ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. October 26, 2016

Do not open exam until instructed to.

Open notes, open books, etc.

You have 1 hr and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

Problem	Points Received	Points Possible
1		15
2a		10
2b		15
2c (218 only)		15 (218)
3a		10
3b		10
3c		10
4		15
5a		7.5 (145) or 12.5 (218)
5b		7.5
total		85 (145) or 105 (218)

Name: Solution.

$\Gamma = -0.5$

$z = \frac{1+\Gamma}{1-\Gamma} = 1/3 \rightarrow Z = \frac{50}{3}\,\Omega$

$\Gamma = +0.8$

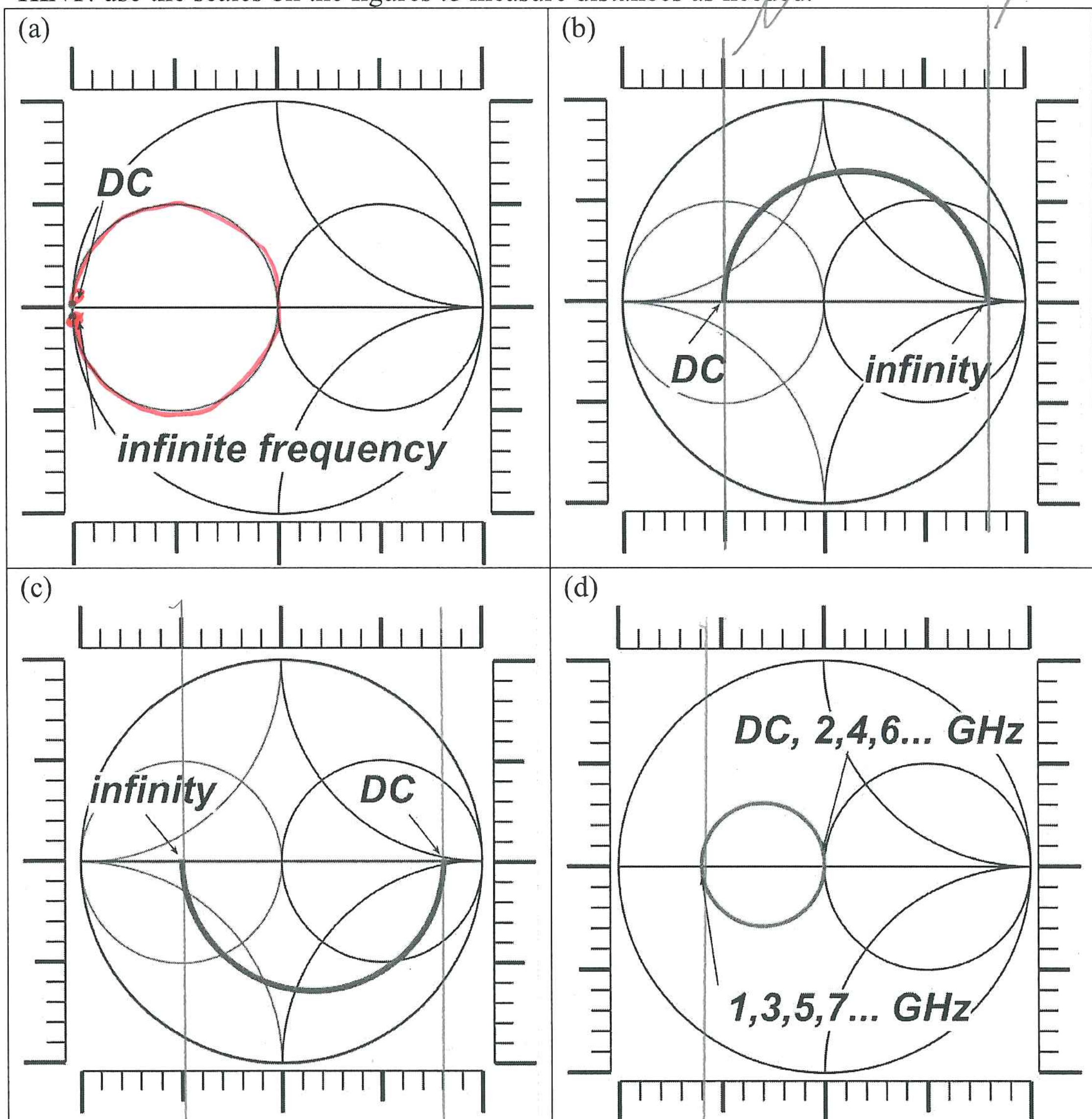
$z = \frac{1.8}{0.2} = 9$

$Z = 450\,\Omega$

Problem 1, 15 points

The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.



