ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. February 11, 2009

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

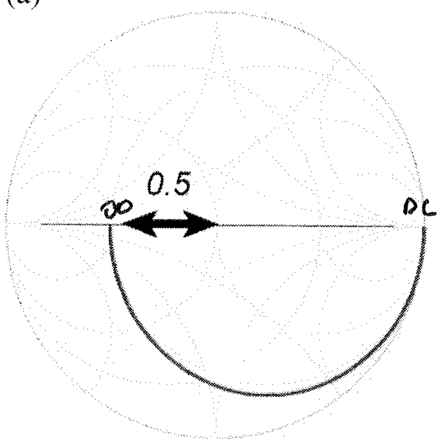
Problem	Points Received	Points Possible
1		20
2		25
3a		10
3b		10
3c		10
4a		5
4b		20
total		100

Name: Answer key

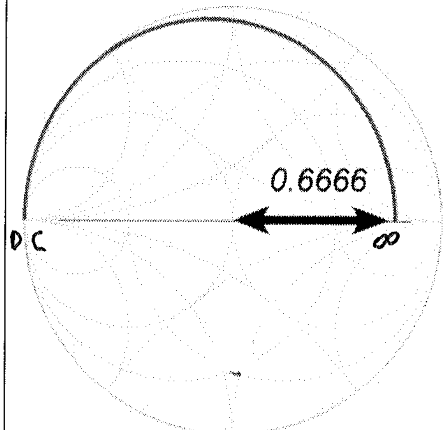
Problem 1, 20 points

The Smith Chart and Frequency-Dependent Impedances.

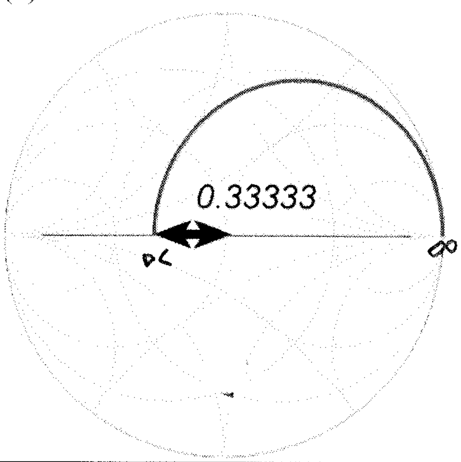
(a)



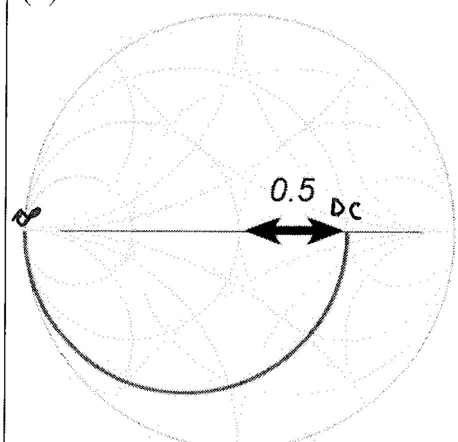
(b)



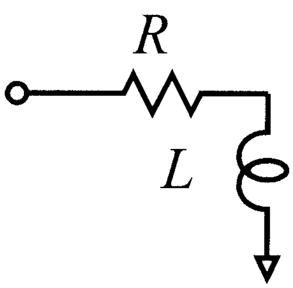
(c)



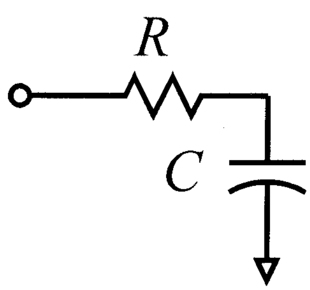
(d)



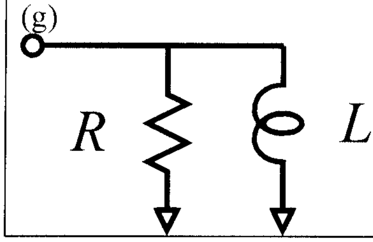
(e)



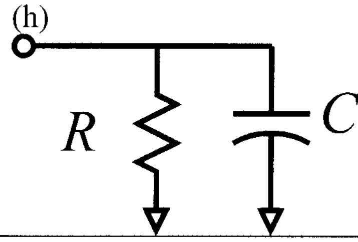
(f)



(g)



(h)



Match each Smith Chart with each circuit, and give all resistor values:

Circuit (e):	Smith chart= C	R= 25 Ω
Circuit (f):	Smith chart= A	R= 16.67 Ω
Circuit (g):	Smith chart= B	R= 250 Ω
Circuit (h):	Smith chart= D	R= 150 Ω

$$Z = Z_0 \cdot \frac{1+\Gamma}{1-\Gamma}$$

E/C

$$\Gamma = \tfrac{1}{3}\angle 180^\circ = -\tfrac{1}{3}$$

$$R = 50 \cdot \frac{1-1/3}{1+1/3} = 25$$

$$Z = R + j\omega L$$

F/A

$$\Gamma = -0.5$$

$$R = 50 \cdot \frac{1-0.5}{1+0.5} = 16.67$$

$$Z = R - j/\omega C$$

G/B

$$\Gamma = +\tfrac{2}{3}$$

$$R = 50 \cdot \frac{1+2/3}{1-2/3} = 250$$

$$Y = \frac{1}{R} - j\frac{1}{\omega L}$$

H/D

$$\Gamma = +0.5$$

$$R = 50\,\frac{1+1/2}{1-1/2} = 150$$

$$Y = \frac{1}{R} + j\omega C$$

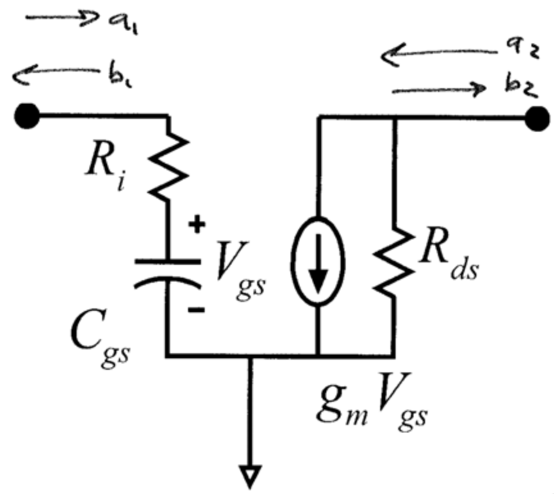
Problem 2, 25 points

Transistor models and S-parameters.

To the left is a simplified model of a MOSFET.

With element values as defined, first give algebraic expressions of the four S-parameters as a function of frequency.

Then using Cgs=31.8 fF, Ri=200 Ohms, gm= 20 mS, Rds=250 Ohms, give numerical values at f=50 GHz.



$$\underline{S_{11}} = \frac{b_1}{a_1} = \Gamma_{in} = \frac{Z_{in}-Z_0}{Z_{in}+Z_0} = \frac{R_i + \frac{1}{j\omega C_{gs}} - Z_0}{R_i + \frac{1}{j\omega C_{gs}} + Z_0} = \frac{j\omega C_{gs}(R_i - Z_0)+1}{j\omega C_{gs}(R_i+Z_0)+1} = \frac{j2\pi\, 50\times10^9 \cdot 31.8\times10^{-15}(150)+1}{j2\pi\, 50\times10^9 \cdot 31.8\times10^{-15}(250)+1}$$

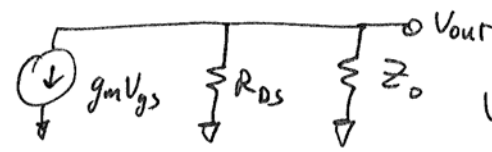
$$\boxed{S_{11} = 0.655 - j \times 0.130}$$

$$\underline{S_{22}} = \Gamma_{out} = \frac{R_{DS}-Z_0}{R_{DS}+Z_0} = \frac{200}{300} = \boxed{\frac{2}{3}}$$

$\underline{S_{12}} = \boxed{0}$ By inspection we see that no signal at Port 2 will couple to Port 1 since $C_{GD}=0$

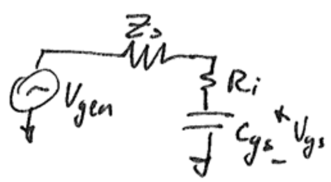
$$\underline{S_{21}} = \frac{2V_{out}}{V_{gen}}$$

Find Vout:



$$V_{out} = -g_m V_{gs} (R_{DS} // Z_0)$$

Find V_{gs} in terms of V_{gen}:



$$V_{gs} = \frac{1/sC_{gs}}{1/sC_{gs} + R_i + Z_0} V_{gen} = \frac{V_{gen}}{1+sC_{gs}(R_i+Z_0)}$$

