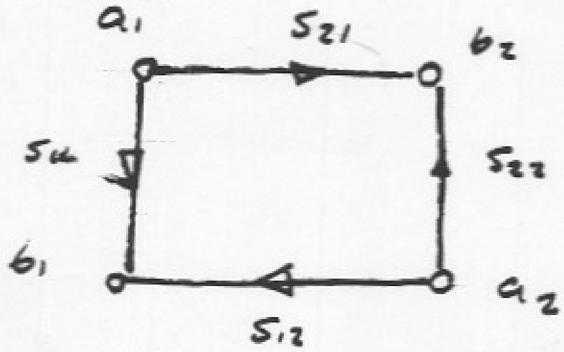
Signal Flow Graphs.

Mason (control system theory)

$$b_1 = S_{11}\, a_1 + S_{12}\, a_2$$
$$b_2 = S_{21}\, a_1 + S_{22}\, a_2$$

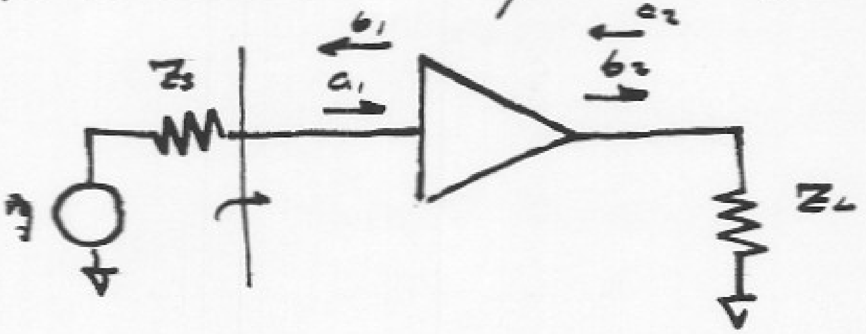
represent as below:



variables denoted as nodes $\overset{\circ}{\ } a_1$

value of variable = sum of entering branches
= sum of values of connecting nodes times weight of branches.

Representation of generator, load

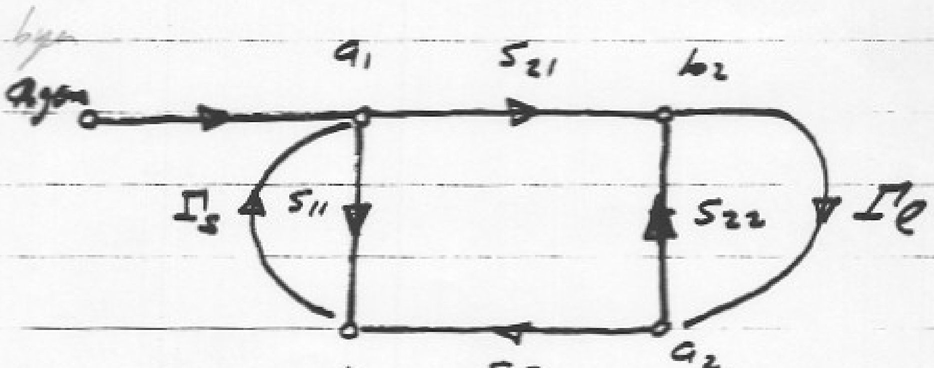


$$v^+ = V_g T_s + \Gamma_s v^-$$

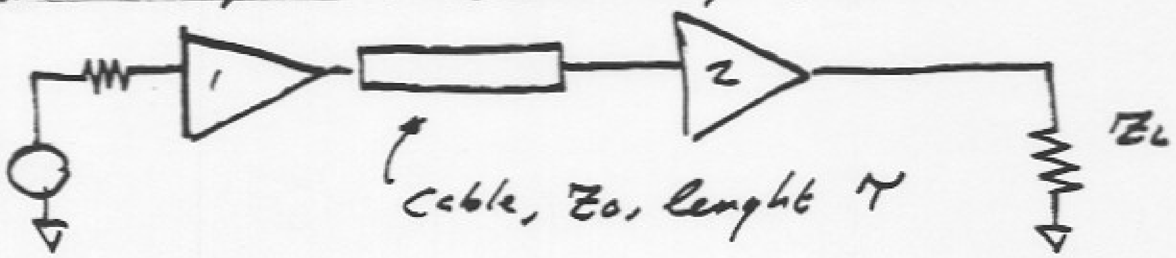
$$v^+/\sqrt{Z_0} = V_g T_s/\sqrt{Z_0} + \Gamma_s \frac{V_1}{\sqrt{Z_0}}$$

