# ECE ECE145A (undergrad) and ECE218A (graduate)
# Final Exam. Monday December 9, 2019, noon - 3 p.m

Open book. You have 3 hrs.
Use all reasonable approximations (5% accuracy is fine.),
***AFTER STATING and justifying THEM.***
***Think before doing complex calculations. Sometimes there is an easier way.***

| Problem | Points Received | Points Possible |
|---|---|---|
| 1A | | 5 |
| 1B | | 5 |
| 1C | | 7 (218A only) |
| 2A | | 5 |
| 2B | | 10 |
| 3A | | 5 |
| 3B | | 10 (218A only) |
| 3C | | 5 |
| 3D | | 10 |
| 4A | | 5 |
| 4B | | 5 |
| 4C | | 5 |
| 4D | | 10 |
| 5A | | 7 |
| 5B | | 5 |
| 5C | | 10 (218A only) |
| total | | 87(145A), 114 (218A) |

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L|^2} \qquad G_P = \frac{1}{1-\|\Gamma_{in}\|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-\|\Gamma_{out}\|^2} \qquad G_{\max} = \frac{|S_{21}|}{|S_{12}|}\cdot\left[K-\sqrt{K^2-1}\right]\text{if } K>1$$

$$G_{MS} = \frac{|S_{21}|}{|S_{12}|}\cdot\text{if } K<1 \qquad K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{21}S_{12}|} \qquad \text{where } \Delta=\det[S]$$

Unconditionally stable if: (1) K>1 **and** (2) ||det[S]||<1

Solution

**Problem 1, 10 points (145A), 17 points (218A)**
*Transistor two-port properties, Gain relationships*

Device model:

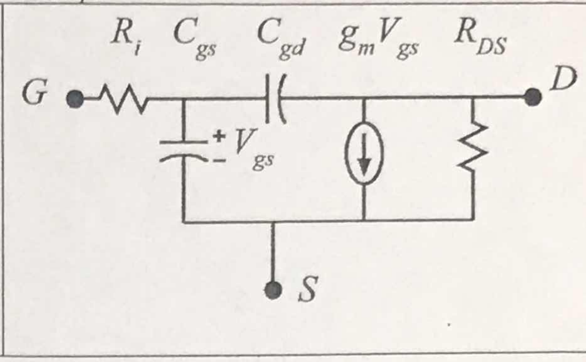
$g_m = 2mS/\mu m \cdot W_g \quad R_i = 1/g_m$

$C_{gd} = 0.0 fF/\mu m \cdot W_g$

$C_{gs} = 0.5 fF/\mu m \cdot W_g$

$G_{ds} = 0.2 mS/\mu m \cdot W_g$

$W_g = 20\mu m$




part a, 5 points

What are $f_\tau$ and $f_{max}$ for this transistor ?

$f_\tau$ = 636 GHz

$f_{max}$ = 1010 GHz

$g_m = 2 mS/\mu m \cdot 20\mu m = 40 mS = 1/25\Omega$

$R_i = 1/g_m = 25\Omega.$

$C_{gs} = 0.5 fF/\mu m \cdot 20\mu m = 10 fF$

$C_{gd} = 0 fF$

$G_{ds} = 0.2 mS/\mu m \cdot 20\mu m = 4 mS = 1/250\Omega.$

2.5 $f_\tau = \frac{g_m}{2\pi(C_{gs}+C_{gd})} = 0.159 \cdot \frac{40mS}{10fF} = 636 GHz$

2.5 $f_{max} = \frac{f_\tau}{2\sqrt{R_i G_{ds}}} = \frac{f_\tau}{0.632} = 1.01 THz.$

For this device model only.

part b, 5 points

You are going to use the transistors at 60 GHz signal frequency.
What power gain would you expect to get after impedance, matching ?
What would be the correct generator impedance and load impedance to obtain this power gain ?

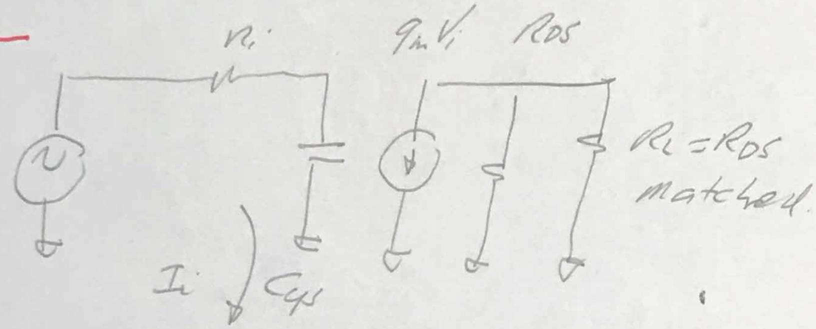
Gain = ___24.5___ dB
Source impedance = ___25 + j265___ Ohms
Load impedance = ___250___ Ohms

2

we can use $MAG = (f_{max}/f)^2 = \frac{1.01\text{ THz}}{60\text{ GHz}} = 283.$

or, from first principles:

Either or.




2

$$P_{in} = I_{in}^2 R_i ; \quad g_m V_i = \frac{g_m}{\omega C_{gs}} \cdot I_{in}$$

$$P_{out} = 1/4 (g_m V_i)^2 R_{DS} = I_{in}^2 \left(\frac{g_m}{\omega C_{gs}}\right)^2 \frac{R_{DS}}{4}.$$

$$\frac{P_o}{P_i} = \frac{R_{DS}}{4R_i}\left(\frac{g_m}{\omega C_{gs}}\right)^2 = \left(\frac{f_{max}}{f}\right)^2 = \left(\frac{1.01\text{ THz}}{60\text{ GHz}}\right)^2$$

$$= 283:1 \quad \text{or} \quad 10\log_{10}(283) = 24.5\text{ dB}$$

optimum load impedance is $R_L = R_{DS} = 250\Omega$.