$\Gamma = -0.5$

$Z = \frac{50}{3}\,\Omega$

$\Gamma = +0.8$

$Z = 450\,\Omega$

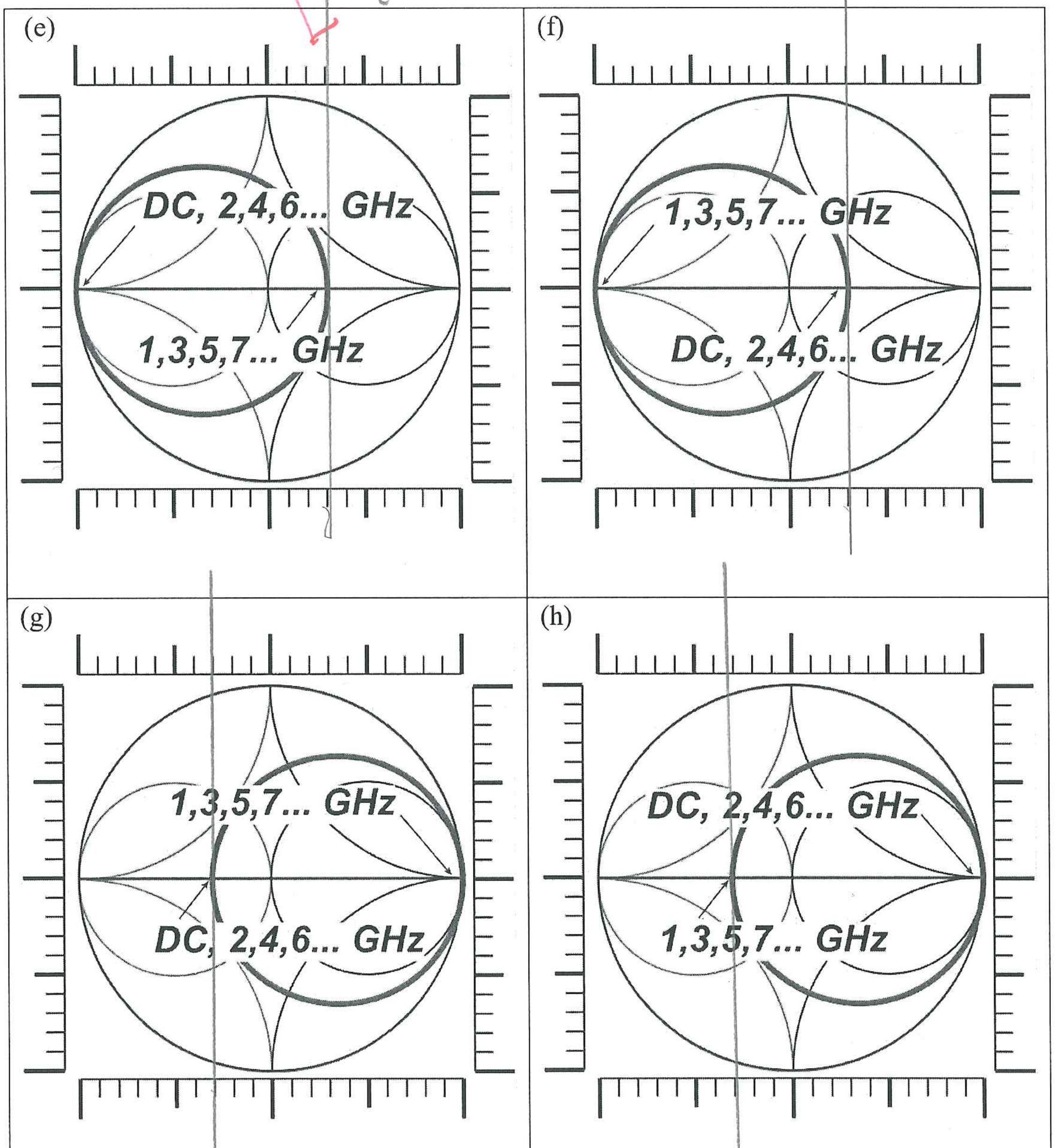
$\Gamma = -0.6$

$z = \frac{1-0.6}{1+0.6} = \frac{0.4}{1.6} = \frac{1}{4}$

$Z = \frac{50}{4}\,\Omega$

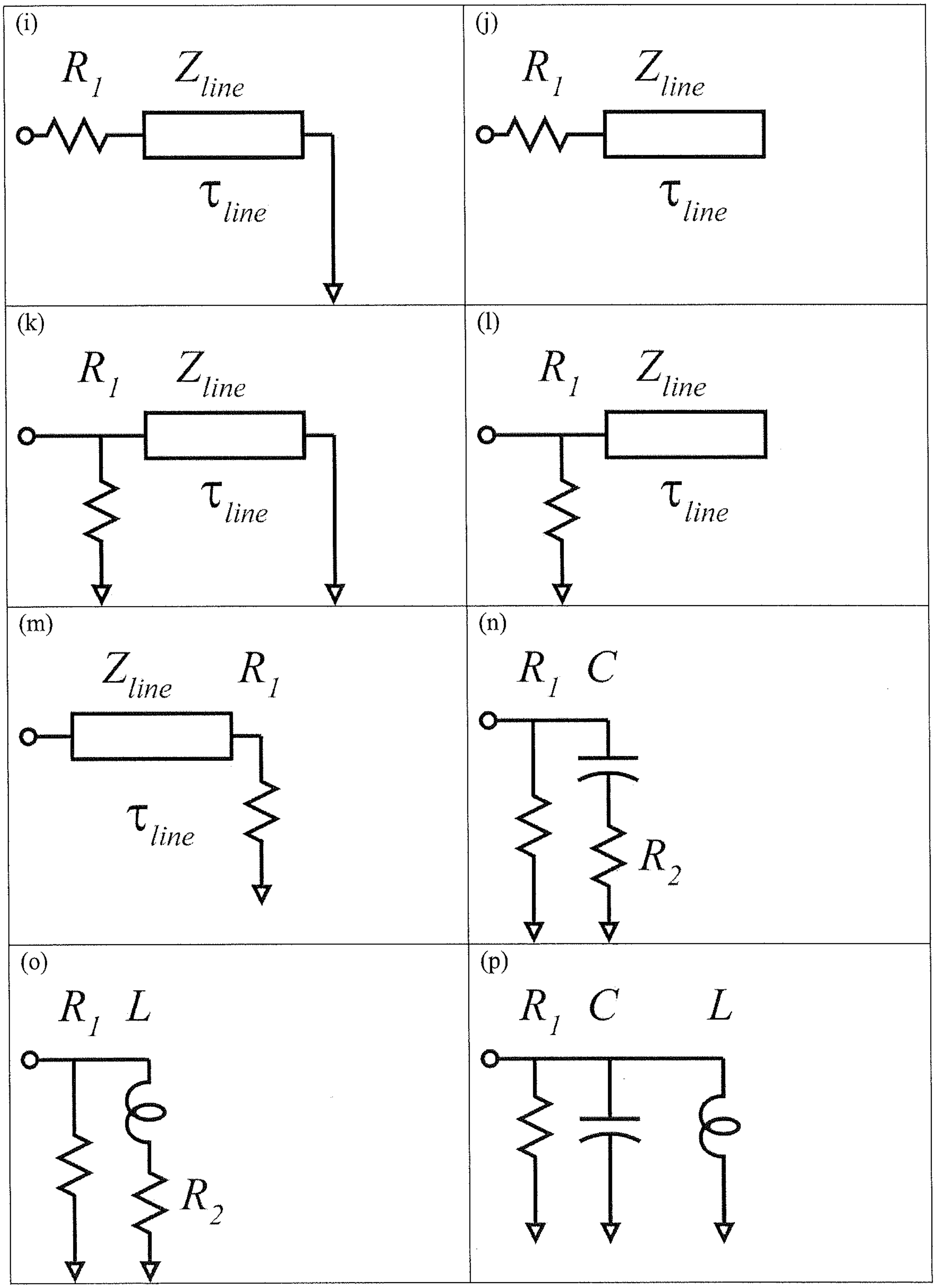
$\Gamma = 0.3$
$z = \frac{1.3}{0.7}$
$Z = 93\,\Omega$

$\Gamma = 0.3$
$Z = 93\,\Omega$



$\Gamma = -0.3$
$z = \frac{0.7}{1.3}$
$Z = 27\,\Omega$

$\Gamma = -0.3$
$Z = 27\,\Omega$


(i)
R_1 Z_{line} τ_{line}
(j)
R_1 Z_{line} τ_{line}
(k)
R_1 Z_{line} τ_{line}
(l)
R_1 Z_{line} τ_{line}
(m)
Z_{line} R_1 τ_{line}
(n)
R_1 C R_2
(o)
R_1 L R_2
(p)
R_1 C L

First match each Smith Chart with each circuit. ***Then determine as many component values as is possible*** (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given. The charts all use 50 Ohm normalization:

2 Smith chart (a). Circuit= P .
Component values: R = 50Ω. , ____ , ____ ,

2 Smith chart (b). Circuit= O .
Component values: R_1 = 450Ω , R_2 = 17.3Ω; ____ ,

2 Smith chart (c). Circuit= N .
Component values: R_1 = 450Ω , R_2 = 17.3Ω. , ____ ,

Smith chart (d). Circuit= M .
2 Component values: R = 50Ω , Z_c = 25Ω , $\tau = \frac{1}{4}$ ns. ,

Smith chart (e). Circuit= k .
2 Component values: R_1 93Ω , $\tau = \frac{1}{4}$ns. , ____ ,

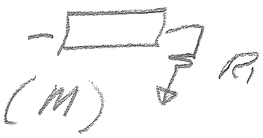
2 Smith chart (f). Circuit= l .
Component values: R = 93Ω , $\tau = \frac{1}{4}$ns. , ____ ,

2 Smith chart (g). Circuit= i .
Component values: R = 27Ω , τ = 1/4ns. , ____ ,

1 Smith chart (h). Circuit= j .
Component values: R = 27Ω , τ = 1/4ns. , ____ ,

Smith Chart D

Impedance is 50Ω @ DC, 2, 4, 6, ... GHz
is $50/4\,\Omega$ @ 1, 3, 5, 7 ... GHz

⇒  (M)

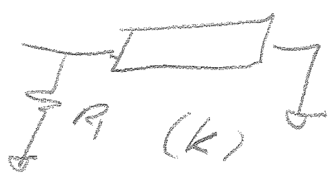
This is a T-line with $R_1 = 50\Omega$,

2 & $Z_0 = \sqrt{50\Omega \cdot \frac{50}{4}\Omega} = \frac{50}{2}\Omega = 25\Omega$.

$\ell = \lambda/4$ @ 1 GHz so $\tau = \frac{1/4}{1\,GHz} = \frac{1}{4}\,ns$.

Smith Chart E

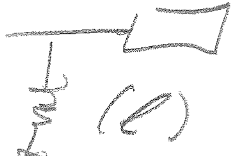
0Ω @ DC, 2, 4, 6 GHz,
93Ω at 1, 3, 5, 7, 9 GHz

2 ⇒  (k)

$R_1 = 93\Omega$, Z_0 unknown.
line is $\lambda/4$ @ 1GHz.
$\tau = (1/4)/1GHz = 1/4\,ns$

Smith Chart F

0Ω @ 1, 3, 5, 7, ... GHz
93 ~~50~~ Ω @ DC, 2, 4 ... GHz

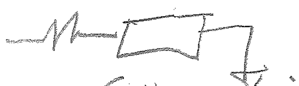
2 ⇒  (ℓ)

$R_1 = 93$ ~~50~~ Ω,
Z_0 = unknown

line is $\lambda/4$ @ 1 GHz
$T = 1/4\,ns$.

Smith chart g

27Ω @ DC, 2, 4, 6 GHz
∞ @ 1 3 5 7 GHz

2  (i)

$R_1 = 27\Omega$, $\tau = (1/4)/1GHz = 1/4\,ns$.

Smith chart h

27Ω @ 1, 3, 5, 7, 9 ... GHz
∞ @ 2, 4, 6 GHz

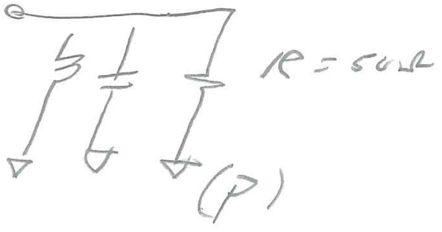
1 (j)

$R_1 = 22\Omega$
$\tau = (1/4)(1GHz) = 1/4\,ns$.

Smith Chart A

2

This goes from 0Ω @ DC to 50Ω at some frequency to 0Ω @ f=∞ Hz.

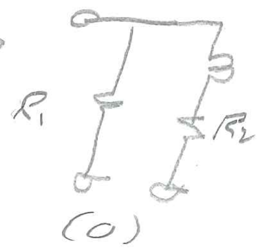
⇒ 

without more information, we can't compute L or C.

Smith Chart B

goes from 16.66Ω (DC) to 450Ω (f=∞) & is inductive

2

⇒ 

$R_1 \parallel R_2 = \cancel{16}\,17\,\Omega \rightarrow G_1 + G_2 = 0.06\,S.$

$R_1 = 450\,\Omega \rightarrow G_1 = 2.22\,mS.$

$G_2 = 60\,mS - 2.22\,mS = 57.77\,mS$

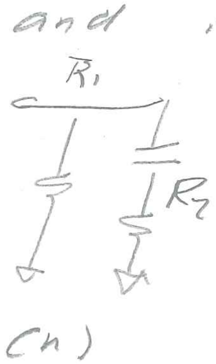
$R_2 = 17.3\,\Omega.$

Can't compute L

Smith Chart C

goes from 450Ω (DC) to 16.66Ω (f=∞) and is capacitive

2

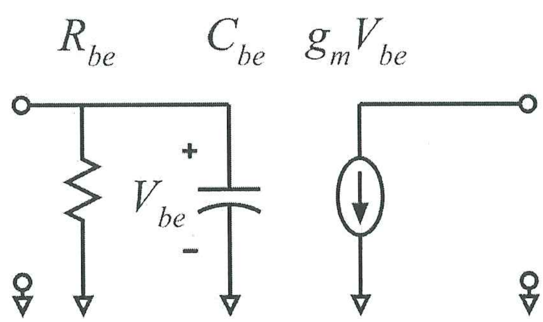
⇒ 

Calculations for R1 & R2 are the same as above.

$R_2 = 17.3\,\Omega$, $R_1 = 450\,\Omega$

Part c, ***ECE218A students only*** 15 points
For the network at the right, give algebraic expressions for the four S-parameters.

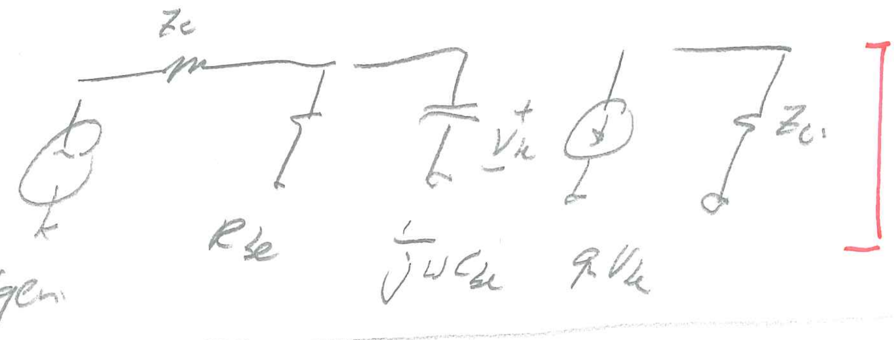
Assume a normalization to impedance Zo for the S parameters.