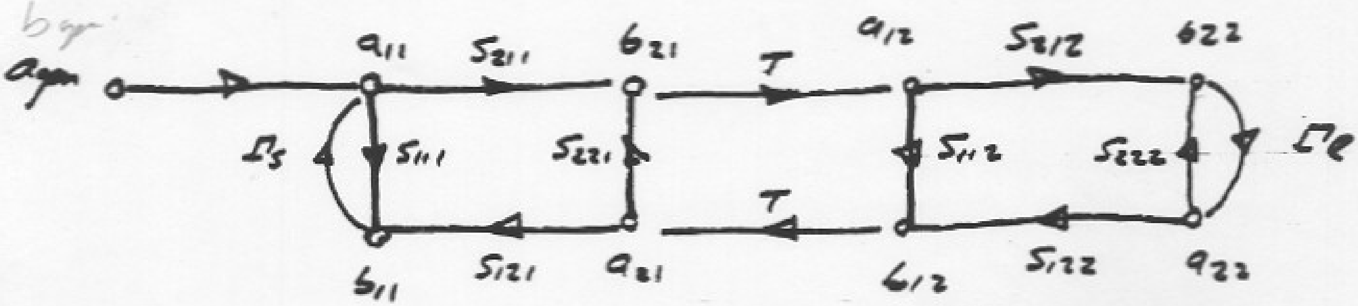
$$a_1 = \overset{b_{gen}}{a_{gen}} + \Gamma_s b_1$$

$$a_2 = \Gamma_\ell b_2$$



Second example: Cascaded amplifiers





$$T = e^{-j\omega\tau}$$

The above ~~relates~~ represents the many equations of the system.

Signal Flow graphs most heavily used in control system theory, where they

1) Provide an organized way of presenting a set of mathematical relationships.

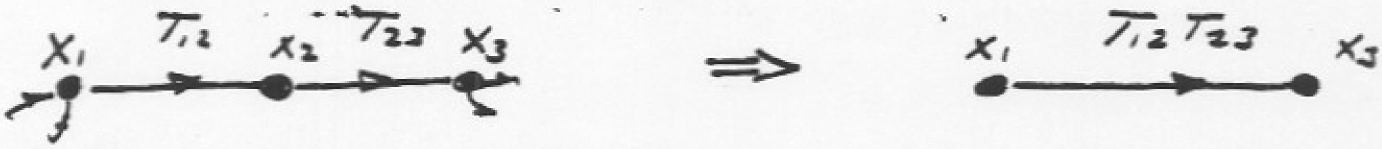
2) Lend some visual intuition to aid in solving otherwise "dry" simultaneous equations. (Kind of like Macintosh vs P.C.)

