{be}}\right)^2 \frac{25\Omega}{1/3\,\Omega}$ but $C_{be} = g_m \tau_f$ so $\frac{g_m}{\omega C_{be}} = \frac{g_m}{\omega g_m \tau_f} = \frac{1}{\omega \tau_f}$

$\frac{P_o}{P_{in}} = \left(\frac{1}{2\pi f \tau_f}\right)^2 \frac{25\Omega}{1/3\,\Omega} = 10.1 \cdot \frac{25\Omega}{1/3\,\Omega} = \underline{\underline{760}}$

100 GHz $\quad$ 1/2 pS

1 $\frac{P_o}{P_{in}} = 760$ quite unrealistic (silly transistor #s).

(29 dB)