2. $$Z_{in} = R_i + \frac{1}{j\omega C_{gs}} = 25\Omega + \frac{0.159}{j\,60\,\text{GHz}\cdot 10\,\text{fF}}$$

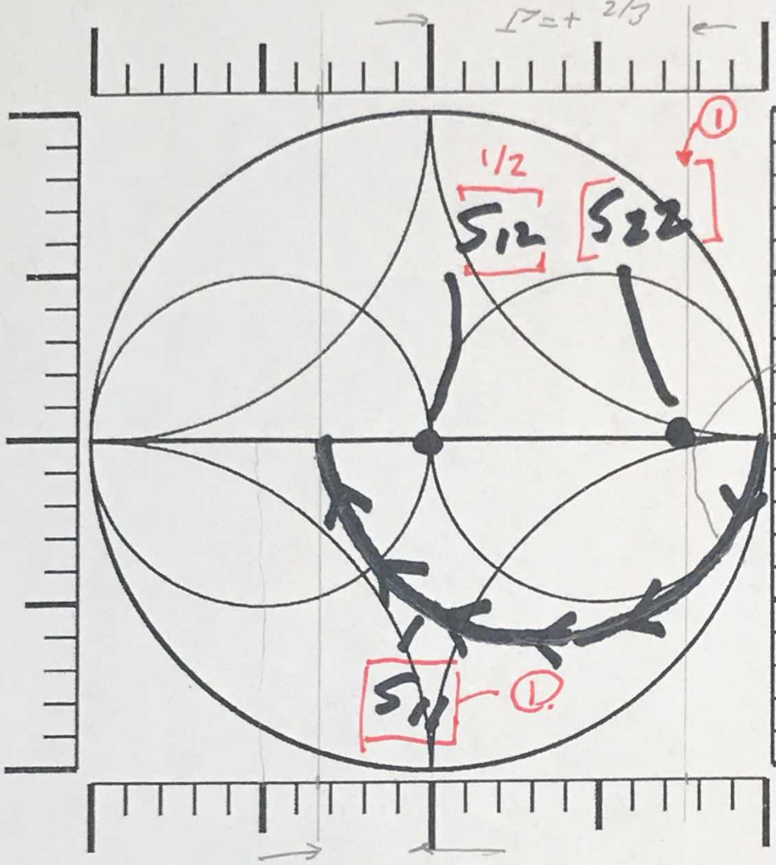
$$= 25\Omega - j\,265\Omega.$$

$$Z_{gen,opt} = Z_{in}^{*} = 25\Omega + j\,265\Omega.$$

1 $$Z_{L,opt} = R_{DS} = 250\Omega.$$

part c, 7 points (218A students only)

With the numerical values given in the equivalent circuit, make clear sketches of S11, S22, S12, and S21, from DC to infinite frequency, on the Smith chart below. This may require that you calculate the S parameters first.

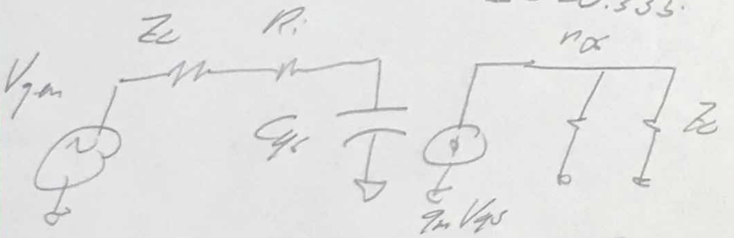



1/2 [ $S_{12} = 0$.

① [ $S_{22} = \frac{250/50 - 1}{250/50 + 1} = \frac{5-1}{5+1} = 4/6 = 2/3.$

① [ $S_{11}$: well $Z_i$ is $25\Omega$ in series with 10 fF, so $S_{11}$ follows $25\Omega$ curve on smith chart.

$\frac{25/50 - 1}{25/50 + 1} = \frac{-1/2}{1.5} = -0.333.$

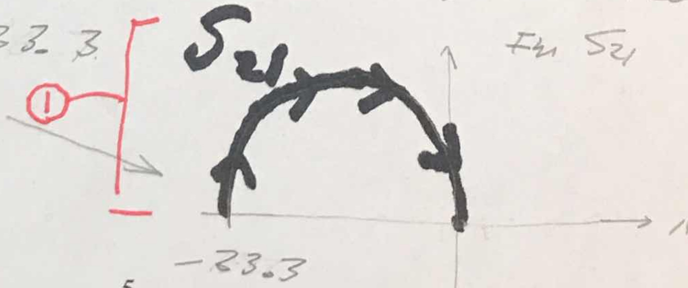



$S_{21} = 2 \frac{V_0}{V_{gen}} \Big/_{Z_S = Z_L = Z_{gen}}$

$= \frac{-2 \cdot g_m \cdot (R_{ds} \| Z_0)}{1 + j\omega C_{gs} \cdot (R_i + Z_0)}$ ①

$2 \cdot g_m \cdot (r_{ds} \| Z_0) = -33.3$

① [

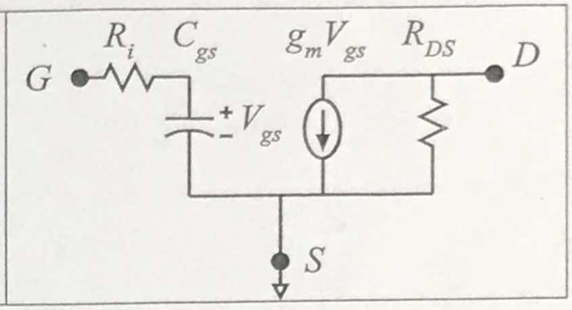





Separate sketch of $S_{21}$.