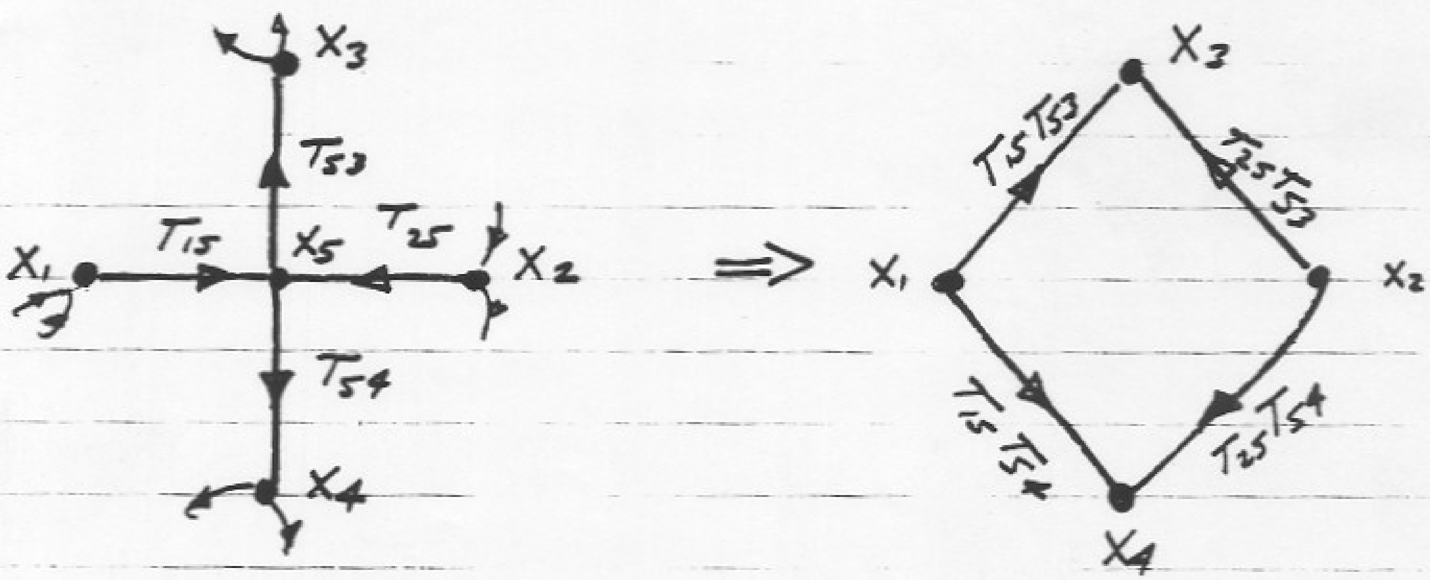
3) Provide efficient solution through "Masons' gain rules"

Ref: S.J. Mason "Feedback theory - Some Properties of Signal Flow Graphs" Proc. IRE, 41, p. 1141, Sept 1953.

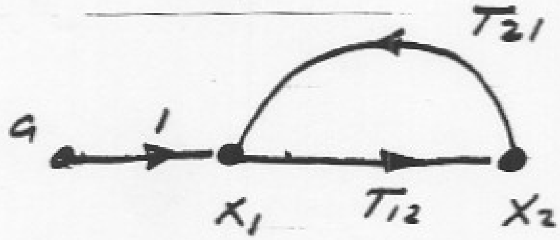
Ref Any text on general control system theory.

Important flow-graph Manipulations



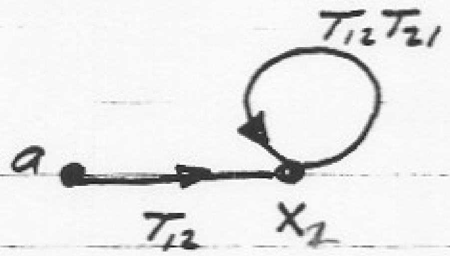


Reducing a feedback loop



$$X_2 = T_{12} X_1$$
$$X_1 = a + T_{21} X_2$$

$$\Rightarrow \quad X_2 = T_{12}\, a + T_{12} T_{21} X_2$$



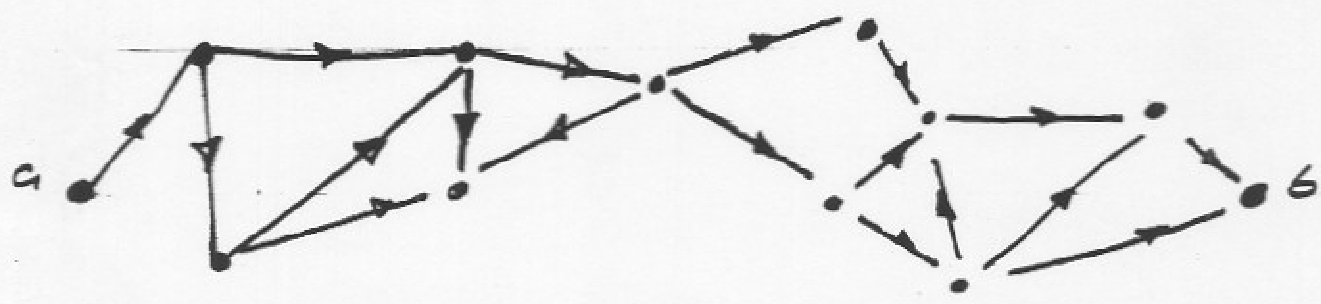
⇐ simpler representation

$$\Rightarrow \quad X_2 = \frac{T_{12}}{1 - T_{12} T_{21}}\, a$$

$$a \xrightarrow{\;\frac{T_{12}}{1 - T_{12} T_{21}}\;} X_2$$

⇐ simpler still.