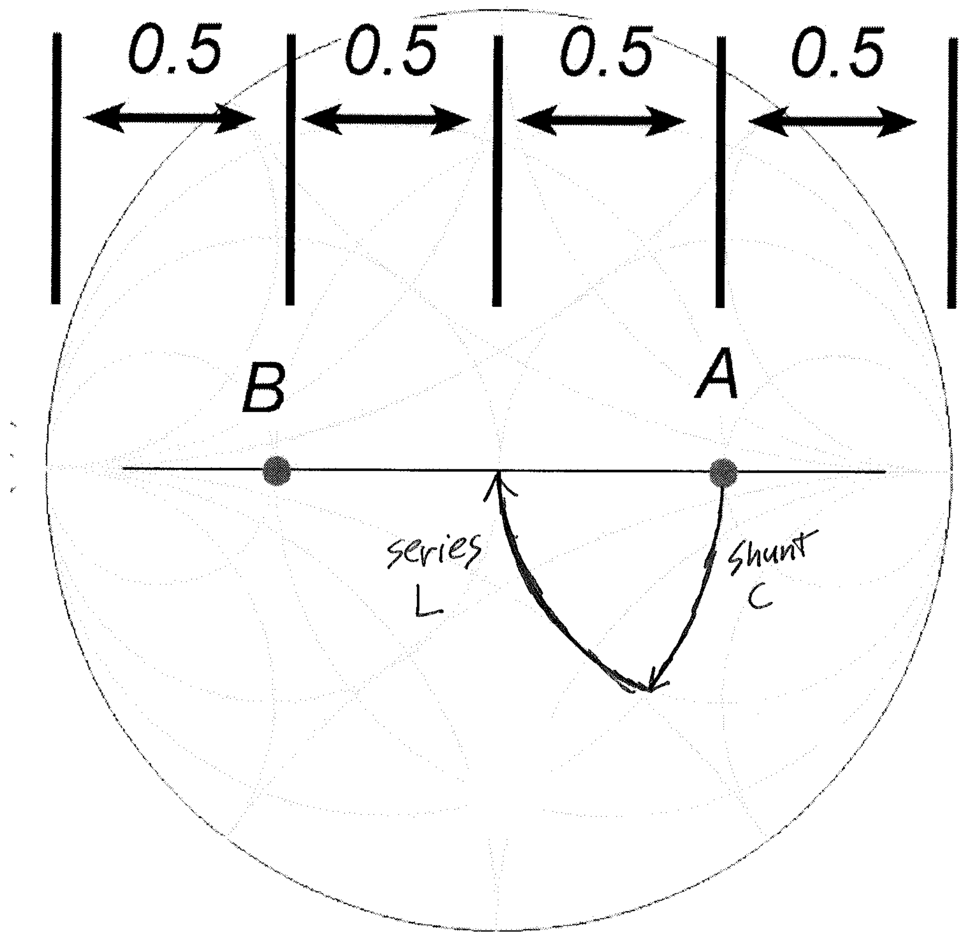
Thus: $$V_{out} = -g_m \cdot \frac{V_{gen}}{1+sC_{gs}(R_i+Z_0)} \cdot (R_{DS} // Z_0)$$

$$S_{21} = \frac{2V_{out}}{V_{gen}} = \frac{-2g_m(R_{DS}//Z_0)}{1+sC_{gs}(R_i+Z_0)} = \frac{-2\times.02\left(\frac{1}{250}+\frac{1}{50}\right)^{-1}}{1+j\,2\pi\cdot 50\times10^9(250)\times 31.8\times10^{-15}}$$

$$\boxed{S_{21} = -0.230 + j \times 0.575}$$

Problem 3, 30 points
Elementary impedance matching network design.

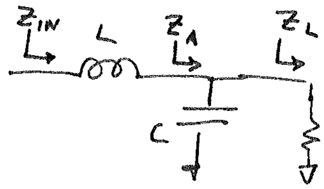
Use the impedance-admittance chart below. Assume a 50 Ohm impedance normalization. To keep the problem simple, if certain Admittance and Impedance coordinate lines *almost* cross, use the approximation that they *exactly* cross.



Part (a), 10 points.

Design a series-L, shunt-C impedance matching network to match impedance "A" to 50 Ohms at 20 GHz. Give the circuit diagram and the element values.

$$Z_L = 50 \cdot \frac{1+0.5}{1-0.5} = 150\Omega$$



$R_L = 150\Omega$, 3Ω normalized

- Use shunt C to bring load to unit resistance circle, i.e. make $Re(Z_A) = 1$ (normalized)

$$Z_A = (Y_A)^{-1} = \left(\frac{1}{R_L} + j\omega C\right)^{-1} = \frac{R_L}{1 + j\omega C R_L} = \frac{R_L(1 - j\omega C R_L)}{1 + (\omega C R_L)^2}$$

$$Re(Z_a) = \frac{R_L}{1 + (\omega C R_L)^2} = 1$$

$$\Rightarrow C\,(\text{norm.}) = \frac{\sqrt{r_L - 1}}{\omega r_L} = \cancel{25}\ \frac{\sqrt{2}}{2\pi \times 20 \times 10^9 \times 3} = 3.75 \times 10^{-12}$$

$$C = \underbrace{\frac{1}{50}}_{Y_0} \times 3.75 \times 10^{-12} = \boxed{75\,fF = C}$$