**Problem 2, 15 points**
*Two-port properties*

part a, 5 points

| *Properties of Y-parameters*<br><br>$C_{gs}$=63.6 fF, $g_m$=50 mS.<br>$R_{ds}$=100 Ohms, $R_i$=50 Ohms,<br><br>Find Y11, Y12, Y21, Y22 at 50 GHz. | <br> |
|---|---|

Y11= 10mS + j 10mS.　　Y12= 0mS.

Y21= 25mS − j 25mS.　　Y22= 10mS.

2. $Y_{11} = \dfrac{1}{R_i + 1/j\omega C_{gs}} = \dfrac{1}{50\Omega - j\,50\Omega}$

$= \dfrac{1}{50\Omega} \dfrac{1}{1-j1} \dfrac{1+j1}{1+j1} = \dfrac{1}{50\Omega} \dfrac{1+j}{2} = 10^{mS} + j\,10mS$

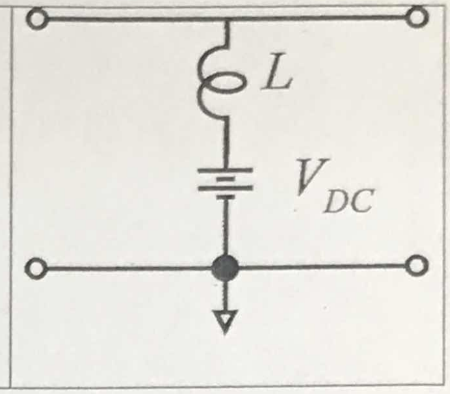
2 $Y_{21} = \dfrac{g_m}{1 + j\omega R_i C_{gs}} = \dfrac{50mS}{1+j} \dfrac{1-j}{1-j} = \dfrac{50mS}{2}(1-j)$

$= 25mS - j\,25mS.$

1/2 $Y_{12} = 0$ mS

1/2 $Y_{22} = 1/R_{DS} = 1/100\Omega = 10mS.$

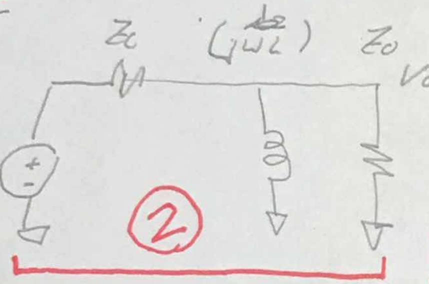
part b, 10 points

| *Properties of S-parameters*<br><br>The network at the right is for a DC bias feed .<br><br>If we want $\|S21\|$> -3 dB at 1GHz, what is the minimum value of the inductor ?<br><br>If we want $\|S11\|$< -40 dB at 1GHz, what is the minimum value of the inductor ?<br><br>Assume a 50 Ohm impedance standard. | <br> |
|---|---|

Minimum value of L to meet S21 specification= 4 nH

Minimum value of L to meet S11 specification= 400 nH.

$S_{21}$




$S_{21} = 2 \dfrac{V_0}{V_{gen}} \Big|_{Z_0 = Z_L = Z_{gen}}$

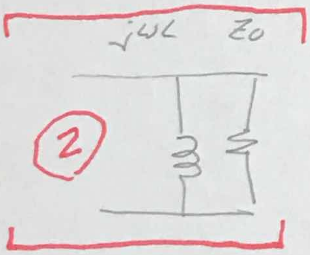
Nodal analysis:

$V_0 \left( Y_0 + Y_0 + \dfrac{1}{j\omega L} \right) = V_{gen} Y_0$

2 $S_{21} = \dfrac{2Y_0}{2Y_0 + \dfrac{1}{j\omega L}} = \dfrac{j\omega L \cdot 2Y_0}{1 + j\omega L 2Y_0} = \dfrac{j\omega L/(Z_0/2)}{1 + j\omega L/(Z_0/2)}$

$\rightarrow$ -3 dB when $\omega = \dfrac{Z_0/2}{L} \rightarrow f = \dfrac{1}{2\pi} \dfrac{Z_0/2}{L} \Rightarrow L = \dfrac{Z_0/2}{2\pi f} = 4\,nH$ 1

$S_{11}$




2 $S_{11} = \dfrac{Z_{in}|_{Z_L = Z_0} - Z_0}{Z_{in}|_{Z_L = Z_0} + Z_0} = \dfrac{Y_0 - Y_{in}|_{Y_L = Z_0}}{Y_0 + Y_{in}|_{Y_L = Z_0}}$

$= \dfrac{Y_0 - (Y_0 + \frac{1}{j\omega L})}{Y_0 + (Y_0 + \frac{1}{j\omega L})} = \dfrac{1/j\omega L}{2Y_0 + 1/j\omega L}$

$S_{11} = \dfrac{1}{1 + j\omega L \cdot 2Y_0} = \dfrac{1}{1 + j\omega L/(Z_0/2)} = \dfrac{1}{1 + j\omega\tau}$ where $\tau = \dfrac{L}{Z_0/2}$

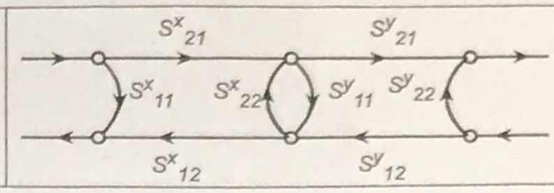
If $S_{11} = -40\,dB \rightarrow \|S_{11}\| = \dfrac{1}{100} \rightarrow \omega\tau \cong 100$.

