ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. October 30, 2018

Do not open exam until instructed to.

Open notes, open books, etc.

You have 1 hour and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.), *AFTER STATING THEM.*

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points Received</th>
<th>Points Possible</th>
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<tbody>
<tr>
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<td>2c</td>
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<td><strong>2d (218 only)</strong></td>
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<td>3a</td>
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<td>total</td>
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<td>85 (145), 107.5 (218A)</td>
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Name: 5/6/20
Problem 1, 15 points
The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.
<table>
<thead>
<tr>
<th>(i)</th>
<th>(j)</th>
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<tbody>
<tr>
<td>( R_1 ) ( R_2 ) ( Z_{\text{line}} ) ( \tau_{\text{line}} )</td>
<td>( R_1 ) ( C ) ( L )</td>
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<td>( R_1 ) ( L ) ( C )</td>
<td>( R_1 ) ( C ) ( L )</td>
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First match each Smith Chart with each circuit. Then determine as many component values as is possible (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given. The charts all use 50 Ohm normalization:

Smith chart (a). Circuit= $\frac{N}{\lambda}$.
Component values: $R_1 = 12 \Omega$, $R_2 = 42 \Omega$.

Smith chart (b). Circuit= $\frac{N}{\lambda}$.
Component values: $R_1 = 27 \Omega$.

Smith chart (c). Circuit= $\frac{N}{\lambda}$.
Component values: $R = 50 \Omega$.

Smith chart (d). Circuit= $\frac{N}{\lambda}$.
Component values: $R = 27 \Omega$.

Smith chart (e). Circuit= $\frac{N}{\lambda}$.
Component values: $R = 30 \Omega$.

Smith chart (f). Circuit= $\frac{N}{\lambda}$.
Component values: $R = 30 \Omega$.

Smith chart (g). Circuit= $\frac{N}{\lambda}$.
Component values: $R = 12 \Omega$, $R = 32 \Omega$.

Smith chart (h). Circuit= $\frac{N}{\lambda}$.
Component values: $R_1 = 47 \Omega$, $R_2 = 17 \Omega$.

4) goal from $I_f = -1/2 \Omega$ to $50 \Omega$.
   $\frac{1}{1+1/2} = \frac{50 \Omega}{3 \Omega}$

6) goes between $I_f = -0.3 \Rightarrow 50 \Omega$ and an open circuit. Does so repeatedly.

2) goal from open @ DC, to $50 \Omega$, then to open.
(D) goes between qa and scr \( \frac{1-0.8}{1+0.3} = 2 \) do so repeatedly. 
\[ \rightarrow (C) \]

\[ (E) \text{ goes from scr } @ \text{ OC to QA qa and scr } @ t=0. \]
O: Oh no! I've made a mistake!
The right solution is:
\[ \begin{align*}
\text{scr}: & \quad u = \frac{7}{1} \\
\beta : & \quad \frac{3}{5}
\end{align*} \]

\[ (F) \text{ goes from QA to scr } \]
O: to scr @ reach (0) to QA @ \( t=0. \)
\[ \rightarrow (P) \quad R_1 = \text{scr.} \]

\[ (G) \quad \text{ goes repetitively between scr } @ \text{ DC and scr } \]
O: \( 50 \% \left( \frac{1-0.5}{1+0.5} \right) = 12.5 \% \)
\[ \text{c}: \quad \text{with } R_1 = 12.5 \]
\[ a \quad R_2 = 50 \% - 12.5 \% = 37.5 \%
\]

\[ (H) \quad \text{ goes from } 50 \% \frac{1+0.8}{1-0.7} = 45 \% \text{ at DC to scr } \]
O: \( 50 \% \left( \frac{1-0.5}{1+0.5} \right) = 17.2 \% @ t=0. \)
\[ \text{Trajectory is } \begin{align*}
\text{capacitiv} & \quad \Rightarrow (m) \\
R_2 & = 17.2 \\
R_1 & = 45 \% - 17.2 \% = 43.8 \%
\end{align*} \]
Problem 2, 25 points (ece145A), 40 points (ece218A)
2-port parameters and Transistor models

Part a, 10 points
For the network at the right, give numerical values for the four S-parameters. Assume that the reference $Z_0$ is 50 Ohms. The signal frequency is 1GHz and the inductance is 8.0nH.

