ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. November 14, 2019

Do not open exam until instructed to.

Open notes, open books, etc.

You have 1 hour and 15 minutes.

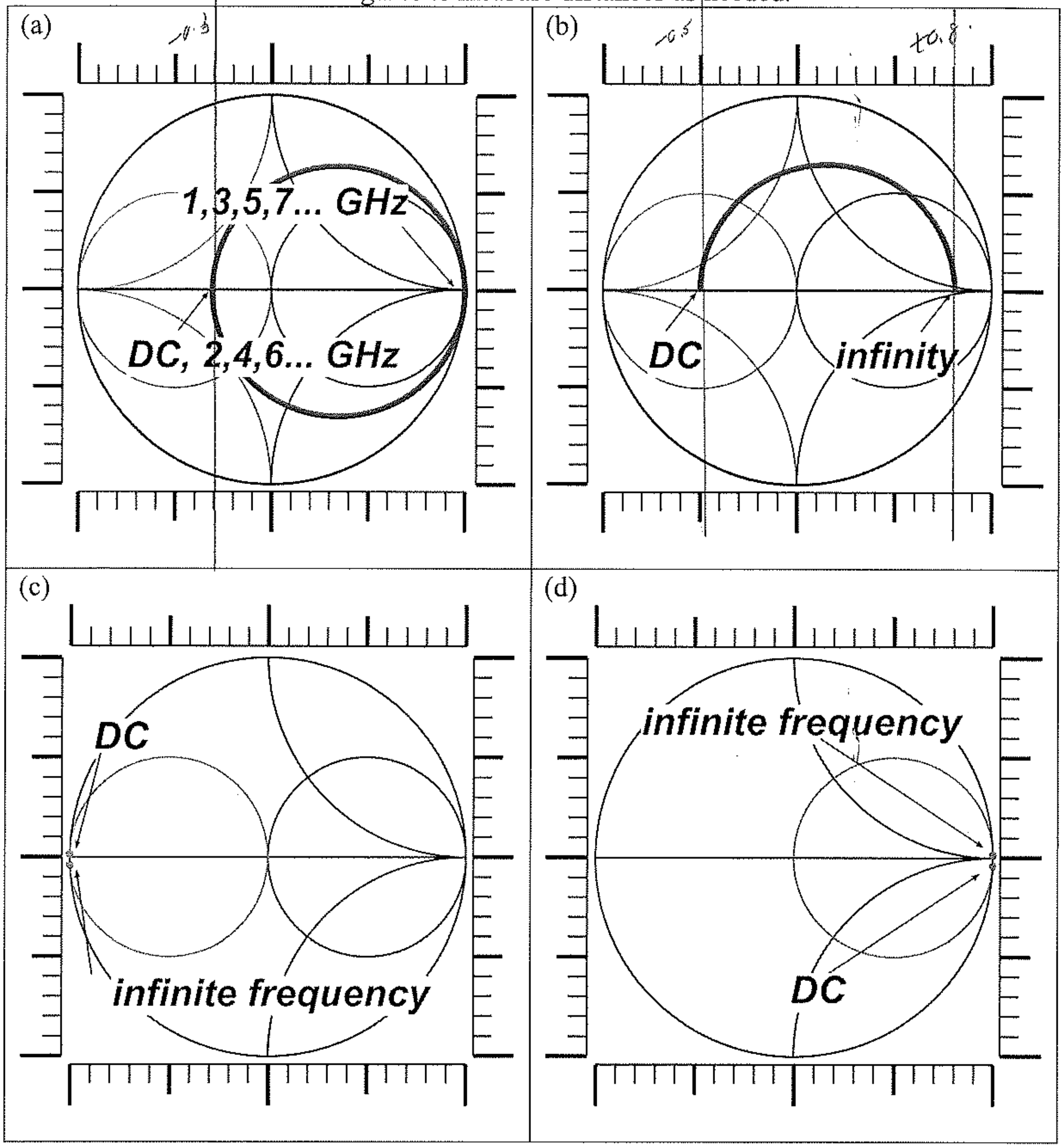
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

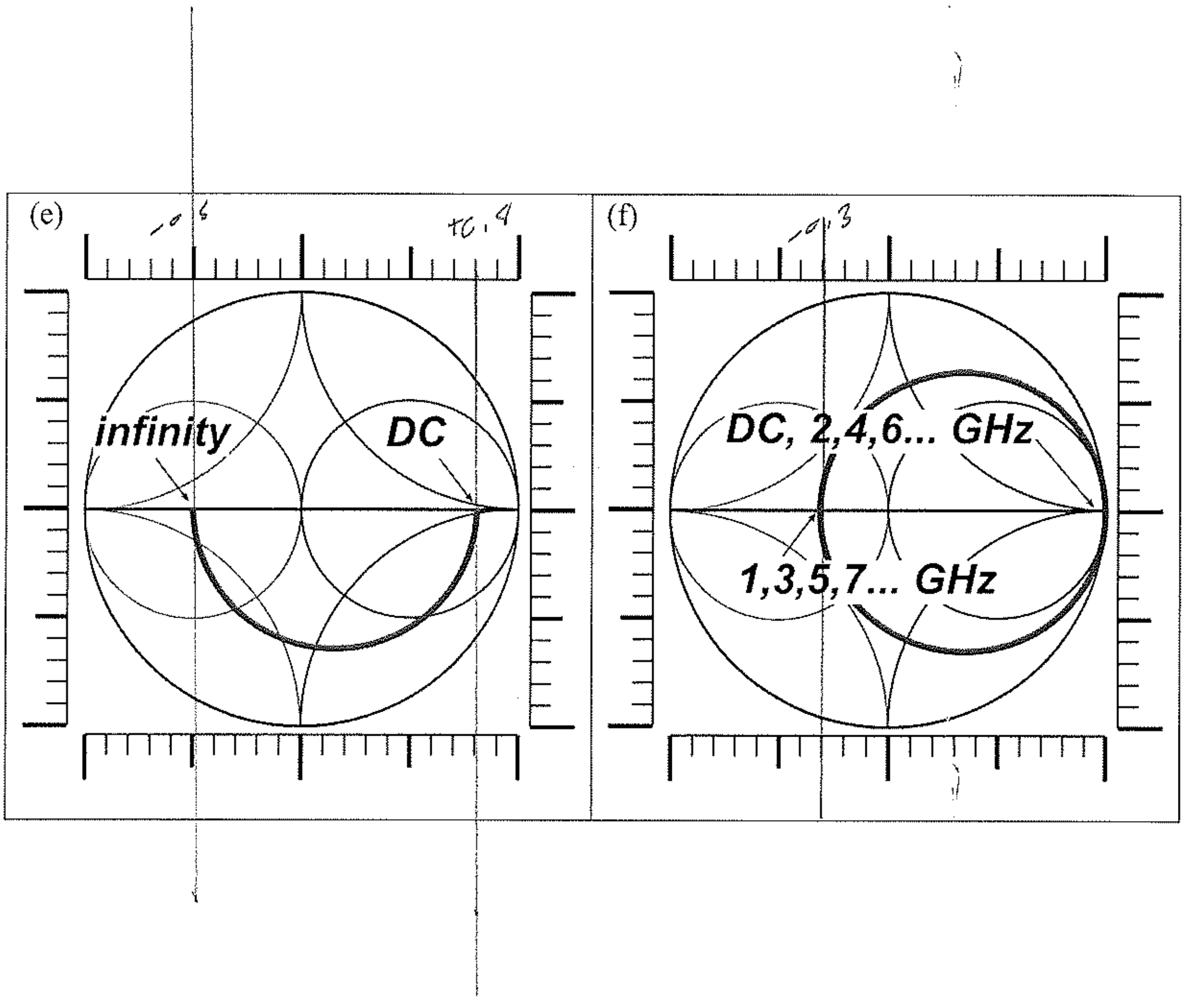
Problem	Points Received	Points Possible
1		15
2a		10
2b		7
2c		8
2d (218 only)		15 (218A only)
3a		7.5
3b		7.5
4		15
5a		5
5b		5
5c		5
5d (218 only)		15 (218A only)
total		85 (145), 115 (218A)