$\rightarrow \omega = 100/\tau = 100 \cdot \dfrac{Z_0/2}{L}$

1 $L = \dfrac{100 \cdot Z_0/2}{2\pi f} = 400\,nH.$

**Problem 3, 20 points (145A), 30 points (218A)**
*2-port parameters and signal flow graphs*

Part a, 5 points

| | |
|---|---|
| The signal flow graph to the right represents the cascade of two-ports "x" and "y". If we call the combined network "z", find $S^z_{21}$, $S^z_{12}$, $S^z_{21}/S^z_{12}$ and $S^z_{11}$ |  |



$S^z_{21}$= $\dfrac{S_{21}^x S_{21}^y}{1 - S_{22}^x S_{11}^y}$ , $S^z_{12}$=__________________

$S^z_{21}/S^z_{12}$=__________________, $S^z_{11}$=__________________

(2)

$S_{21}^z$: there is one path $S_{21}^x \cdot S_{21}^y$

there is one loop $S_{22}^x S_{11}^y$ -- and it touches the path.

$\hookrightarrow S_{21}^z = \dfrac{S_{21}^x S_{21}^y}{1 - S_{22}^x S_{11}^y}$

(1/2)

by symmetry: $S_{12}^z = \dfrac{S_{12}^x S_{12}^y}{1 - S_{22}^x S_{11}^y}$

(1/2)

so: $\dfrac{S_{21}^z}{S_{12}^z} = \dfrac{S_{21}^x S_{21}^y}{S_{12}^x S_{12}^y}$.

(2)

$S_{11}^z$: the first path is $S_{11}^x$.

this does not touch the loop $S_{22}^x S_{11}^y$.

the second path is $S_{21}^x S_{11}^y S_{12}^x$

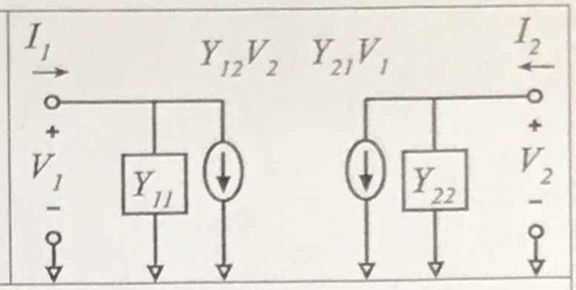
this <u>does</u> touch the only loop.

$$S_{11}^z = \frac{S_{11}^x(1 - S_{22}^x S_{11}^y) + S_{21}^x S_{11}^y S_{12}^x}{1 - S_{22}^x S_{11}^y}$$

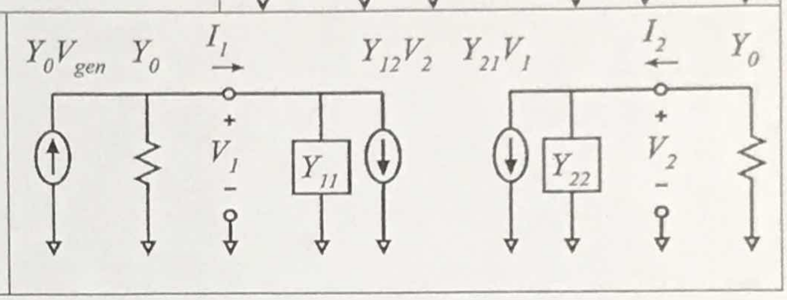
$$= S_{11}^x + \frac{S_{21}^x S_{11}^y S_{12}^x}{1 - S_{22}^x S_{11}^y}.$$

part b, 10 points (218A only)

We can represent a 2-port network having Y-parameters $Y_{ij}$ by the circuit to the right




Given that $S_{21} = 2(V_o / V_{gen})\big|_{Z_L = Z_{gen} = Z_o}$, we have $S_{21} = 2V_{out} / V_{gen}$ in the circuit to the right, where $Y_0 = 1/Z_0$




Prove that $S_{21} / S_{12} = Y_{21} / Y_{12}$. This involves nodal analysis of the above circuit.

$\Sigma I = 0$ @ $V_1$ $\quad V_1(Y_{11} + Y_0) + V_2(Y_{12}) = Y_0 V_{gen}$.

$\Sigma I = 0$ @ $V_2$ $\quad V_1(Y_{21}) + V_2(Y_{22} + Y_0) = 0$. (3)

In matrix form: $\begin{bmatrix} Y_{11} + Y_0 & Y_{12} \\ Y_{21} & Y_{22} + Y_0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_0 \\ 0 \end{bmatrix} V_{gen}$. (2)

so $S_{21} = 2V_0 / V_{gen} \big|_{Z_L = Z_0 = Z_{gen}} = 2 \cdot \dfrac{N_{21}}{D}$ where $N_{21} = \begin{vmatrix} Y_{11} + Y_0 & Y_0 \\ Y_{21} & 0 \end{vmatrix} = -Y_0 Y_{21}$

$D = \begin{vmatrix} Y_{11} + Y_0 & Y_{12} \\ Y_{21} & Y_{22} + Y_0 \end{vmatrix}$ (2)

(2) by symmetry,

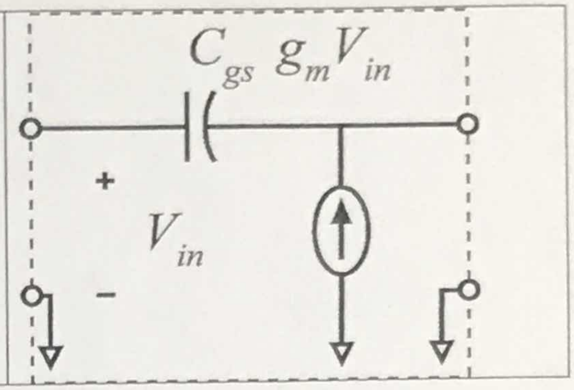
$S_{12} = \dfrac{2 \cdot N_{12}}{D}$; where $N_{12} = -Y_0 Y_{12}$

(1) so

$\dfrac{S_{21}}{S_{12}} = \dfrac{N_{21}}{N_{12}} = \dfrac{Y_{21}}{Y_{12}}$

part c, 5 points (BOTH 218A and 145A)

Given that $S_{21}/S_{12} = Y_{21}/Y_{12}$, for the circuit to the right, find $Y_{21}/Y_{12}$. After finding an exact answer, assume that $g_m >> \omega C_{gs}$ to find a simpler answer applicable at lower frequencies.