\[
\begin{align*}
\text{Inductive reactance} &= -j\omega L = \sqrt{2\pi}\left(\frac{1}{6}\right) \Omega = -j50\Omega.
\end{align*}
\]

\[
\begin{align*}
S_{11} &= \frac{Z_L}{Z_0} = \frac{50\Omega}{j50\Omega} = 50\Omega \left(\frac{1-j}{1+j}\right) \\
S_{21} &= \frac{Z_0}{Z_L} = \frac{50\Omega}{50\Omega} = 1 \\
S_{12} &= \frac{1}{Z_0} \left(\frac{Z_L}{Z_0} + Z_0\right) = \frac{1}{50\Omega} \left(\frac{1-j}{1+j} + 1\right) = \frac{1}{j - 1 + j} \\
S_{22} &= \frac{1}{Z_0} \left(\frac{Z_0}{Z_L} + Z_0\right) = \frac{1}{50\Omega} \left(1 + 1\right) = 2 \\
\end{align*}
\]

\[
\begin{align*}
S_{21} &= \frac{1}{j - 1 + j} = \frac{1}{j + 1 + j} = \frac{1}{2j} = \frac{1}{\sqrt{2j}} = \frac{1}{\sqrt{2}} \angle 117^\circ,
\end{align*}
\]

all ok.

\[
\begin{align*}
\text{by Symmetry, } S_{22} = S_{11}
\end{align*}
\]
Part b. 7 points
Compute the \( Z \) parameters for this network

\[
\begin{align*}
Z_{11} &= Z_{22} = 150 \Omega \\
Z_{21} &= Z_{12} = 100 \Omega & \text{[by inspection, } \eta = I_1(150\Omega) + I_2(100\Omega)\text{]} \\
\text{so this gives } Z_{11} \text{ and } Z_{12} \\
\text{[ } Z_{22} \text{ and } Z_{21} \text{ by symmetry]}
\end{align*}
\]
Part c. 7 points
Compute the Y parameters for this network.

We could just invert the Z matrix...

...or...

we don't know \( V_x \), so:

\[
2 \left[ \begin{array}{c}
\frac{V_x - V_1}{50} + \frac{V_x - V_2}{100} + \frac{V_x}{100} = 0 \\
V_x (5 \text{ms}) = V_1 (2 \text{ms}) + V_2 (2 \text{ms})
\end{array} \right]
\]

\[
V_x = 0.4V_1 + 0.4V_2
\]

So

\[
I_1 = \frac{V_1 - V_x}{50} = (V_1 - V_x) 2 \text{ms} = 2 \text{ms} (V_1 - 0.4V_1 - 0.4V_2)
\]

\[
I_1 = 2 \text{ms} (0.6V_1 - 0.4V_2) = 1.2 \text{ms} \cdot V_1 - 0.8 \text{ms} \cdot V_2
\]

\[
I_1 = Y_{11} V_1 + Y_{12} V_2 = 1.2 \text{ms} \cdot V_1 - 0.8 \text{ms} \cdot V_2
\]

So

\[
Y_{11} = 1.2 \text{ms}, \quad Y_{12} = -0.8 \text{ms}
\]

by symmetry, \( Y_{22} = Y_{11} \) and \( Y_{21} = Y_{12} \)
Part d. **ECE218A students only** 15 points
For the network at the right, give an
algebraic expressions for $Y_{11}$ and $Y_{21}$.
Please write as a Taylor series in $j\omega$, omitting terms of power $(j\omega)^3$ and higher.

This is an exercise in device model extraction from measured S/Y/Z parameters.