Name: solution

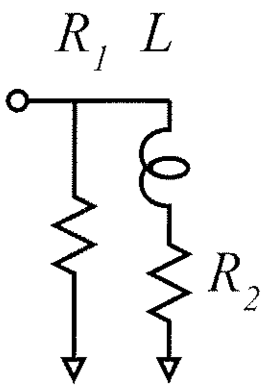
Problem 1, 15 points
The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.

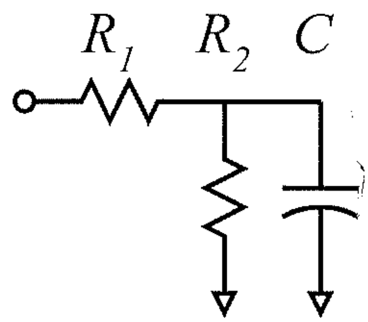



(e)
-0.5
+0.4
infinity
DC
(f)
-0.3
DC, 2,4,6... GHz
1,3,5,7... GHz

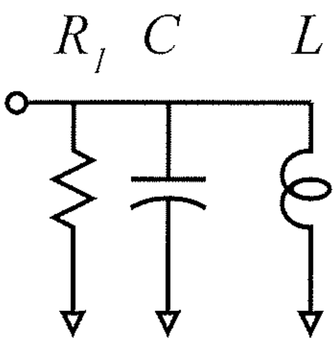
(i)



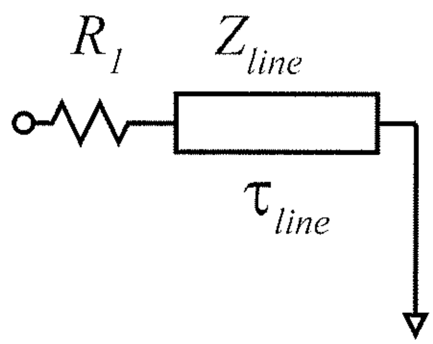
(j)



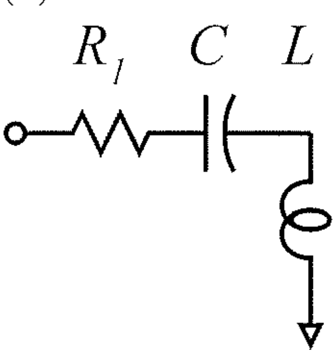
(k)



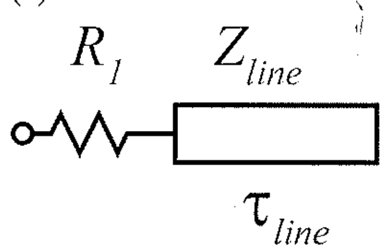
(l)



(m)



(n)



First match each Smith Chart with each circuit. ***Then determine as many component values as is possible*** (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given. The charts all use 50 Ohm normalization:

Smith chart (a). Circuit= e.
Component values: $R = 26.9\Omega$, $\tau = 1/4$ ns., ______,
Smith chart (b). Circuit= i.
Component values: $R_1 = 450\Omega$, $R_2 = 17.3\Omega$., ______,
Smith chart (c). Circuit= k.
Component values: $R = 50\Omega$., ______, ______,
Smith chart (d). Circuit= m.
Component values: $R = 50\Omega$., ______, ______,
Smith chart (e). Circuit= j.
Component values: $R_1 = 16.7\Omega$, $R_2 = 433\Omega$., ______,
Smith chart (f). Circuit= N.
Component values: $R = 27\Omega$, $\tau = 1/4$ ns., ______,

a) $\Gamma = -0.3 \rightarrow Z = Z_c \frac{1-0.3}{1+0.3} = 26.9\Omega$ (e)

impedance is ∞ at 1, 3, 5... GHz.

$\rightarrow$ network e with $R = 26.9\Omega$.

line is $\lambda/4$ @ 1 GHz $\rightarrow$ is λ @ 4 GHz $\rightarrow \tau = 1/4$ GHz $= 250$ ps

b) impedance increases with frequency $\rightarrow$ (i)

$R = 50\Omega (1-0.5)/(1+0.5) = 16.7\Omega$

to $R = 50\Omega (1+0.8)/(1-0.8) = 450\Omega$

$R_1 = 450\Omega$

$R_2 = (1/16.7\Omega - 1/450\Omega)^{-1} = 17.3\Omega$.

c) impedance increases from 0Ω @ DC then reaches 50Ω, drops to 0Ω as $f \rightarrow \infty$. (k)

50Ω

k. 1

D) goes from open @ DC to open @ $f = \infty$.