1 [$S_{12} = 0$ (no connection)

1 [$S_{22} = 1$ ($Z_{out} = \infty$)

S_{21}:



3

3
$$S_{21} = 2\frac{V_o}{V_{gen}}\Bigg|_{Z_S = Z_L = Z_{o}} = -2 g_m Z_o \cdot \frac{R_{be}}{R_{be} + Z_o} \cdot \frac{1}{1 + j\omega C_{be}(R_{be} \| Z_o)}$$

S_{11}

$$S_{11} = \frac{Z_{in}/Z_o - 1}{Z_{in}/Z_o + 1} = \frac{1 - Z_o/Z_{in}}{1 + Z_o/Z_{in}} = \frac{1 - Z_o Y_{in}}{1 + Z_o Y_{in}}$$ 2

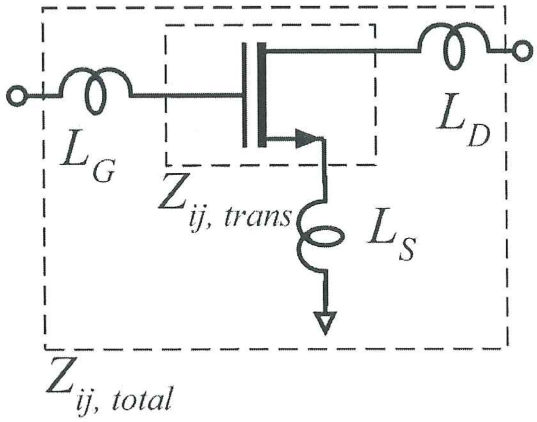
$Y_{in} = G_{be} + j\omega C_{be}$ 2

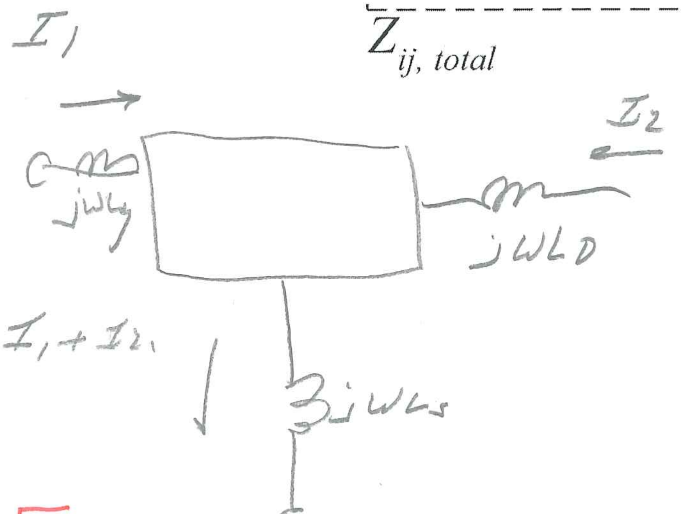
3
$$S_{11} = \frac{1 - G_{be}Z_o - j\omega C_{be}Z_o}{1 + G_{be}Z_o + j\omega C_{be}Z_o}$$

$$= \frac{1 - G_{be}Z_o}{1 + G_{be}Z_o} \times \frac{1 - j\omega C_{be} Z_o/(1 - G_{be}Z_o)}{1 + j\omega C_{be} Z_o/(1 + G_{be}Z_o)}$$

Part b, 15 points

A transistor has four Z-parameters $Z_{ij,trans}$. Derive algebraic expressions for the four Z-parameters of the overall network $Z_{ij,total}$





before

5

$$V_1^{TRANS} = Z_{11}^T I_1 + Z_{12}^T I_2$$
$$V_2^{TRANS} = Z_{21}^T I_1 + Z_{22}^T I_2$$

but

5

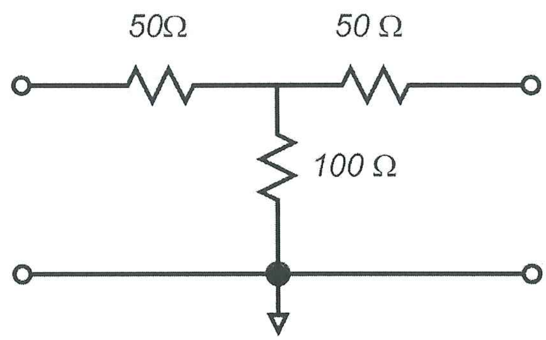
$$V_1^{Total} = (j\omega L_g + j\omega L_s) I_1 + j\omega L_s I_2 + V_1^{TRANS}$$
$$V_2^{Total} = j\omega L_s I_1 + (j\omega L_d + j\omega L_s) I_2 + V_2^{Trans}$$

5.

$$\Rightarrow Z_{ij}^{Total} = \begin{bmatrix} Z_{11}^{TRANS} + j\omega L_g + j\omega L_s & Z_{12}^{Trans} + j\omega L_s \\ Z_{21}^{trans} + j\omega L_s & Z_{22}^{Trans} + j\omega L_s + j\omega L_d \end{bmatrix}$$

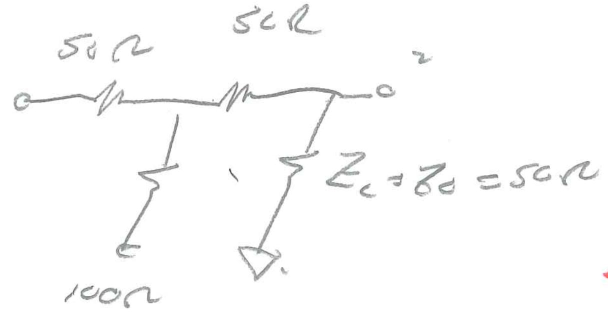
Problem 2, 25 points (ece145A), 40 points (ece218A)
2-port parameters and Transistor models

Part a, 10 points
For the network at the right, give numerical values for the four S-parameters. Assume that the reference Zo is 50 Ohms.



$$S = \begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix}$$

S_{11}:

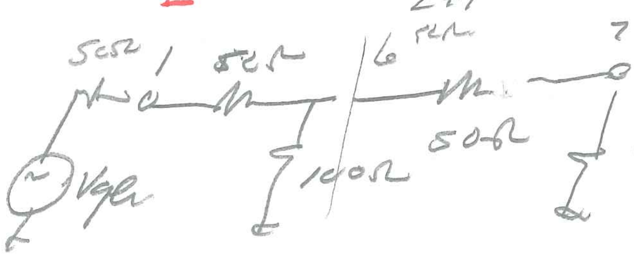


2

1 $Z_i\big|_{Z_L = Z_0} = 50\Omega + 100\Omega \parallel (50\Omega + 50\Omega)$
$= 50\Omega + 50\Omega$

1 $S_{11} = \frac{2-1}{2+1} = 1/3.$ 1 $S_{22} = 1/3$ by symmetry.