\[
\begin{align*}
I_1 &= Y_{11}V_1 = \frac{V_1}{R_i + j\omega C_{GS}} = \frac{V_1}{1 + j\omega C_{GS}} \\
\Rightarrow Y_{11} &= \frac{j\omega C_{GS}}{1 + j\omega C_{GS}} = j\omega C_{GS} \left(1 + (j\omega C_{GS}) + O((j\omega)^3)\right) \\
Y_{11} &= j\omega C_{GS} + \omega^2 C_{GS}^2 \left(R_i + O(\omega^3)\right) \\
\frac{V_2}{V_1} &= \frac{1}{G_{m} + j\omega C_{GS}} = \frac{1}{1 + j\omega C_{GS}} \\
I_2/V_1 &= Y_{21} = \frac{G_{m}}{1 + j\omega C_{GS}} = \frac{G_{m} \left(1 - j\omega C_{GS} R_i\right)}{1 + \omega^2 C_{GS}^2 R_i^2} \\
&= G_{m} \left(1 - j\omega C_{GS} R_i\right) \left(1 - \omega^2 C_{GS}^2 R_i^2 + O(\omega^4)\right) \\
&= G_{m} \left(1 - j\omega C_{GS} R_i - \omega^2 C_{GS}^2 R_i^2 + O(\omega^3)\right)
\end{align*}
\]
Problem 3, 15 points
Transmission lines in the time domain.

Part a, 7.5 points
V_{gen} is a 1 V step-function.
R_{gen} is zero Ohms and R_{L} is 50 Ohms.
tau is 1 ns.
Z_{line} is 50 Ohms.

Plot below V_{out}(t) and V_{in}(t)
Part b, 7.5 points
Vgen is a 1 V step-function.
Rgen is 50 Ohms and RL is infinite.
tau is 1 ns.
Zline is 50 Ohms.

Plot below Vout(t) and Vin(t)

Please comment on the pulse response of this circuit and that of part a.

\[
\begin{align*}
I_L &= \frac{\infty - 1}{\infty + 1} = 1 \\
I_S &= 0 \\
I_S &= \frac{Z_0}{Z_0 + nRL} = 1/	au
\end{align*}
\]
\[
I_c = 3, \quad I_S = 1/2, \quad I_S = 0.1
\]

Comment:

part (a) is called sending and termination, signal is undistorted at both ends of line.

part (b) is called receiving and termination, signal is undistorted only at receiving end of line.
Problem 4, 15 points

Impedance-matching exercise.

The (50 Ohm normalization) Smith chart gives the input impedance of a circuit at 1 GHz signal frequency. Design a lumped-element matching network which converts this impedance to **50 Ohms**. Give all element values. Use the full impedance-admittance chart which has been provided to you.

There are two different

lumped element solutions.

Both are shown.

Neither one is acceptable.
Point "A" is at an impedance normalized of \[ Z_A = 0.5 - j0.3. \]

We move to point "B", which has a normalized impedance of \[ Z_B = 0.5 + j0.5. \]

This movement from "A" to "B" corresponds to the addition of \[ \Delta Z = 0.5 j \text{ ohms}. \]

This is an inductor, \[ \Delta Z = j \omega L = j 40 \Omega \]

\[ \Rightarrow L = \frac{40 \Omega}{\omega} = \frac{40 \Omega}{2\pi (16 \Omega)} = 6.37 \text{ mH}. \]

Point "B" has a normalized admittance of \[ Y_B = 1.0 - j1.0. \]

Point "C" has a normalized admittance of \[ Y_C = 1.0 + j0. \]

This movement from "B" to "C" corresponds to the addition of \[ \Delta Y = j1. \]

This is a capacitor, \[ \Delta Y = j \omega C = j 2 \text{ mF}. \]

\[ C = \frac{2 \text{ mF}}{2\pi (16 \Omega)} = 3.18 \text{ pF}. \]
Second Solution

Point "A" is at the normalized impedance

\[ Y_A = 0.5 - j0.3 \]

We move to point "B", a normalized impedance of

\[ Y_B = 0.5 - j0.5 \]

This movement from "A" to "B" corresponds to the addition of \( \Delta Y = -j0.5 - (\sqrt{1.03}) = -j0.2 \)

on an un-normalized impedance of

\[ \Delta Z = -j0.2 \cdot 50\Omega = -j10\Omega \]

This is a series capacitor \( \Delta Z = \frac{1}{j\omega C} = -j10\Omega \)

\[ \rightarrow C = \frac{10.2 \cdot 2\pi (6kHz)}{15.9} = 15.9 \mu F \]

Point "B" has a normalized admittance of

\[ Y_B = 1 + j1 \]

Point "C" has a normalized admittance of

\[ Y_C = 1 + j0 \]

The movement from "B" to "C" corresponds to the addition of \( \Delta Y = j0 - (j1) = -j1 \)

an un-normalized admittance of \( \Delta Y = \frac{-j1}{50\Omega} = -j(20\mu S) \)

This is an inductor \( \Delta Y = \frac{1}{\sqrt{L}} = -j(20\mu S) \)

\[ L = \frac{50\mu H}{2\pi (6kHz)} = 7.95 \text{ nH} \]
Problem 5, 15 points (ece145A), 20 points (218A)
Transmission-line properties.