1 [50Ω at some intermediate frequency.

] [OM] 1

6] goes from $\Gamma = +0.8 \Rightarrow R = 50\Omega \,\frac{1+0.8}{1-0.8} = 450\Omega$ @ DC

to $\Gamma = -0.5 \Rightarrow R = 50\Omega \,\frac{1-0.5}{1+0.5} = 16.\bar{6}\,\Omega$ @ $f = \infty$

$R_1 + R_2 = 450\Omega$

$R_1 = 16.\bar{6}\,\Omega$

$R_2 = 450\Omega - 16.6\Omega = 433.\bar{3}\,\Omega$

(i)] 1

2

7). Impedance is infinity @ 2, 4, 6 GHz

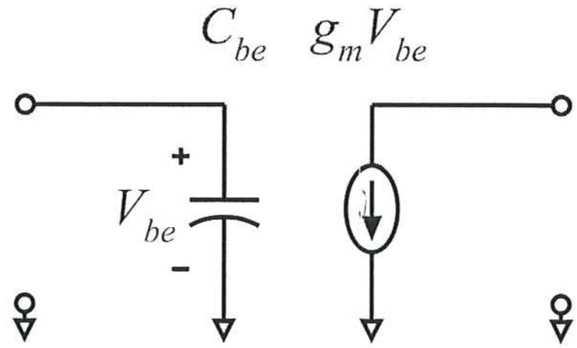
Line is $\lambda/4$ @ 1GHz $\rightarrow \lambda$ @ 4GHz $\rightarrow \tau = 250$ps

1 [@ 1GHz, $\Gamma = -0.3$, $\Rightarrow R = 50\Omega \,\frac{1-0.3}{1+0.3} = 26.9\Omega$

network N.] 1

Problem 2, 25 points (ece145A), 40 points (ece218A)
2-port parameters and Transistor models

Part a, 10 points
For the network at the right, give numerical values for the four S-parameters. Assume that the reference Zo is 50 Ohms. The signal frequency is 10GHz, gm=1 mS, and Cbe=1fF.



$$S_{11} = \left.\frac{Z_{in}/Z_o - 1}{Z_{in}/Z_o + 1}\right|_{Z_L = Z_o}$$

$$\left. Z_{in}\right|_{Z_L = Z_o} = 1/j\omega C$$

$$= \frac{1/j\omega C Z_o - 1}{1/j\omega C Z_o + 1} = \frac{1 - j\omega C Z_o}{1 + j\omega C Z_o} = \frac{(1 - j\omega C Z_o)^2}{(1 + \omega^2 C^2 Z_o^2)}$$

3

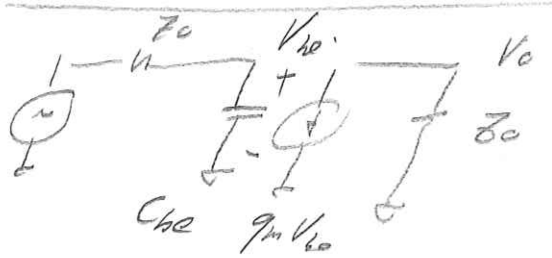
by inspection, $|S_{11}| = 1$

and $\angle S_{21} = 2\cdot(-1)\cdot \arctan(\omega C Z_o)$

$\doteq -2\cdot(89.8°) = -179.6°$

1

$S_{22} = 1$, $S_{12} = 0$



$$S_{21} = \left.\frac{2V_o}{V_{gen}}\right|_{Z_o = Z_L = Z_{gen}} \doteq \frac{-g_m Z_o \cdot 2}{1 + j\omega C Z_o} = \frac{-2 g_m Z_o}{1 + \omega^2 C^2 Z_o^2}(1 - j\omega C Z_o)$$

3

1

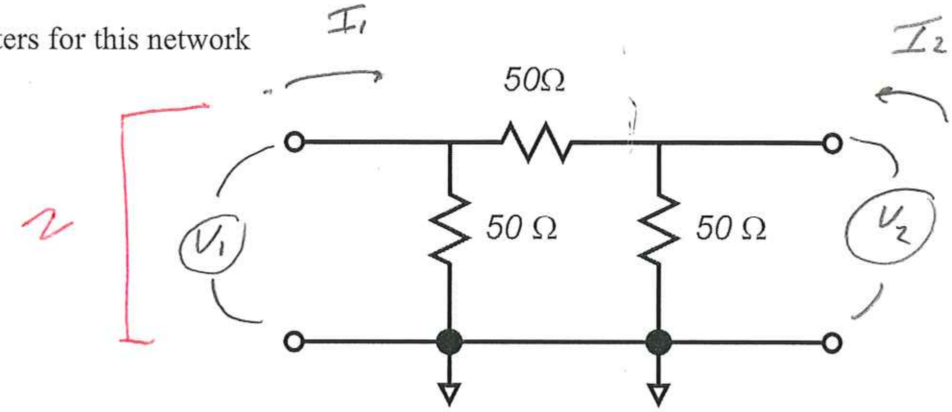
$$S_{21} = \frac{-50\Omega\cdot 1mS\cdot 2}{1 + 9.9\cdot 10^{-6}}\cdot(1 - j\,3.1\cdot 10^{-3})$$

$$= 0.1$$

$Y_o = 1/50\Omega.$

Part b, 7 points

Compute the Y parameters for this network



2 $I_1 = V_1(2Y_o) - V_2 Y_o.$

2 $Y_{11} = 2Y_o = 2/50\Omega = 40mS$

$Y_{12} = -Y_o = -20mS.$

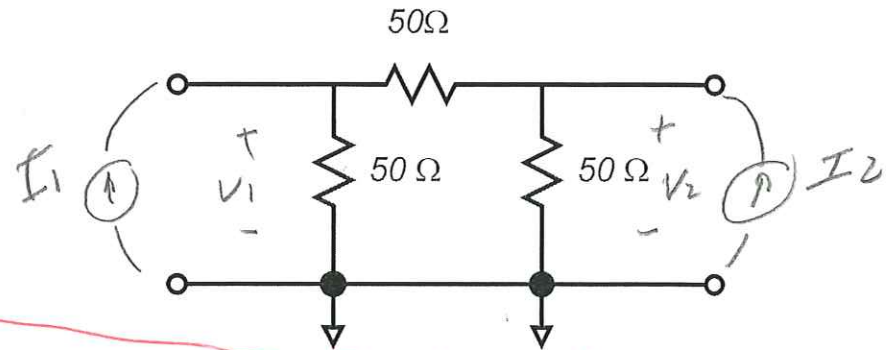
1 by Symmetry $Y_{22} = Y_{11} = 40mS$

$Y_{21} = Y_{12} = -20mS.$

$Z_0 = 50\Omega$

Part c, 7 points

Compute the ***Z *** parameters for this network



problem is matrix inversion

4 pts for recognizing this
3 pts for correct math.