S_{21}



2

$S_{21} = 2\frac{V_L}{V_{gen}}\Big|_{Z_L = Z_0 = Z_{gen}}$

$= 2(1/2) \cdot 1/3$

$= 1/3.$

2

1 $S_{12} = 1/3$ by symmetry

1/2 [$\Gamma_2 = 1$

1/2 [$T_5 = \frac{50}{50 + 50/3} = \frac{1}{4/3} = 3/4$

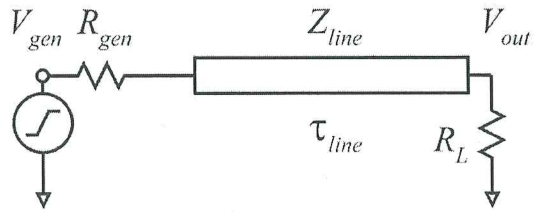
1/2 [$\Gamma_3 = \frac{1/3 - 1}{1/3 + 1} = \frac{1-3}{1+3} = -1/2.$

Part b, 7.5 points

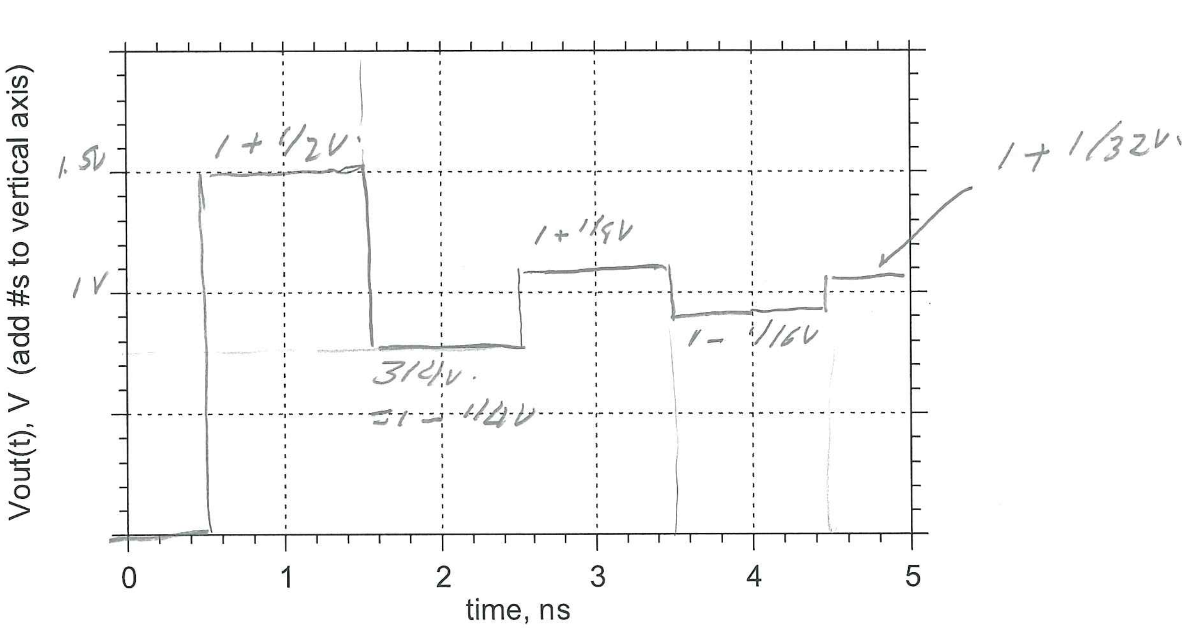
RL is infinite. Rgen is (50/3) Ohms.

Plot Vout (t) on the graph below.

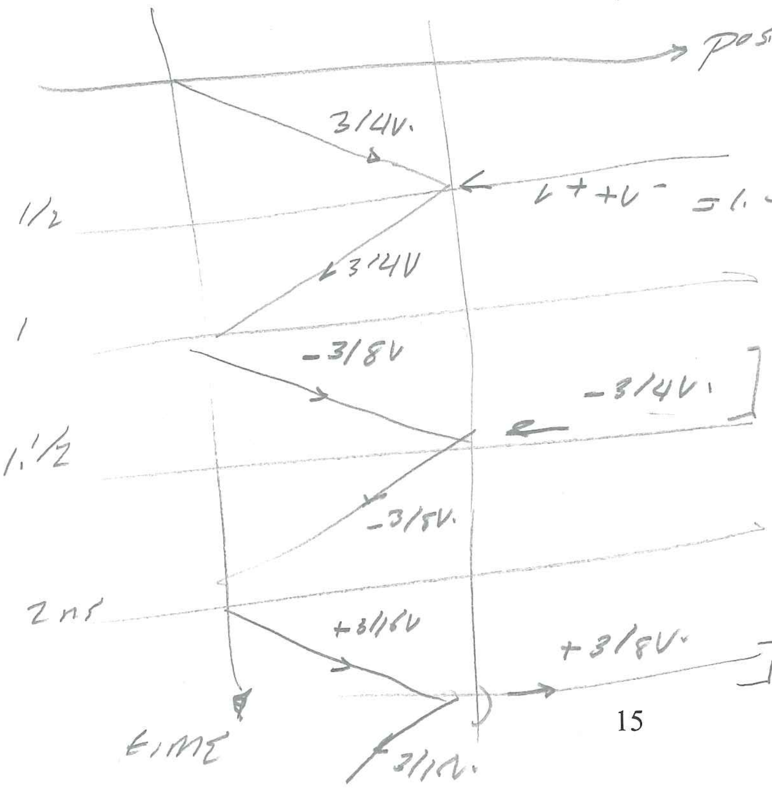
Does the step response of the line appear inductive, capacitive, both, or neither ?



3



3.



1.5 - 3/4 = 3/4 V.

1.5 - 3/4 + 3/8 V = 6/8 + 3/8 = 9/8 V

$\frac{1}{2}\ [\ \Gamma_L = 1$

$\frac{1}{2}\ [\ T_S = \frac{50\Omega}{150\Omega + 50\Omega} = 1/4.$

$\frac{1}{2}\ [\ \Gamma_S = \frac{3-1}{3+1} = 1/2.$

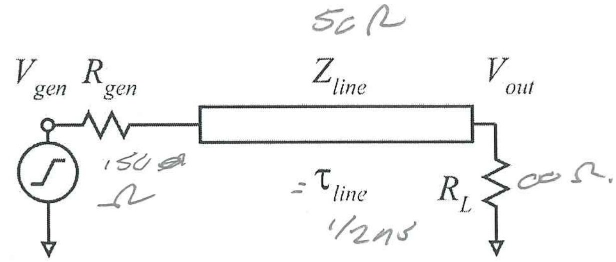
Problem 3, 15 points
Transmission lines in the time domain.