$Y_{21}/Y_{12}=$____________________

$Y_{21}/Y_{12} \cong$____________________

by inspection:

$$Y_{21} = -g_m - j\omega C_{gs} \quad 1.5$$

$$Y_{12} = -j\omega C_{gs} \quad 1.5$$

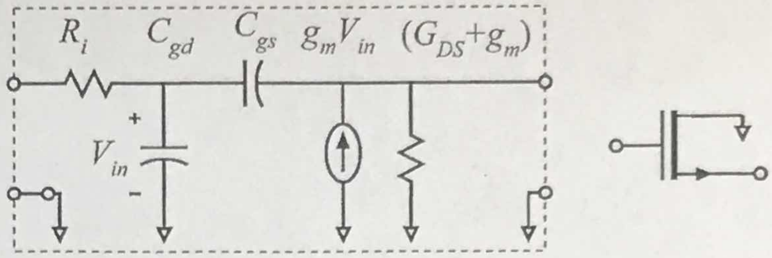
$$Y_{21}/Y_{12} = \frac{g_m + j\omega C_{gs}}{j\omega C_{gs}} = \frac{g_m}{j\omega C_{gs}} + 1 \quad 1$$

at low frequencies, this is approximately:

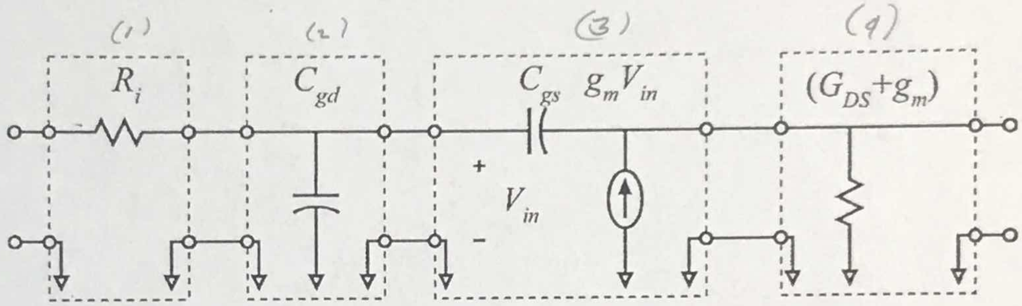
$$Y_{21}/Y_{12} \cong \frac{g_m}{j\omega C_{gs}} \quad 1$$

part d, 10 points

(A)




(B)




The network (A), which represents the hybrid-pi FET model in source-follower operation, can be represented as the cascaded network (B) below.
*If we assume* that the network is potentially unstable (it will be at lower frequencies), find an expression for the maximum stable gain. This derivation shows why source followers have difficulties with stability.

MSG= $g_m/\omega C_{gs}$ or $\sqrt{1+\left(\frac{g_m}{\omega C_{gs}}\right)^2}$

3 [ " networks (1), (2), (4) are symmetric and have $Y_{21}=Y_{12}$ (we can also invoke reciprocity for these) ]

So, from part a of the problem set:

$$3\ \left[\left.\frac{Y_{21}}{Y_{12}}\right|_{overall} = \left.\frac{Y_{21}}{Y_{12}}\right|_{part\ 3}\right] = \left[1 + \frac{g_m}{j\omega C_{gs}} \cong \frac{g_m}{j\omega C_{gs}}\right]\ 1$$

2 [ But $MSG = \left|\frac{S_{21}}{S_{12}}\right| = \left|\frac{Y_{21}}{Y_{12}}\right|$ if $k<1$ ]

$$= \begin{cases} \dfrac{g_m}{\omega C_{gs}} & \text{approximate} \\ \sqrt{1+\left(\dfrac{g_m}{\omega C_{gs}}\right)^2} & \text{exact} \end{cases}\ \ 1$$