We know that.

$$\begin{bmatrix} I_1 = Y_{11} V_1 + Y_{12} V_2 \\ I_2 = Y_{21} V_1 + Y_{22} V_2 \end{bmatrix}$$

so

$$Y_{22} I_1 = Y_{22} Y_{11} V_1 + Y_{22} Y_{12} V_2$$

$$Y_{12} I_2 = Y_{12} Y_{21} V_1 + Y_{12} Y_{22} V_2$$

subtract these:

$$Y_{22} I_1 - Y_{12} I_2 = \underbrace{(Y_{22} Y_{11} - Y_{12} Y_{21})}_{\Delta Y} V_1$$

$\Delta Y = (Y_{11}^2) - (Y_{12})^2 = 4Y_0^2 - Y_0^2 = 3Y_0^2$

or

$$V_1 = \underbrace{\frac{Y_{22}}{\Delta Y}}_{Z_{11}} I_1 - \underbrace{\frac{Y_{12}}{\Delta Y}}_{Z_{12}} I_2$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{2Y_0}{3Y_0^2} = \frac{2}{3}\frac{1}{Y_0} = \frac{2}{3} \cdot Z_0 = \frac{2}{3} \cdot 50\Omega = \underline{33.33\ \Omega}$$

$$Z_{12} = \frac{-Y_{12}}{\Delta Y} = \frac{+Y_0}{3Y_0^2} = \frac{1}{3} Z_0 = \frac{1}{3} \cdot 50\Omega = 16.\bar{6}\ \Omega.$$

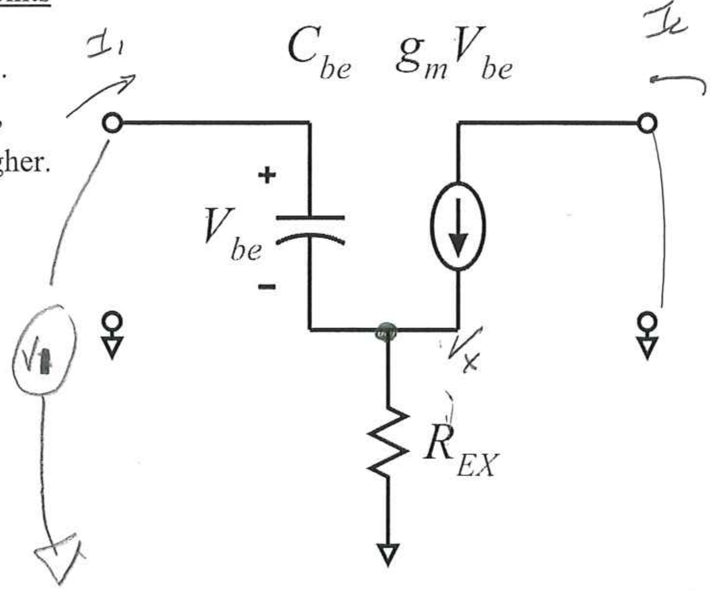
by symmetry, $Z_{22} = Z_{11} = 33.3\ \Omega$

$\underline{Z_{21} = Z_{12} = 16.6\ \Omega}$

Part d, ***ECE218A students only*** 15 points

For the network at the right, give an algebraic expressions for Y_{11} and Y_{21}. Please write as a Taylor series in $j\omega$, omitting terms of power $(j\omega)^3$ and higher.

This is an exercise in device model extraction from measured S/Y/Z parameters.



take V_x as the only unknown:

5pts for correct circuit solution

ΣI @ $V_x = 0$.

$$(V_x - V_1) j\omega C_{be} + V_x/R_{ex} + g_m (V_x - V_1) = 0.$$

$$v_x (g_m + 1/R_{ex} + j\omega C_{be}) = v_1 (g_m + j\omega C_{be}).$$

$$v_x = v_1 \frac{g_m + j\omega C_{be}}{g_m + 1/R_{ex} + j\omega C_{be}}.$$

5pts

$$(v_1 - v_x) = \frac{1/R_{ex}}{g_m + 1/R_{ex} + j\omega C_{be}} \cdot v_1$$

$$I_1 = j\omega C_{be}(V_1 - V_x) =$$

$$= \frac{j\omega C_{be}/R_{ex}}{g_m + 1/R_{ex} + j\omega C_{be}} \cdot v_1$$

$$\frac{I_1}{v_1} = Y_{11} = \frac{j\omega C_{be}/R_{ex}}{g_m + 1/R_{ex} + j\omega C_{be}}$$

$$= j\omega\left(\frac{C_{be}}{1+g_m R_{ex}}\right)\frac{1}{1 + j\omega\dfrac{C_{be}}{1+g_m R_{ex}}}$$

but
$$\frac{1}{1+j\varepsilon} = \frac{1}{1+j\varepsilon}\,\frac{1-j\varepsilon}{1-j\varepsilon} = \frac{1-j\varepsilon}{1+\varepsilon^2}$$

$$= (1-j\varepsilon)(1-\varepsilon^2+O(\varepsilon^4))$$

$$= 1 - j\varepsilon - \varepsilon^2 + j\varepsilon^3 + O(\varepsilon^4))$$

$$= 1 - j\varepsilon - \varepsilon^2 + O(j\varepsilon^3)$$

so
$$Y_{11} = j\omega\frac{C_{be}}{1+g_m R_{ex}}\left[1 - j\omega\frac{C_{be}}{1+g_m R_{ex}} - \frac{\omega^2 C_{be}^2}{(1+g_m R_{ex})^2} + O(\omega^3)\right]$$

$$\boxed{Y_{11} = j\omega\frac{C_{be}}{1+g_m R_{ex}} + \frac{\omega^2 C_{be}^2}{(1+g_m R_{ex})^2} + O(j\omega^3)}$$ *