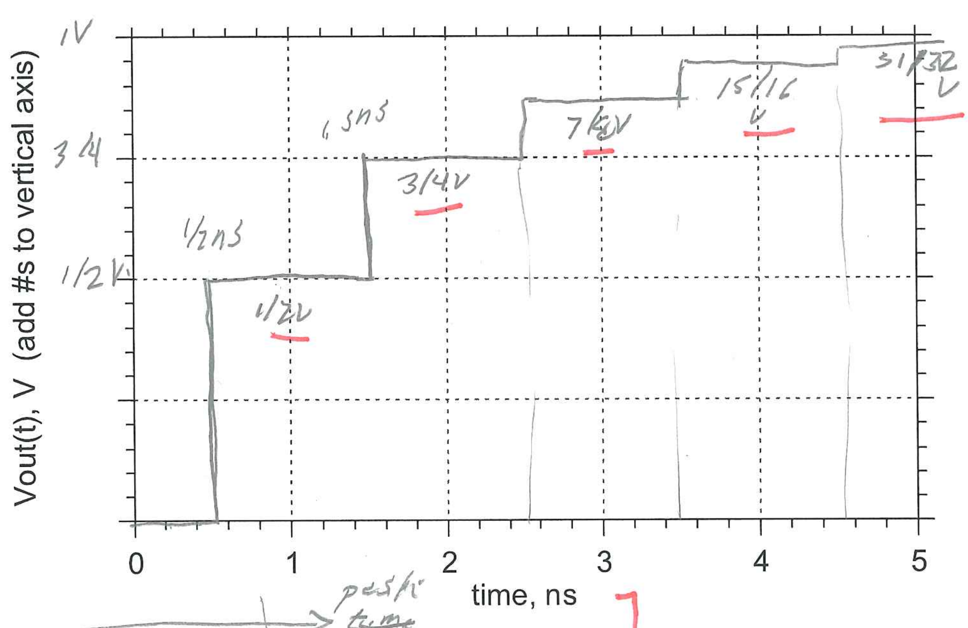
<u>Part a, 7.5 points</u>
Vgen is a 1V step-function occurring at t=0 seconds. Zline is 50 Ohms. τ_{line} is 1/2 ns.

RL is infinite. Rgen is 150 Ohms.

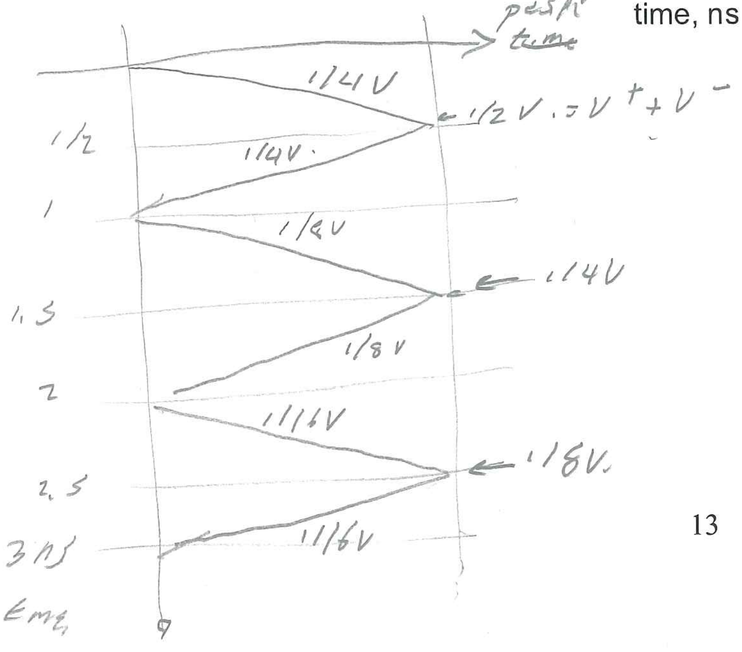
Plot Vout (t) on the graph below.

Does the step response of the line appear inductive, capacitive, both, or neither ?