## Comment

Here we encounter a common silly error

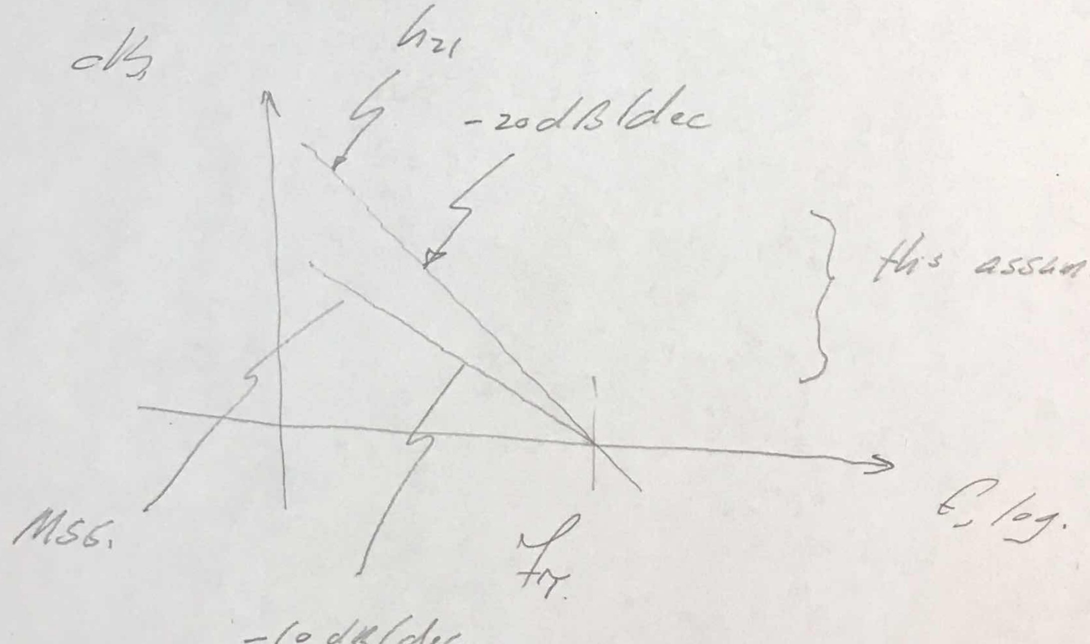
$h_{21}$ is current gain, so

$$dB(h_{21}) = 20 \cdot \log_{10}\left(\frac{g_m}{\omega C_{gs}}\right)$$

MSG is power gain, so

$$dB(MSG) = 10 \cdot \log_{10}\left(\frac{g_m}{\omega C_{gs}}\right)$$

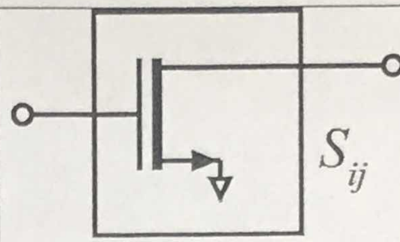
If $h_{21}$ is 20 dB, then MSG is only 10 dB!!




this assumes simple FET model.

**Problem 4, 25 points**

*gain definitions*

| | |
|---|---|
| At a signal frequency of 10 GHz, a two-port has $S_{11}=0.7071$, $S_{12}=0$, $S_{21}=5$ and $S_{22}=0.5$, as defined with a 50 Ohm impedance reference. |  |

part a, 5 points

The device is connected to a 50 Ohm generator with 1 microwatt available power, and is connected via a conjugate impedance-matching network to a 50 Ohm load. Find the power in the load.

$P_{Load} =$ 33 μW.

2 the output is matched
The input is not
This is the AVAILABLE Gain.

2 $G_a = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2} |S_{21}|^2 \cdot \frac{1}{1-||\Gamma_{out}||^2}$ where $\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s}$

but $Z_{gen} = 50\Omega = Z_0$ so $\Gamma_s = 0$

1/2 $G_a = |S_{21}|^2 \cdot \frac{1}{1-||\Gamma_{out}||^2}$ where $\Gamma_{out} = S_{22} + 0$

$= \frac{|S_{21}|^2}{1-||S_{22}||^2} = \frac{|5|^2}{1-||0.5||^2} = \frac{25}{1-1/4} = \frac{25}{3/4} = 25 \cdot \frac{4}{3}$

$= \frac{100}{3} = 33.333$

1/2 $P_{load} = 33.3 \cdot 1\mu W$
$= 33 \mu W.$

part b, 5 points

The device is directly connected to a 50 Ohm generator with 1 microwatt available power, and is directly connected to a 50 Ohm load. Find the RF power in the load.

$P_{Load}$ = 25 μW.

4

$\Gamma_s$ & $\Gamma_L$ are both zero.

Power gain = $|S_{21}|^2$

1

$P_{Load}$ = 1 μW · $|S_{21}|^2$ = 1 μW · 25

= 25 μW.

part c, 5 points

The device is connected via a conjugate impedance-matching network to a 50 Ohm generator with 1 microwatt available power, and is connected via a conjugate impedance-matching network to a 50 Ohm load. Find the power in the load. Find the source and load impedances presented to the transistor.

$P_{Load}$ = 66.6 μW. $Z_{source}$ 291Ω $Z_{Load}$ = 150Ω.

1 This is the MAG

1 Because $S_{12} = 0$, $MAG = \frac{1}{1-\|S_{11}\|^2} \|S_{21}\|^2 \frac{1}{1-\|S_{22}\|^2}$

$$MAG = \frac{1}{1-\|\frac{1}{\sqrt{2}}\|^2} \|5\|^2 \frac{1}{1-\|1/2\|^2}$$

1 $$= 2 \cdot 25 \cdot 4/3 = 100 \cdot 2/3 = 66.\overline{66}$$

$P_{Load} = 1\,\mu W \cdot 66.66 = 66.6\,\mu W.$

Because $S_{12} S_{21} = 0$, $S_{11} = \Gamma_{in}$ $S_{22} = \Gamma_{out}$

1 so $$Z_{source} = 50\Omega \frac{1+S_{11}}{1-S_{11}} = 50\Omega \cdot \frac{1+1/\sqrt{2}}{1-1/\sqrt{2}}$$

$$= 50\Omega \cdot 5.83 = 291\Omega.$$

1 and $$Z_{Load} = 50\Omega \frac{1+S_{22}}{1-S_{22}} = 50\Omega \frac{1+1/2}{1-1/2}$$

$$= 50\Omega \cdot 3 = 150\Omega$$

part d, 10 points

Using the impedance-matching networks of part C (they are NOT CHANGED for part d), the device is now connected to a 100 Ohm generator with 1 microwatt available power, and is directly connected to a 25 Ohm load. Find the RF power in the load.