*we can see that this is $C_{be}/(1+g_m R_{ex})$ in series with R_{ex}

5pts for Y_{11}

5 $V_2 = I_1 \cdot \dfrac{g_m}{j\omega C_{be}}$

so $Y_{21} = Y_{11} \cdot \dfrac{g_m}{j\omega C_{be}}$

$$Y_{21} = \cancel{\frac{g_m}{j\omega C_{be}}} \left[\frac{\cancel{j\omega C_{be}}}{1 + g_m R_{ex}} \right] \left[1 - \frac{j\omega C_{be}}{1 + g_m R_{ex}} - \frac{\omega^2 C_{be}^2}{(1 + g_m R_{ex})^2} \right] + O(\omega^3)$$

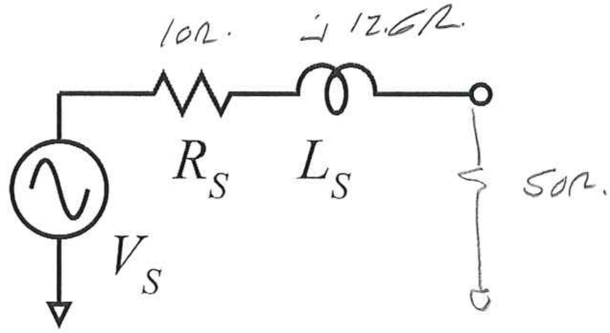
$$\boxed{Y_{21} = \frac{g_m}{1 + g_m R_{ex}} \left[1 - \frac{j\omega C_{be}}{1 + g_m R_{ex}} - \frac{\omega^2 C_{be}^2}{(1 + g_m R_{ex})^2} + O(\omega^3) \right]}$$

5 pts for Y_{21}

Problem 3, 15 points
Available source power relationships, lumped/distributed relationships.

Part a, 7.5 points
Vs is 0.1V RMS at 2GHz
Rs is 10 Ohms, Ls is 1nH.

At 2GHz, what power would be delivered into a 50 Ohm load ? What is the available signal power ?



1 $X_L = \omega L = 2\pi f \cdot L = 12.57\Omega.$

$V_L = 0.1V \cdot \frac{50\Omega}{60 + j12.6\Omega} \Rightarrow \|V_L\| = 0.1V \frac{50\Omega}{\|60\Omega + j12.6\Omega\|}$

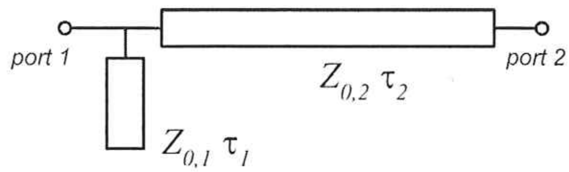
4.5 $\|V_L\| = 0.1V \frac{50\Omega}{\sqrt{60^2 + 12.6^2}} = 0.1V \cdot 0.8155 = 0.08155V.$

$P_L = \frac{\|V_L\|^2}{50\Omega} = 0.133\,mW.$ into 50Ω.

2 $P_{AVG} = \frac{(0.1V)^2}{4(10\Omega)} = \underline{0.25\,mW}.$

Part b, 7.5 points

In the network to the right, Zo2=100 Ohm, Zo1=25 Ohms, tau_2=50ps, tau_1=50ps.



Representing line #2 as a pi-section and line #1 as a T-section, give an approximate lumped equivalent circuit model, with element values, for the network.

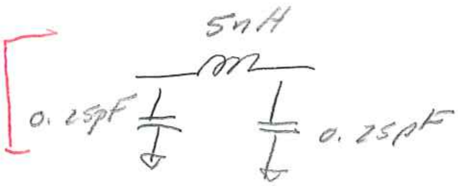
Approximately what would be the highest frequency at which the lumped network might reasonably approximate the distributed network (rough answer only) ?

Line 2:

1/2 $L = \tau_2 Z_{02} = 50ps \cdot 100\Omega = 5000pH = 5nH$

1/2 $C = \tau_2 / Z_{02} = 50ps/100\Omega = 0.5pF$

1

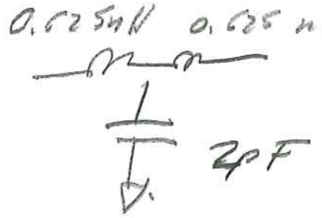


Line 1:

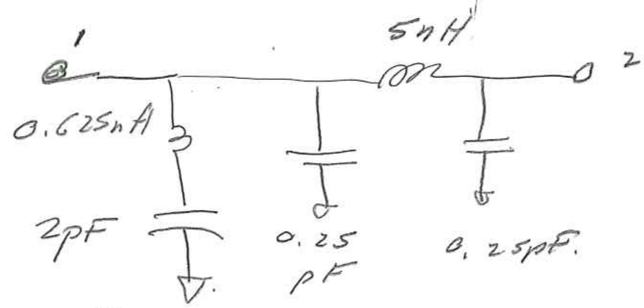
$L = \tau_1 Z_{01} = 50ps \cdot 25\Omega = 1250\,pH = 1.25\,nH.$ 1/2

$C = \tau_1 / Z_{01} = 50ps/25\Omega = 2pF$ 1/2

1



Combined network:



1.5

T and π equivalent ccts are approximate for any ratio of ℓ to λ. The approximations become extremely poor for $\ell > \lambda/4$.

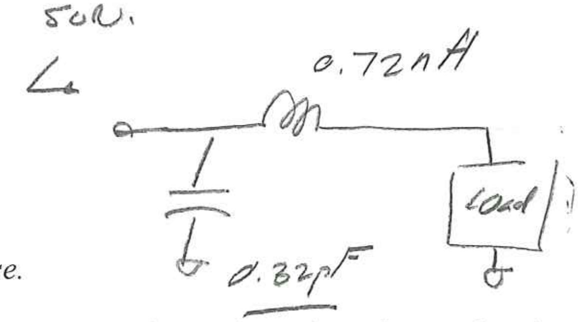
$\ell = \lambda/4$ corresponds to $f = 1/4\tau$

$f = 1/4\tau = \dfrac{1}{4\,(50\,ps)} = 5\,GHz$

1.