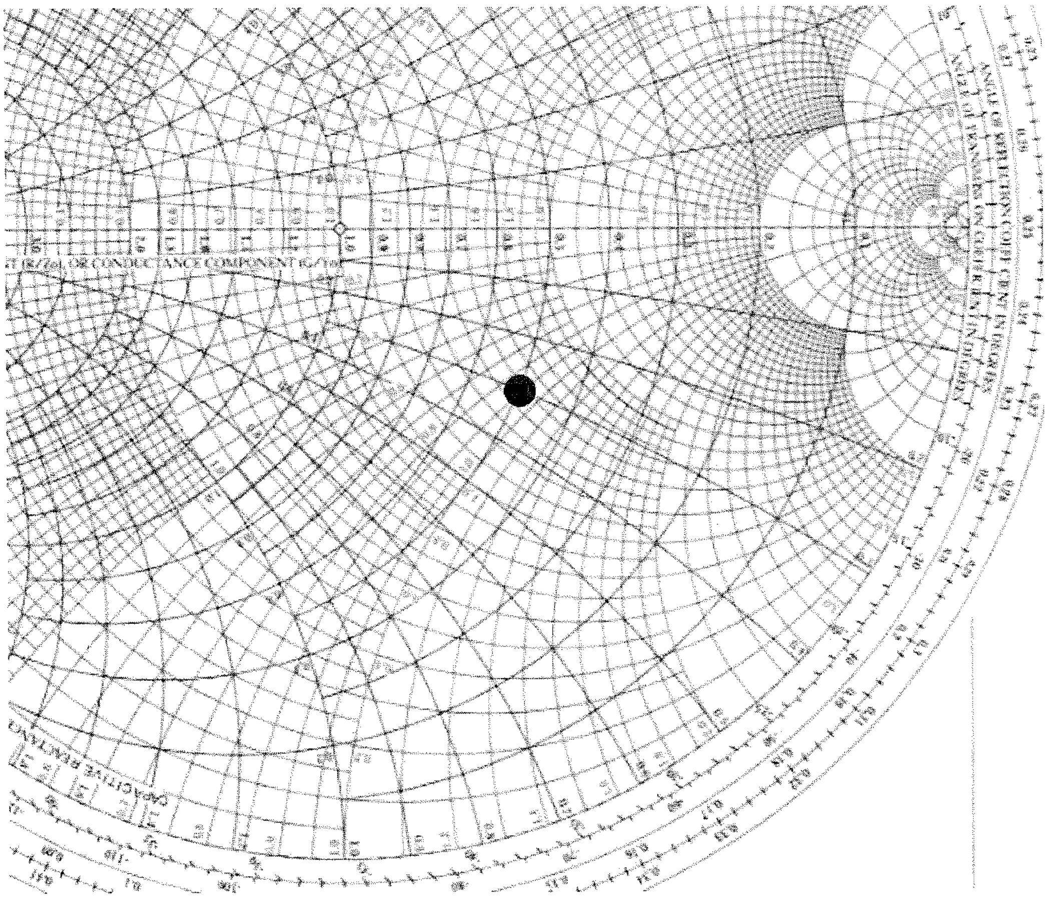
3



3

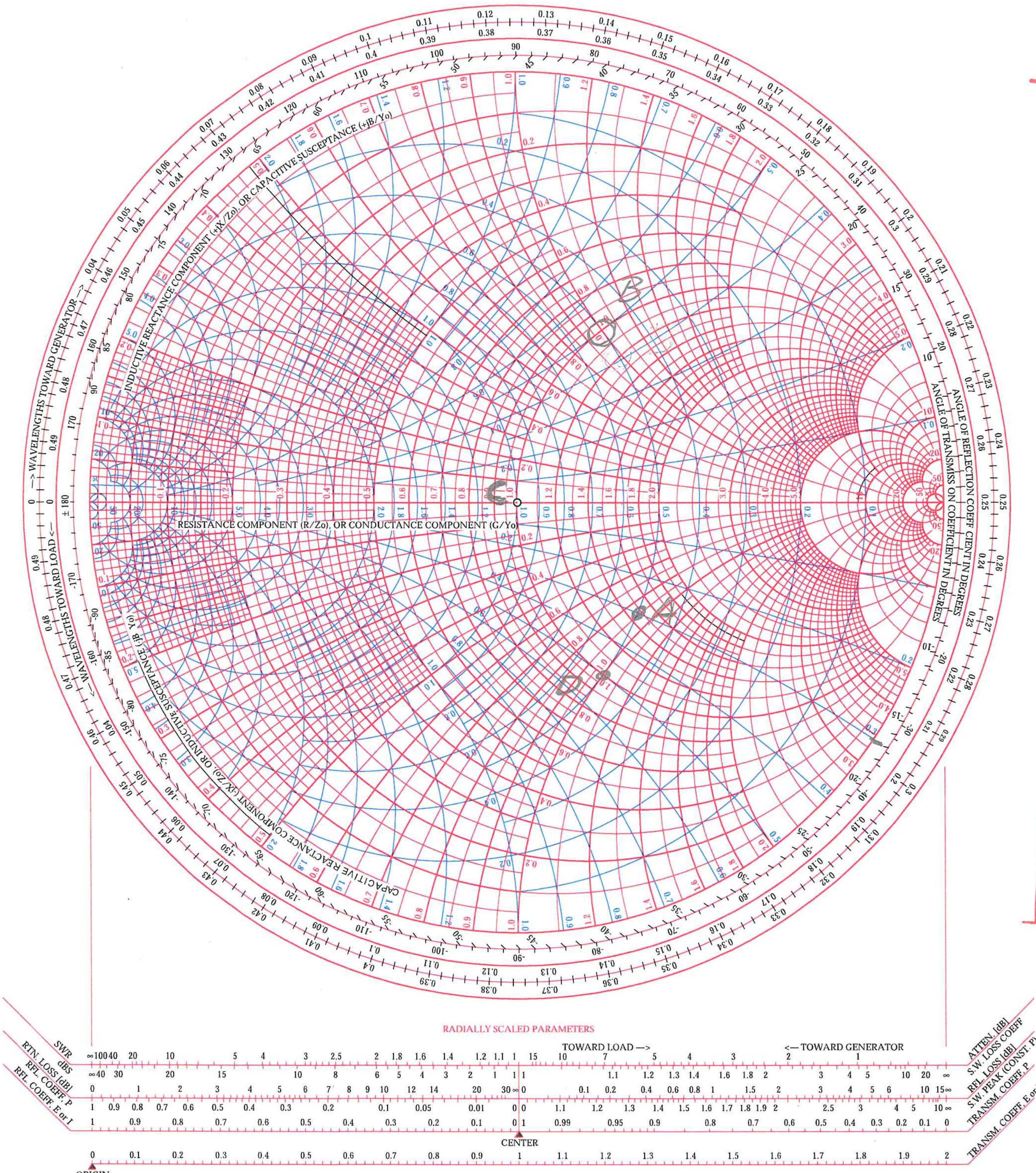
Problem 4, 15 points
Impedance-matching exercise.

The (50 Ohm normalization) Smith chart gives the input impedance of a circuit at 10 GHz signal frequency. Design a lumped-element matching network which converts this impedance to **50Ohms** at 10 GHz. Give all element values. Use the full impedance-admittance chart which has been provided to you.



NAME	TITLE	DWG. NO.
		DATE
SMITH CHART FORM ZY-01-N	Microwave Circuit Design - EE523 - Fall 2000	

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



179

There are two solutions. A-B-C & A-D-C

A-B-C

point (A): $y = 0.5 + j\,0.3$]'

point (B) $y = 0.5 - j\,0.5$]'

$\Delta y = -j\,0.8$]'

$\Delta Y = \Delta y / Z_0 = -j \cdot 16\,mS = \frac{1}{j\omega L}$]'

$\Rightarrow L = \frac{1}{16\,mS}\,\frac{1}{(2\pi \cdot 10\,GHz)} = 0.99\,nH. \cong 1\,nH.$]'

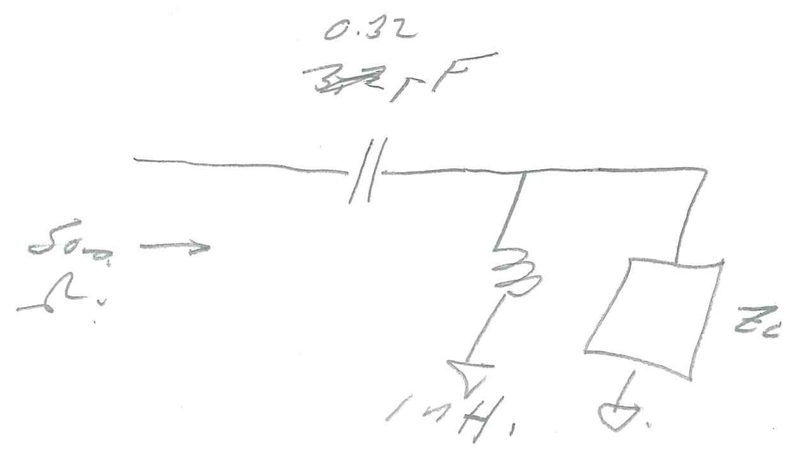
point (B) $z = 1 + j1$]'

point (C) $z = 1 + j0.$]'

$\Delta z = -j1$]'

$\Delta Z = \Delta z \cdot Z_0 = -j \cdot 50\Omega = \frac{1}{j\omega C}$]'

$\Rightarrow C = \frac{1}{50\Omega}\,\frac{1}{2\pi(10\,GHz)} = 0.32\,pF$]'



]'