Part a. 5 points

\[ Z_{in} \]

In an undergraduate class, a 1 meter length of 50 Ohm cable connects a circuit under test to an oscilloscope. The input impedance to the oscilloscope is extremely high. The propagation velocity of the transmission line is \(2 \cdot 10^8 \text{ m/s}\). At a signal frequency of 1 MHz, what would the input impedance be, approximately?

\[
\text{speed of light} = \gamma = \frac{c}{v} = \frac{1 \text{ m}}{2 \cdot 10^8 \text{ m/s}}
\]

\[
\gamma = 5 \text{ ns}
\]

The period of 1 m/\gamma is 0.2 \text{ ns}, so 5/\gamma << 1, or equivalently, \(l << \lambda\).

Use lumped element model: R-L-C

\[
\begin{align*}
C &= \frac{5 \text{ ns}}{50 \text{ Ohm}} = 100 \text{ pF} \\
L &= 7 \text{ H} = 545.5 \text{ mH} \\
Z &= \sqrt{\omega L + \frac{1}{\omega C}} = \sqrt{(1.62) - \sqrt{15702}} \\
\approx \frac{1}{\sqrt{\omega C}} = -j \cdot 1570 \text{ Ohm}
\end{align*}
\]
Part b. 5 points

$Z_{in}$

At 1 MHz, if we short-circuit the far end of the cable, what would be the approximate input impedance?

\[ Z = \frac{j \omega L}{\sqrt{\omega C/2}} = \frac{1}{1(6.2)\parallel (-13.8\pi \Omega)} \]

\[ \approx j 1.6 \Omega \]
Part c. 5 points

\[ Z_{in} \]

At what frequencies is the input impedance infinite?
At what frequencies is the input impedance zero?

\[ \text{Line has } L = 5 \text{ m} \]
\[ \text{so line is one wavelength long } @ f = \frac{1}{5} = 200 \text{ Hz} \]
\[ \text{line is } 1 \text{ ft } \text{ long } @ f = 50 \text{ kHz} \]
\[ \text{line is } \frac{1}{2} \text{ ft } \text{ long } @ f = 100 \text{ kHz} \]

Input impedance will be zero when
\[ f = 0, 100 \text{ kHz}, 200 \text{ kHz}, 300 \text{ kHz}, \ldots \]

Input impedance will be infinite when
\[ f = 50 \text{ kHz}, 150 \text{ kHz}, 250 \text{ kHz}, 350 \text{ kHz}, \ldots \]
We will make a 50 Ohm microstrip line on a circuit board of 1mm thickness having a dielectric constant of 4. If we ignore fringing fields, how wide must the conductor be?

Using transmission-line relationships, and ignoring the fringing fields, derive expressions for the inductance and capacitance of a conductor of length L and width W, on a circuit board of thickness T, where the circuit board has a ground plane on the back surface.

If we ignore fringing fields, then

\[ \varepsilon_0 = \frac{\text{377} \omega}{\sqrt{\varepsilon_r}} \frac{1}{\sqrt{\varepsilon_r}} \]

\[ \frac{H}{w} = \frac{30 \varepsilon_r}{377} - 1.47 = 0.265 \]

\[ w = \frac{H}{0.265} = \frac{1\text{ mm}}{0.265} = 3.77\text{ mm} \]

If we ignore fringing fields, then the capacitance must be

\[ C = \frac{\varepsilon_0 \varepsilon_r W L}{H^2} \]

But \( \varepsilon_0 = \sqrt{\frac{\mu_0}{\varepsilon_r}} = \sqrt{\frac{M_{\text{ho}} W}{H}} \rightarrow C = \left( \frac{M_{\text{ho}} W}{\varepsilon_0 \varepsilon_r} \right) \frac{H^2}{W^2} \cdot \mu_0 \]

\[ \text{In} \frac{M_{\text{ho}} W}{W^2} \frac{H}{H^2} \]