1

5 GHz would be the upper useful frequency of this approximate model for even rough approximation



Problem 4, 15 points
Impedance-matching exercise.

At 10GHz signal frequency, an antenna has an input impedance of 25-j20 Ohms. Design a matching network, using a series inductor and a shunt capacitor, which matches this impedance to 50 Ohms at 10GHz.

Give all element values. Use the full impedance-admittance chart which has been provided to you.

2 Normalized $z_L = \frac{25 - j20}{50} = 0.5 - j0.4$

2 We then move to point "x" on the smith chart

at point x, $z = 0.5 + j0.5$

2 so, change in $z = (0.5 + j0.5) - (0.5 - j0.4) = j0.9$

change in $\Delta Z = j0.9 \cdot 50\Omega = j45\Omega$.

This is a series inductor $\Delta Z = j45\Omega = j\omega L$

2 $\Rightarrow L = 45\Omega / 2\pi f = 45\Omega / 2\pi(10GHz)$

$= 0.716\ nH$.

2 We then move to point "y"; $z = 1. + j0$, $y = 1 + j0$.

2 $\left.\begin{aligned} y_y &= 1 + j0 \\ y_x &= 1 - j1 \end{aligned}\right\} \quad \Delta y = (1 + j0) - (1 - j1) = j1$

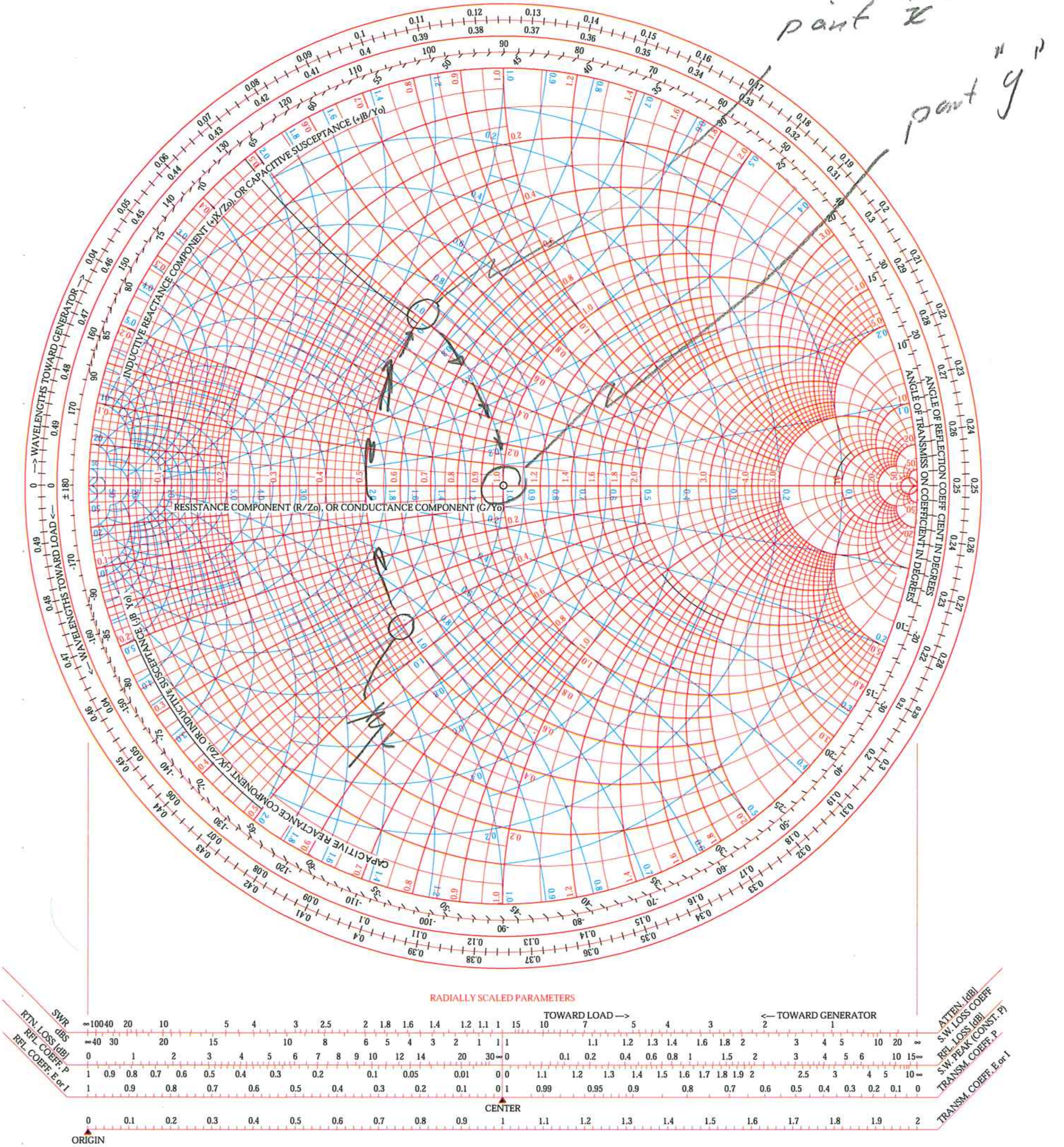
$\Delta Y = \Delta y / Z_0 = \frac{j1}{50\Omega} = j\,20mS$

This is a shunt capacitor $\Delta Y = j20mS = j\omega C$

3. $\Rightarrow C = \frac{20mS}{2\pi f} = \frac{20mS}{2\pi(10GHz)} = 0.318pF$

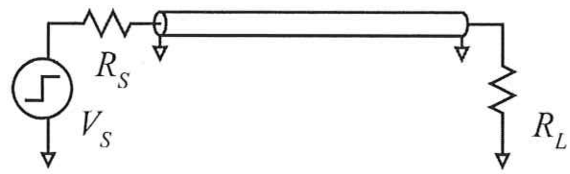
NAME	TITLE problem 4.	DWG. NO.
		DATE
SMITH CHART FORM ZY-01-N	Microwave Circuit Design - EE523 - Fall 2000	