- Use series L to cancel remaining complex component of Z_A

$$Z_{in} = 50\Omega = j\omega L + Z_A = 50 + j\Big(\underbrace{\omega L - \frac{\omega C R_L^2}{1 + (\omega C R_L)^2}}_{\text{set to } 0}\Big)$$

$$L = \frac{C R_L^2}{1 + (\omega C R_L)^2} = \frac{75\,fF \times (150\Omega)^2}{1 + (2\pi\, 20 \times 10^9 \times 75\,fF \times 150\Omega)^2}$$

$$\boxed{L = 563\,pH}$$

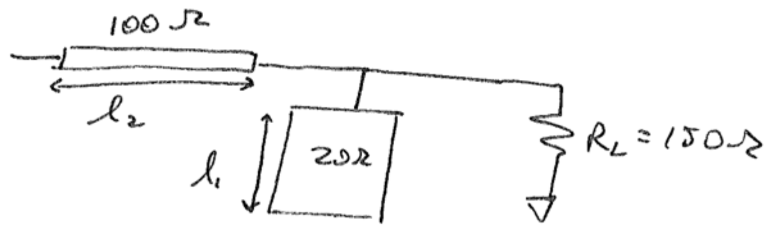
Part (b), 10 points

Ideal inductors are not available to you. Assuming high-impedance lines of 100 Ohm impedance and low-impedance lines of 20 Ohms impedance, give an approximate distributed-element design to this matching network. The lines' effective dielectric constant is 2.0

$L = \tau Z_0$

$C = \tau / Z_0$

$V_p = c/\sqrt{\varepsilon_R} = 2.12 \times 10^8 \, m/s$



- High-Z lines are almost like inductors
- Low-Z lines are almost like capacitors

$\tau_1 = Z_0 C = 20\Omega \times 75\,fF = 1.5\,ps$

$l_1 = \tau_1 V_p$

$\boxed{l_1 = 318\,\mu m}$

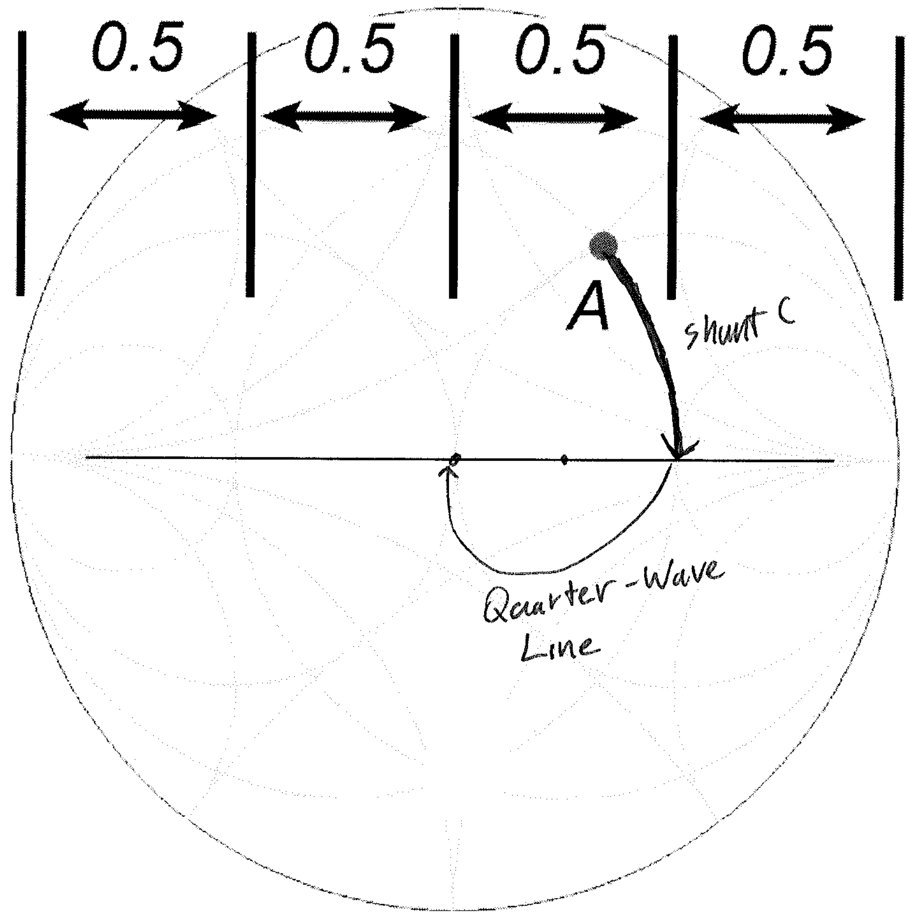
$\tau_2 = L/Z_0 = 563\,pH / 100\Omega = 5.63\,ps$