A D C

point A: $y = 0.5 + j0.3$

point D: $y = 0.5 + j0.5$

$\Delta y = +j0.2$

$\Delta Y = \Delta y / Z_0 = \frac{j0.2}{50\Omega} = +j.4mS = j\omega C$

$\Rightarrow C = \frac{4mS}{2\pi(10GHz)} = 64fF$

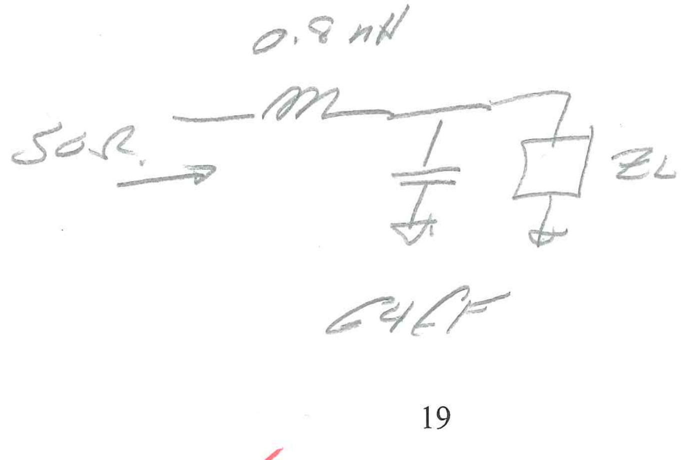
point D $\Rightarrow$ $z = 1 + j1$

point C $\Rightarrow$ $z = 1 + j0$

$\Delta z = +j1$

$\Delta Z = j50\Omega = j\omega L$

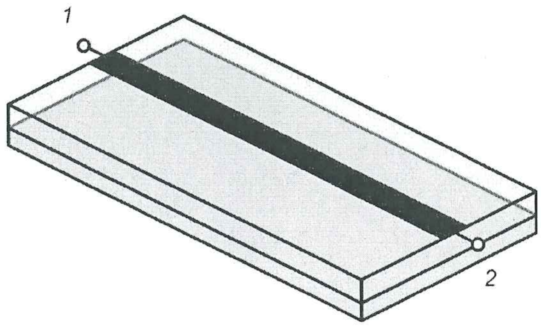
$L = \frac{50\Omega}{2\pi(10GHz)} = 0.80nH.$



Problem 5, 15 points (ece145A), 20 points (218A)
Transmission-line parasitics.

Part a, 7.5 points (145A), 12.5 points (218A)

We are designing a microstrip line and calculating its properties. We will assume dimensions typical of a dielectric stack on an IC: the signal and ground planes are separated vertically by 5 μm, and the dielectric constant is 3.8.



If we approximate the effective conductor width as being the physical conductor width plus the dielectric thickness then (1) what width is required for a 50 Ohm characteristic impedance ?

If the line were 300 microns length, what is the total wiring inductance and total wiring capacitance in that length ?

W = 14.33μm
L = 97.5pH
C = 38.9fF

ECE 218 students only (5 more points)

The conductivity of copper is $59.6*10^{6}$ Siemens/meter and $\mu_0 = 4\pi * 10^{-7} H/m$. Find the skin depth, the attenuation constant α, and the total line attenuation at 60GHz signal frequency.

Hint----the skin depth is $\delta = \sqrt{2/\omega\mu_0\sigma}$

2 line width required for Z0=50 Ohms; W= 14μm

2.75 total inductance in 100 microns L= 97.5pH

2.75 total capacitance in 100 microns C= 39fF

1 skin depth delta = 0.266μm

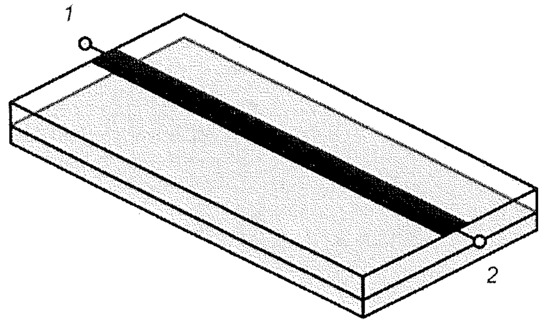
1 attenuation constant alpha = 160. (nepers/meter)

2 total attenuation, S21 = (dB)

1384 dB/meter or 1.38 dB/mm.

Part b, 7.5 points

Using the very crude approximation that the effective conductor width is the physical conductor width plus the substrate thickness, what is the highest characteristic impedance which we can obtain ?



Assuming a signal frequency of 60 GHz, what is the maximum conductor width allowable to suppress propagation of parasitic *transverse* modes on the conductor ?

Setting a practical limit of width 2:1 smaller than that, what would be the resulting Zo.

Maximum feasible Z0= 193 Ω.

~~Minimum feasible~~ Z0=

Maximum width to prevent transverse modes at 60 GHz W= 1.3 mm.

Characteristic impedance with width set to 1/2 of the above, Z0= 1.5 Ω very approximately

2 $$Z_0 \cong \frac{377\Omega}{\sqrt{\epsilon_r}} \cdot \frac{H}{H+W} = 193.4\Omega \frac{1}{1+W/H} = 50\Omega$$

$\sqrt{\mu_0 / \epsilon_0}$

$$1+W/H = \frac{193.4\Omega}{50\Omega} \qquad W/H = 2.87 \rightarrow W = 14.33\mu m.$$

$$\text{velocity} \cong \frac{c}{\sqrt{\epsilon_r}} \Rightarrow \text{delay} = \tau = \frac{l}{c} \cdot \sqrt{\epsilon_r} = 1.95 pS.$$ 1.5

2 $$L = Z_0 \tau = 50\Omega \cdot 1.95pS = 97.5pH$$

2 $$C = \tau / Z_0 = 38.9 fF$$

---

1 $$\delta = \sqrt{2/\omega\mu\sigma} = 0.266\mu m. \ @ \ 60 GHz$$

1 $$R_{series}/l = \frac{1}{\sigma\delta} \cdot \left[\frac{1}{W} + \frac{1}{W+H} \right]$$

14 µm, 19 µm

$$= 15.9 k\Omega/\text{meter}.$$

1 $$\alpha = \frac{R_{series}}{L} \cdot \frac{1}{2Z_0} = 78.2 \ 1/\text{meters}.$$

2 attenuation in 300 µm

$$S_{21} = e^{-\alpha l} = 0.977$$

$$dB \ S_{21} = 20 \log_{10}(0.977) = -0.20 dB$$

ok for a short line...

2.5

$$Z_0 \simeq 193\,\Omega \cdot \frac{1}{1 + w/H} \rightarrow \text{approximately } 193\,\Omega$$

when $w \rightarrow 0\,\mu m$

(actual variation of Z_0 with $w/H \ll 1$ is $\cong$ logarithmic in w/H)

2.5

Maximum width: $\lambda_g/2 = \frac{1}{2}\,\frac{c}{60\,GHz}\,\frac{1}{\sqrt{\epsilon_r}} = 1.3\,mm.$

practical width: $640\,\mu m$

2.5.

$$Z_0 = 193\,\Omega \cdot \frac{H}{W} = 193\,\Omega \cdot \frac{5\,\mu m}{\cancel{1500\,\mu m}\ 640\,\mu m} = 1.5\,\Omega$$

this has turned out to be a <u>bad</u> <u>example</u>

— we would never make a line this wide with $5\,\mu m$ thickness