Mason's Gain Rule



- define $T = b/a$ "transmission"

- define a path P_i as any route following the direction of the arrows taking you from input to outpu without going through any node twice.

- define a loop  as the product (ABC) of the transmission coefficients around any loop.

-

$$T = \frac{b}{a} = \frac{P_1\left[1 - \Sigma' L(1)^{(1)} + \Sigma' L(2)^{(1)} - \ldots\right] + P_2\left[1 - \Sigma' L(1)^{(2)} + \ldots\right]}{1 - \Sigma' L(1) + \Sigma' L(2) - \Sigma' L(3)}$$

here $\Sigma' L(1)$ means sum of all loops.

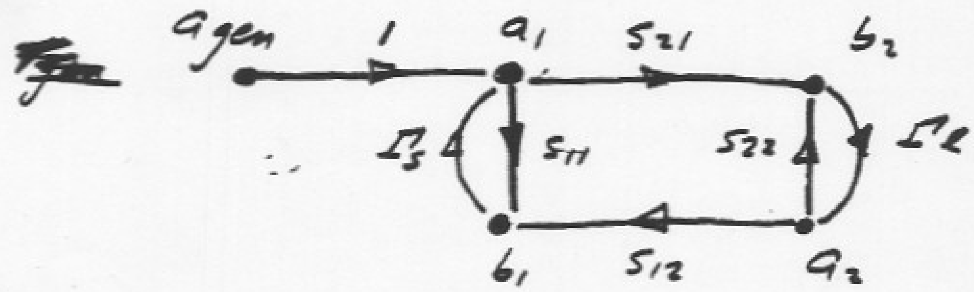
$\Sigma' L(1)^{(1)}$ means sum of all loops which don't touch path P_1

$\Sigma' L(2)$ = sum of all second-order loops

~~[illegible]~~ second-order loops = product of <u>any</u> <u>two</u> nontouching loops.

similarly $L(3)$, $L(2)^{(1)}$, etc.

Back to simple amplifier



find $\frac{b_2}{a_{gen}} = T$

no second path

$$T = \frac{S_{21}\left[1 - \cancel{\sum L(1)^{(1)}} + \cancel{\sum L(2)^{(1)}} - \dots\right]}{1 - \underbrace{\Gamma_s S_{11} - \Gamma_\ell S_{22} - S_{21}\Gamma_\ell S_{12}\Gamma_s}_{\sum L(1)} + (\Gamma_s S_{11})(\Gamma_\ell S_{22})}$$

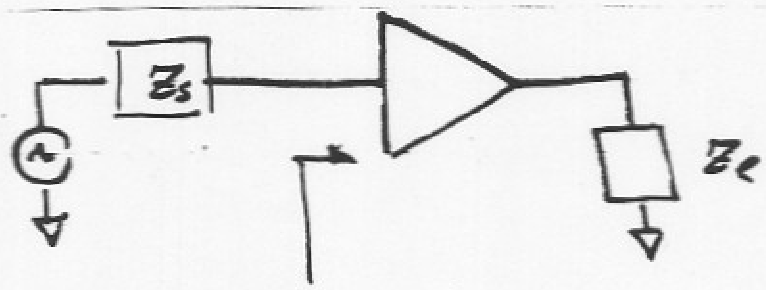
$$= \frac{S_{21}}{1 - \Gamma_s S_{11} - \Gamma_\ell S_{22} + (\Gamma_s S_{11})(\Gamma_\ell S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L}$$

$$\cancel{= \frac{S_{21}}{(1 - \Gamma_s S_{11})(1 - \Gamma_\ell S_{22})}}$$

Easy, wasn't it? — well maybe: as you can see, I initially missed a loop.