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



Part b, 5 points

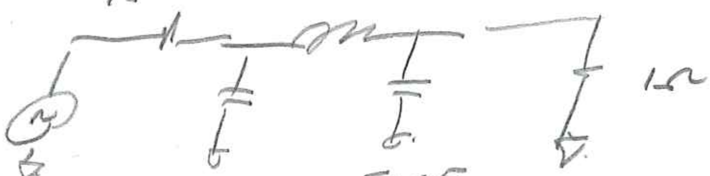
Using the parameters from part a, if we drive the cable with a step-function and with Rs=RL=1 Ohm, what will be, approximately, the 10%-90% risetime of the voltage waveform at the load ?



with $R_S = R_L \ll Z_0$

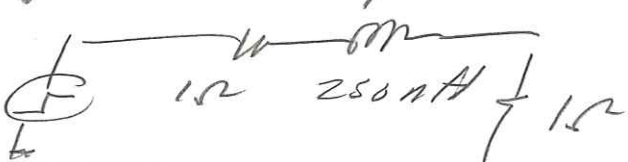
lets treat the line as a lumped approximation

use a π section



but the RC time constants are $R \cdot C = 1\Omega \cdot 50pF = 50ps$.
while the L/R time constants are $L/R = \frac{250nH}{2\Omega} = 125ns$.
So the RC time constants are negligible →

— neglect the capacitors. —



$L/R = 125ns \Rightarrow T_{10-90} = 2.2\tau$

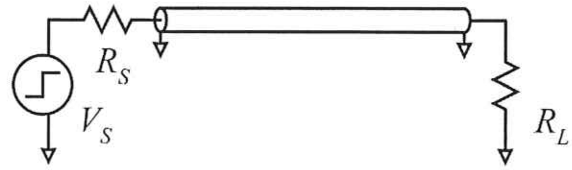
$= 2.2\,(250ns)$

$= \cancel{}ns$

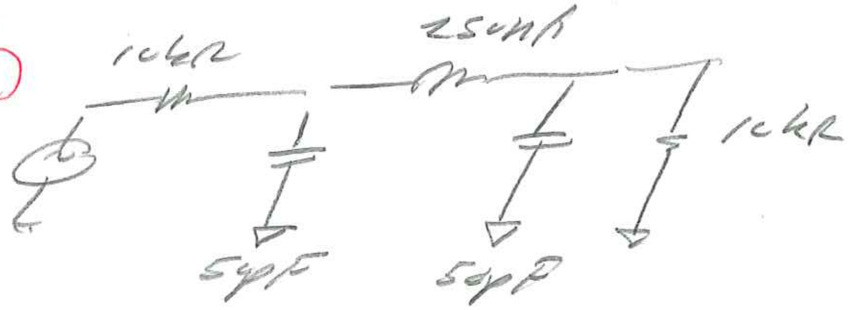
$= \underline{550\ ns}$

Part c, 5 points

Using the parameters from part a, if we drive the cable with a step-function and with Rs=RL=10 kOhm, what will be, approximately, the 10%-90% risetime of the voltage waveform at the load ?



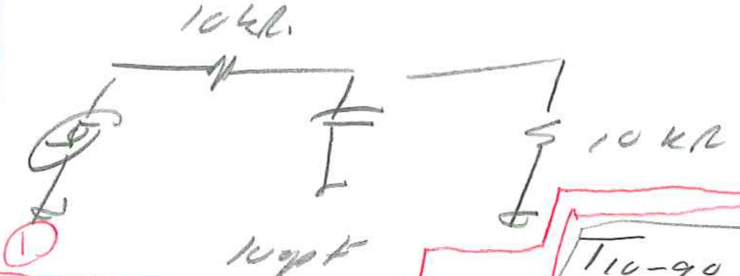
With $R_S = R_L >> Z_0$
let's treat the line as a lumped approximation
use a π section



Now the $\tau_{RC} \approx 50pF \cdot 10k\Omega = 500\ ns$.

Now the $\tau_{L/R} = \frac{250nH}{20k\Omega} = 12.5ps$

⇒ inductive effects are negligible ⇒



$\tau_{RC} = 100pF \cdot 5k\Omega$
$= 500\ nS$

$T_{10-90} = 2.2\tau$
$= 1.1\ \mu s$

Problem 5, 15 points (ece145A), 20 points (218A)
Transmission-line properties.

Part a, 5 points
Coaxial cable uses a signal conductor of round cross-section with an insulating dielectric and a ground conductor both wrapped around it. The characteristic impedance is
$Z_0 = (1/2\pi)(\mu_0/\varepsilon_r\varepsilon_0)^{1/2}\ln(D/d)$
where $(\mu_0/\varepsilon_0)^{1/2} = 377\Omega$, ε_r is the insulator dielectric constant, and d and D are the diameters of the inner and outer conductors.

The velocity is $v = c/\varepsilon_r^{1/2}$, where c is the speed of light.

Suppose that the dielectric is Polyethylene, which has ε_r=2.25, and that the inner conductor is 1mm diameter.
✓For 50 Ohms characteristic impedance, what must be the outer conductor diameter ?
✓What is the wave velocity on the transmission line ?
If the cable is 1 meter long ,what is the total line capacitance and inductance ?

$$50\Omega = \frac{377\Omega}{2\pi}\,\frac{1}{\varepsilon_r^{1/2}}\,\ln\left(\frac{D}{d}\right)$$

$$\ln\left(\frac{D}{d}\right) = 2\pi\cdot\varepsilon_r^{1/2}\,\frac{50\Omega}{377\Omega} = 1.25.$$

2.25

2

$$\Rightarrow \frac{D}{d} = e^{1.25} = 3.50 \rightarrow D = 3.5\cdot(1mm) = 3.5mm$$

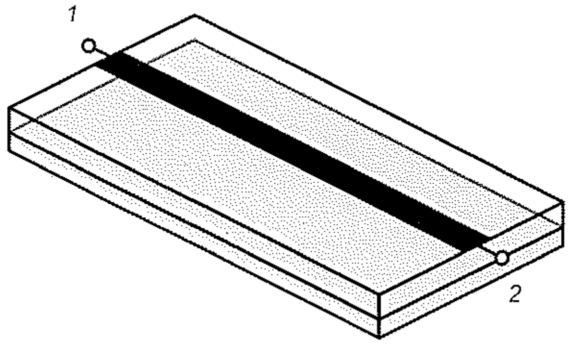
1 $$v = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\cdot10^8 m/s}{1.5} = 2\cdot10^8 m/s.$$

$$l = 1m;\quad \tau = l/v = 1m/2\cdot10^8 m/s = 5ns.$$

1 $$C = \tau/Z_0 = \frac{5ns}{50\Omega} = 0.1nF = 100pF.$$

1 $$L = \tau Z_0 = 5ns\cdot50\Omega = 250nH$$

Part d, 15 points ECE 218 students only



We will make a 50 Ohm microstrip line on commercial board material (like that used in the class) with a dielectric constant of 2.2.