$P_{Load}$ = ____________________

the amplifier matched to $50\Omega$ has, from part c:

1.5 $$|S_{21}| = \sqrt{66.66} \qquad S_{12} = 0.$$

$$S_{11} = 0, \quad S_{22} = 0.$$

this is a new 2-port

we are now calculating the transducer gain of this new 2-port with,

1 $$\Gamma_S = \frac{100\Omega/50\Omega - 1}{100\Omega/50\Omega + 1} = \frac{2-1}{2+1} = 1/3$$

and $$\Gamma_L = \frac{25\Omega/50\Omega - 1}{25\Omega/50\Omega + 1} = \frac{1/2 - 1}{1/2 + 1} = -1/3.$$

given that $S_{12} = 0$:

1 $$G_T = \frac{1 - \|\Gamma_S\|^2}{\|1 - \Gamma_S S_{11}\|^2} \|S_{21}\|^2 \frac{1 - \|\Gamma_L\|^2}{\|1 - \Gamma_L S_{22}\|^2}$$

but $S_{11} = S_{22} = 0$

1 $$G_T = [1 - \|\Gamma_S\|^2][\|S_{21}\|^2][1 - \|\Gamma_L\|^2]$$

$$= (1 - \tfrac{1}{9}) \cdot 66.66 \, (1 - 1/9)$$

$$= (\tfrac{8}{9})^2 \cdot 66.6 = \underline{52.7}$$



1/2 $$P_{Load} = 52.7 \cdot 1\mu W = 52.7\mu W.$$

**Problem 5, 15 points (145A), 25 points (218A)**
*Potentially unstable amplifier design*

potentially unstable

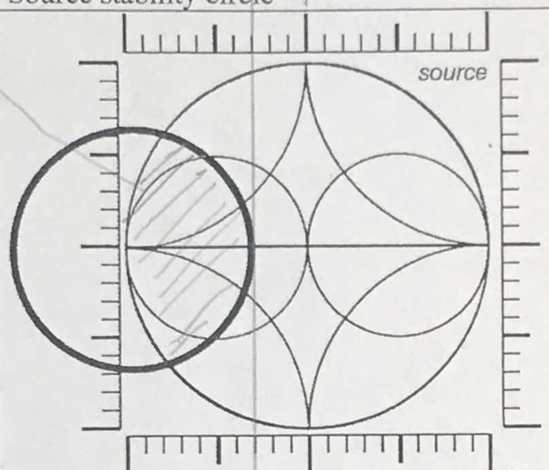
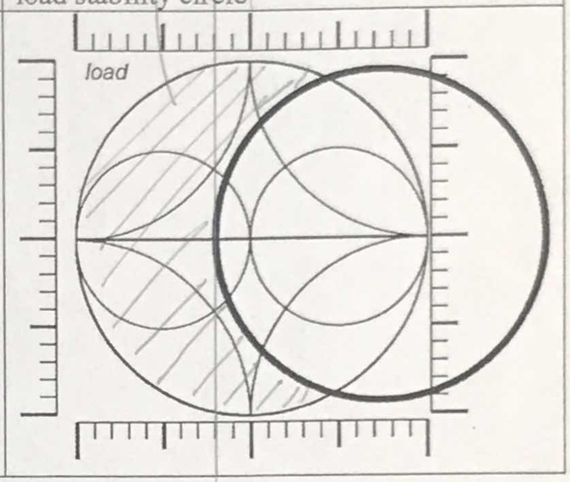
$\Gamma = -0.3$

potentially unstable

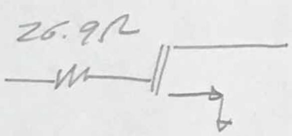
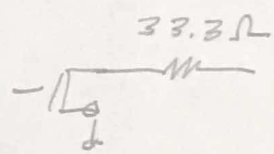
$\Gamma = -0.2$

part a, 7 points

At a design frequency of 10 GHz, a common-source FET has source and load stability circles as below

| Source stability circle | load stability circle |
|---|---|
|   |   |

Given that S11=0.5 and S22=0.9 at 10 GHz, draw two stabilization circuits in the boxes below, giving element values

| Solution 1 | Solution 2 |
|---|---|
|   |   |

1 because $\|S_{11}\|, \|S_{22}\| < 1$ the centers of the smith charts are stable.

2 unstable regions are shaded on diagrams.

2 on the input, we can force $\Gamma_S$ to be more positive than $-0.3$.

→ series resistor of $R = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\Omega \frac{1-0.3}{1+0.3} = 50\Omega \frac{0.7}{1.3}$

$= 26.9\Omega$.

2 on the output, we can force $\Gamma_L$ to be more positive than $-0.2$

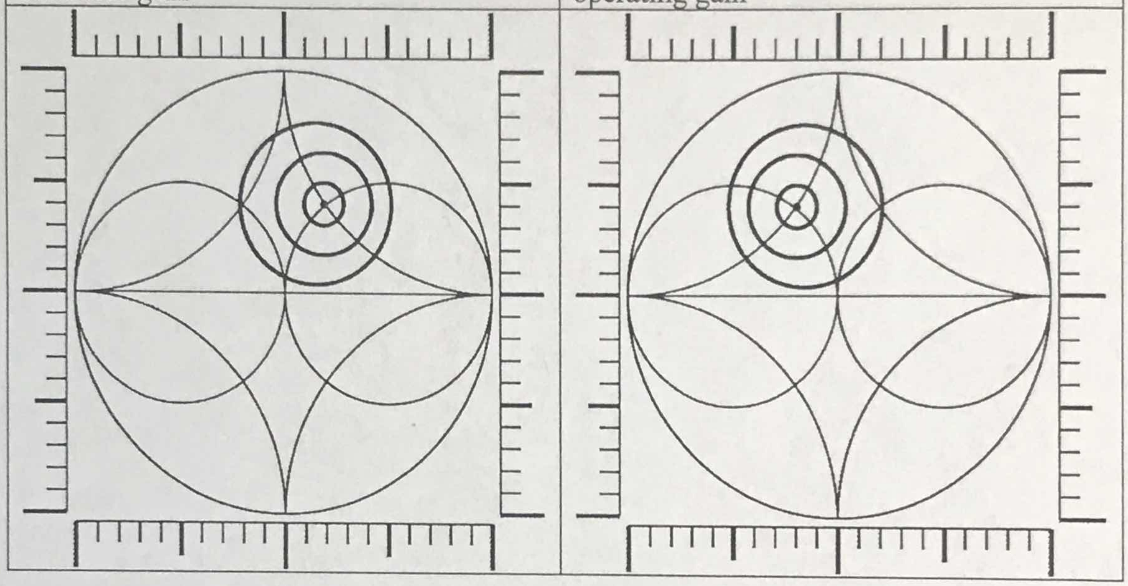
→ series resistor of $R = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50\Omega \frac{1-0.2}{1+0.2} = 50\Omega \frac{0.8}{1.2}$