(tricks from the days before Mathematica)

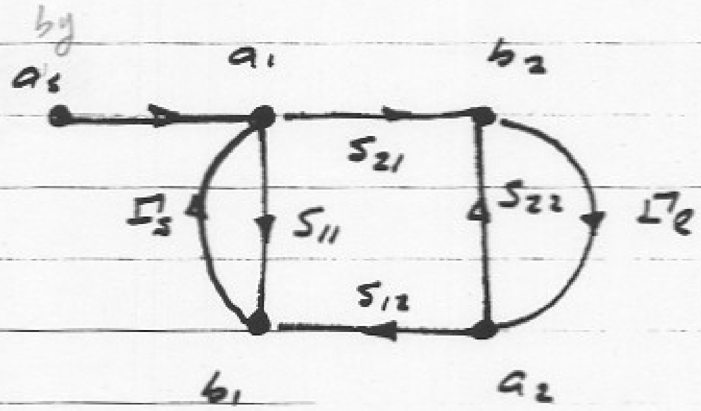
Input Reflection coefficient:



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \; ; \; Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}}$$

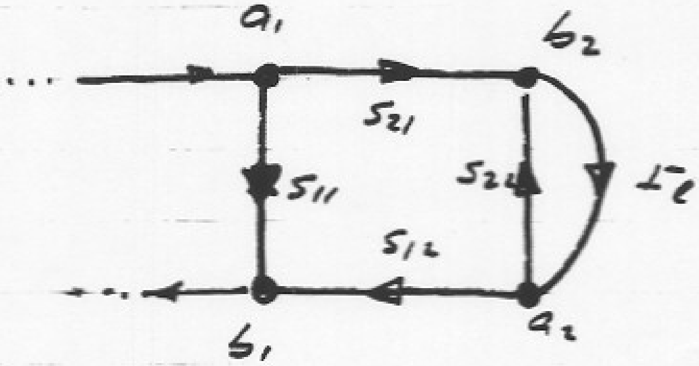
$$\left.\begin{array}{l} \Gamma_{in} = S_{11} \\ Z_{in} = Z_0 \dfrac{1+S_{11}}{1-S_{11}} \end{array}\right\} \underline{\underline{\text{if}}} \; Z_L = Z_0, \text{ but not in general}$$

(unless $S_{12} = 0$).



Block diagram

looking at the relationship between b_1 & a_1:



$$\frac{b_1}{a_1} = \Gamma_{in} = \frac{S_{11}\left[1 - S_{22}\Gamma_L\right] + S_{21}\Gamma_\ell S_{12}}{1 - S_{22}\Gamma_L}$$

$$\boxed{\Gamma_{in} = S_{11} + \Gamma_\ell \cdot \frac{S_{12}S_{21}}{1 - S_{22}\Gamma_\ell}}$$

Input $\left\{\begin{array}{l}\text{reflection coefficient}\\ \text{impedance}\end{array}\right\}$ depends somewhat on

load $\left\{\begin{array}{l}\text{reflection coefficient}\\ \text{impedance}\end{array}\right\}$ unless $S_{12}S_{21} = 0$

Similarly:

$$\boxed{\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{12}S_{21}}{1 - S_{11}\Gamma_S}}$$

If $S_{12}S_{21} = 0$ then either $S_{12} = 0$ or $S_{21} = 0$.

In other words the amplifier cant' pass signals both ways (input → output, output → input).

Such an amplifier is unilateral.

If an amplifier is unilateral then $\Gamma_{in} = S_{11}$ and $\Gamma_{out} = S_{22}$. Matching an input & output is easy.

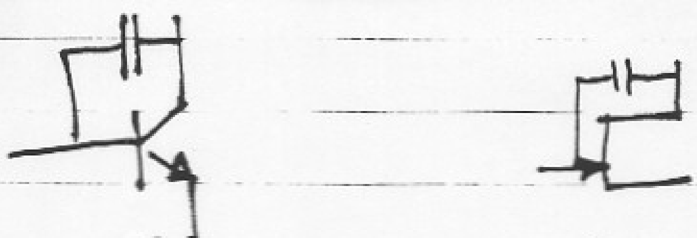
If the amplifier is bilateral $(S_{12}S_{21} \neq 0)$ (passes signals both ways) then

Input $\left\{\begin{matrix}\text{impedance} \\ \text{reflection coeff}\end{matrix}\right\}$ depends on load $\left\{\begin{matrix}\text{impedance} \\ \text{reflection coeff.}\end{matrix}\right\}$, and vice-versa.