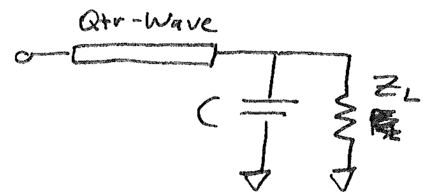
$l_2 = \tau_2 V_p$

$\boxed{l_2 = 1190\,\mu m}$

Part (c), 10 points

Load impedance "A" is to be matched to Zo=50 Ohms at 20 GHz (the chart below uses 50 OHms normalization) using a shunt capacitance and a series quarter-wave line. Find the value of the shunt capacitance and the required line impedance.





- From the symmetry with part (A) it is clear we will use the same capacitor $\boxed{C = 75\,fF}$

same as load resistor in part A : $R = 50 \cdot \frac{1+0.5}{1-0.5}$

$R = 150\,\Omega$

- To match the resultant $150\,\Omega$ impedance to $50\,\Omega$ we use a quarter-wave line with impedance:

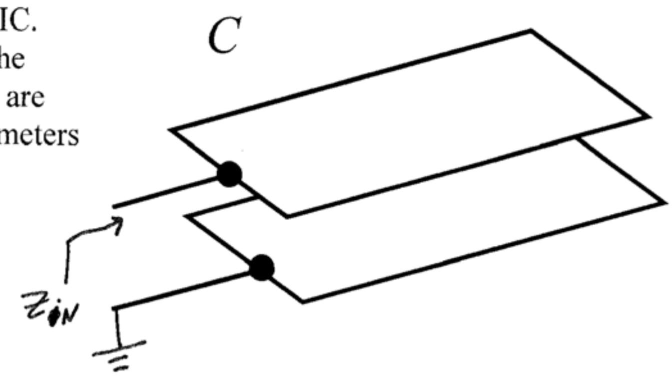
$$Z_0 = \sqrt{Z_L \times Z_{in}} = \sqrt{150 \times 50} = 86.6\,\Omega$$

$$\boxed{Z_0 = 86.6\,\Omega}$$

Problem 4, 25 points

Lumped vs distributed elements

We are designing a bypass "capacitor" for use in a 60 GHz wireless transceiver IC. The plate separation is 100 nm, and the dielectric constant is 4.0. The plates are 500 micrometers long and 100 micrometers wide.



Part (a), 5 points

What is the low-frequency capacitance C ?

$$C = \frac{\varepsilon A}{d} = \frac{4 \times 8.854 \times 10^{-12}\ F/m \times 500\mu m \times 100\mu m}{100nm} = \boxed{17.7pF}$$

Part (b) 20 points

Using transmission-line relationships, what is the element's impedance at 37.5 GHz and at 75 GHz ?

$V_p = c/\sqrt{\varepsilon_R} = 1.5E8\ m/s$

37.5 GHz: $\lambda_{37.5\,GHz} = V_p/f = \frac{1.5E8\ m/s}{37.5E9\ /s} = 4mm$

@ 37.5 GHz $L = \lambda/8$ so $Z_{in} = -jZ_o$ *

Since this is a low-Z line, $Z_o = \tau/C$

$\tau = l/v_p = 500\mu m / 1.5E8\ m/s = 3.33ps$

So $Z_o = 0.188\ \Omega$, $Z_{in} = -j \times 0.188$

or $Z_o \approx \frac{\eta}{\sqrt{\varepsilon_r}} \cdot \frac{H}{W}$

$Z_o = 0.188$

75 GHz: Now $L = \lambda/4$, so the open circuit ends appear to be short circuit

$Z_{in} = 0$

* $Z_{in} = -jZ_o$ because $\Gamma_{in} = \Gamma_L e^{-2j\beta l} = 1 \cdot e^{-2j\frac{2\pi}{\lambda}\frac{\lambda}{8}} = e^{-j\pi/2} = -j$

$Z_{in} = Z_o \cdot \frac{1+\Gamma_{in}}{1-\Gamma_{in}} = Z_o \cdot \frac{1-j}{1+j} = -jZ_o$