The design frequency is 140GHz.

You are to design a 50 Ohm transmission-line. To approximately model the effect of fringing fields, assume that

$Z_0 \cong (\mu_0 / \varepsilon_r \varepsilon_0)^{1/2}(H/(H+W))$

where W is physical conductor width and H is the board thickness.

To adequately suppress coupling to dielectric slab modes, keep the board thickness less than or equal to the 1/4 of a wavelength *in the dielectric* at the design frequency.

To adequately suppress lateral transmission-line models, keep the conductor width less than or equal to 1/4 of a wavelength *in the dielectric* at the design frequency.

The conductivity of (very pure) copper is 5.96×10^7 (1/Ohm/meter) at 20C.

-Determine the board thickness and the conductor width.
-Determine the skin effect loss, in dB/mm, at 140GHz.
(hint: the skin depth is $\delta = (2/\omega\mu_0\sigma)^{1/2}$, where $\mu_0 = 4\pi\cdot10^{-7}$ H/m)

1

$$Z_0 = \frac{377\Omega}{\sqrt{\epsilon_r}} \cdot \frac{H}{H+W} = 50\Omega \,, \quad \epsilon_r = 2.2.$$

$$\Rightarrow \frac{H}{H+W} = \frac{50\Omega \cdot \sqrt{2.2}}{377\Omega} = 0.197 \qquad \frac{H+W}{H} = 5.08$$

$$\Rightarrow \boxed{W/H = 4.08}$$

2

We need $H \leq \lambda_d/4$, but also need $W \leq \lambda_d/4$.
Since $H < W$, clearly we set $W = \lambda_d/4$

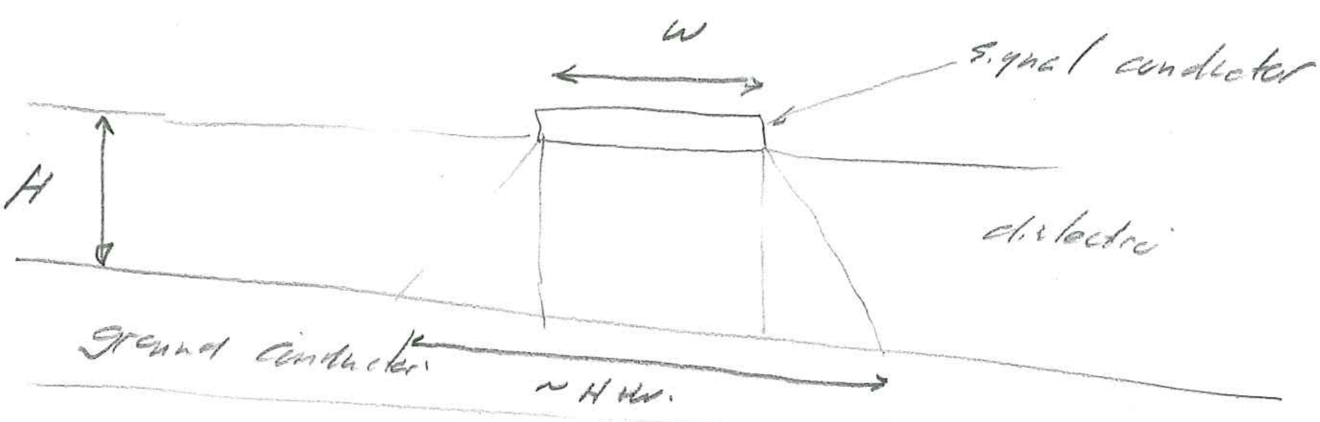
$$\lambda_d = \frac{c}{f\sqrt{\epsilon_r}} = \frac{3 \cdot 10^8 \, m/s}{140\,GHz \, \sqrt{2.2}} = 1.44\,mm.$$

$$\Rightarrow \left[\begin{array}{l} W = \lambda_d/4 = 1.44\,mm/4 = 361\,\mu m. \\ H = W/4.08 = 88\,\mu m \end{array} \right.$$

2

$$\text{skin depth} = \delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} = \sqrt{\frac{2}{2\pi(140\,GHz) \cdot 4\pi \cdot 10^{-7}\,H/m \cdot 6 \cdot 10^7 \, \frac{S}{m}}}$$

$$= 0.174\,\mu m.$$



Now, the signal current is carried across the full conductor width w.

$\Rightarrow$ Signal conductor resistance per unit length

2 $= R_{sig}\ (\Omega/m) \cong \frac{1}{\sigma w \delta} = 267\ \Omega/m.$

While the ground current is carried over an effective conductor width of $\sim(w+H)$

2 $\Rightarrow$ ground conductor resistance per unit length

$= R_{ground}\ (\Omega/m) \cong \frac{1}{\sigma \delta (w+H)} = 215\ \Omega/m$

Total resistance per unit length

2 $$R_{Tot}\ (\Omega/m) \cong \frac{1}{\sigma \delta}\left(\frac{1}{w} + \frac{1}{w+H}\right) = 482\ \Omega/m$$

2 Waves decay in voltage as $e^{-\alpha z}$ where $\alpha = \frac{R_{Tot}}{2 Z_0}$

$$\alpha = \frac{R_{Tot}}{2 Z_0} = \frac{482\ \Omega/m}{100\ \Omega} = 4.82\ m^{-1}$$

In one meter, the signal has attenuated in voltage by $e^{-\alpha L} = e^{-4.82}$.

2 In dB, this is dBloss $= 20 \cdot \log_{10}\left[e^{-4.82}\right]$

$= -41.9\ dB$

$\Rightarrow$ Loss is 41.9 dB/meter or 0.04 dB/mm.