This is a problem: design an output matching network, and you have changed the load impedance on the device, hence the input impedance also changes, disturbing the input match.

Process of matching becomes iterative in the presence of nonzero $S_{12} \cdot S_{21}$. In some cases, it is not possible to match on both input & output.

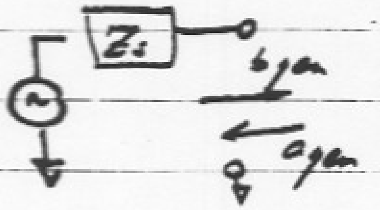
S_{12} arises mostly from C_{gd} C_{bc}



Rf in the feedback amplifier also introduces small S_{12}. Be aware of this in adding matching to lab # 3.

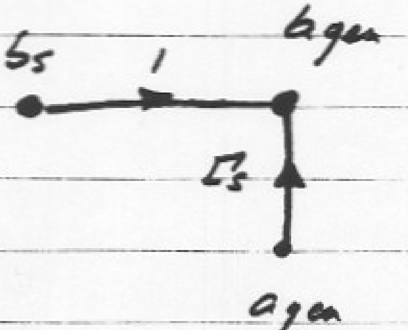
Available source power from a different viewpoint

$$P_{avg} = \|V_{open\ circuit}\|^2 / 4\,Re\{Z_s\} \qquad R.m.s.$$



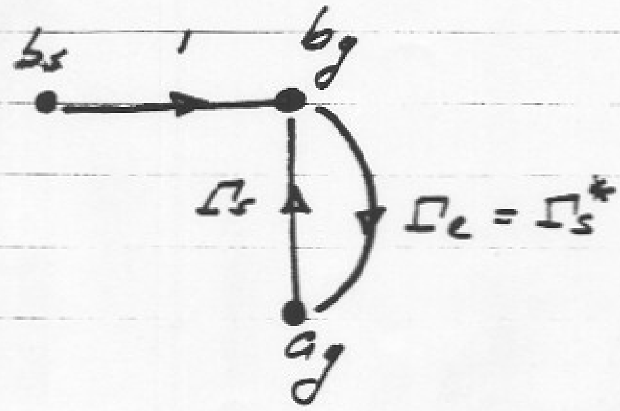
$$b_{gen} = \frac{T_s}{\sqrt{Z_0}} V_g + \Gamma_s a_{gen}$$

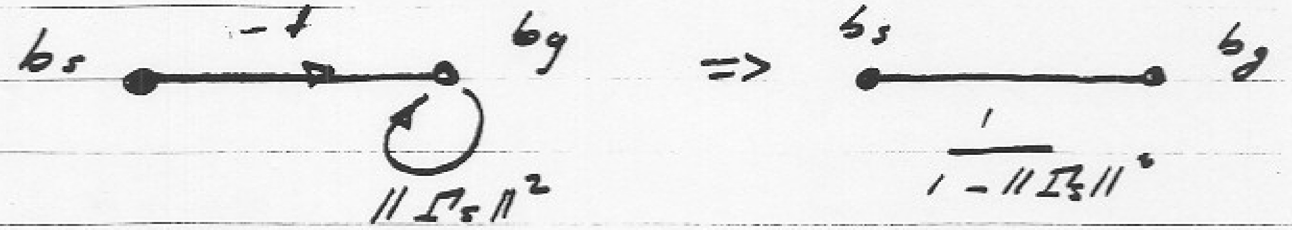
$$b_{gen} = b_s + \Gamma_s a_{gen}$$



b_s is the wave amplitude launched into $Z_L = Z_0$

Now connect conjugate-matched load Z_s^* ($\Rightarrow \Gamma_s^*$)





reverse power $= |b_s|^2 \dfrac{\|\Gamma_e\|^2}{\left[1 - \|\Gamma_s\|^2\right]^2} = \dfrac{\|\Gamma_s\|^2 \, |b_s|^2}{\left[1 - \|\Gamma_s\|^2\right]^2}$

forward power $= \dfrac{|b_s|^2}{\left[1 - \|\Gamma_s\|^2\right]^2}$

Load power = available power $= \dfrac{|b_s|^2}{1 - \|\Gamma_s\|^2}$

$$P_{avail} = \frac{|b_s|^2}{1 - |\Gamma_s|^2}$$ ← Power launched into Z_0