$= 50\Omega \frac{8}{12} = 50\Omega \cdot \frac{2}{3} = 33.33\Omega$

part b, 5 points

A FET has available and operating gain circles as below at 1 GHz.

| available gain | operating gain |
|---|---|



Assuming a 50Ohm impedance normalization, what are the optimum generator and load impedances ?

$Z_{gen,opt}$ = ____________________ $Z_{l,opt}$ = ____________________

1 [ $G_a$ is a function of $\Gamma_s \rightarrow$ optimum source impedance

1 [ From $G_a$ circles above, $z_{s,opt} = 1 + j1$

1/2 [ $Z_{gen,opt} = 50\Omega + j\,50\Omega$.

1 [ $G_p$ is a function of $\Gamma_2 \rightarrow$ optimum load impedance.

[ From $G_p$ circles above:

1

$$y_{L,opt} = 1 - j1$$

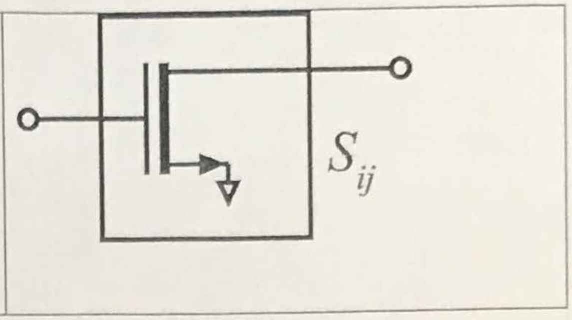
$$z_{L,opt} = \frac{1}{1-j1} \cdot \frac{1+j1}{1+j1} = \frac{1+j1}{2} = 1/2 + j\,1/2$$

1/2 [ $Z_{L,opt} = 25\Omega + j\,25\Omega$.

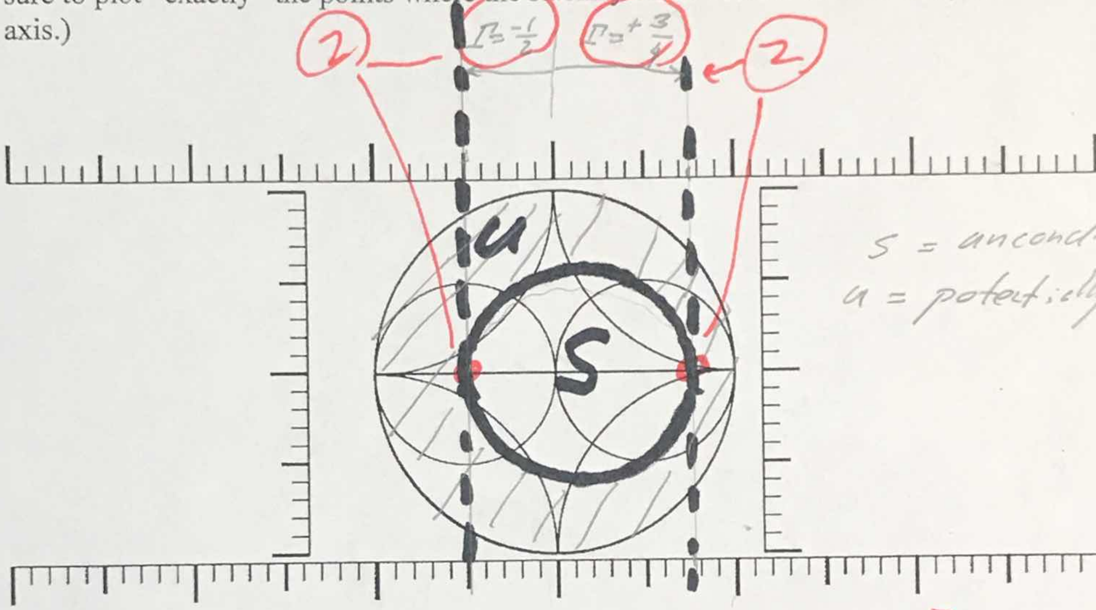
part c, 10 points (218A only)

At 10GHz, a transistor has
S11=0, S12=0.1,
S21=20, S22=-0.5.

These S-parameters are normalized to a 50 Ohm reference impedance




Draw the *source* stability circle on the graph below:
(to do this perfectly, you would need a compass: you can sketch most of the curve, but be sure to plot *exactly* the points where the stability circle crosses the real axis, i.e. the x-axis.)




s = unconditionally stable
u = potentially unstable.

$$2\left[\, e^{j\theta} = \Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11}\Gamma_s} = -0.5 + 2\Gamma_s \,\right]$$

$$2\Gamma_s = 0.5 + e^{j\theta} \longrightarrow \boxed{\Gamma_s = 0.25 + \frac{e^{j\theta}}{2}}\ 2$$

2 [ The real-axis intercepts are when $e^{j\theta} = \pm 1$, so $\Gamma_s = \begin{cases} 0.25 + 0.5 = 0.75 \\ 0.25 - 0.5 = -0.5 \end{cases}$ ]