ECE 202A   11/10/89

stability

Can we have an output without an input?

stability theory: many (equivalent) versions

Control system theory → ~~fast~~ Loop transmission, phase margin, poles & zeroes

Network theory: do nodal analysis, find poles in transfer function, are there any in the right half of the s-plane?

Impedance point of view: is an input impedance negative (its real part)?

s-parameter point of view: Is a reflection coefficient > 1?

here

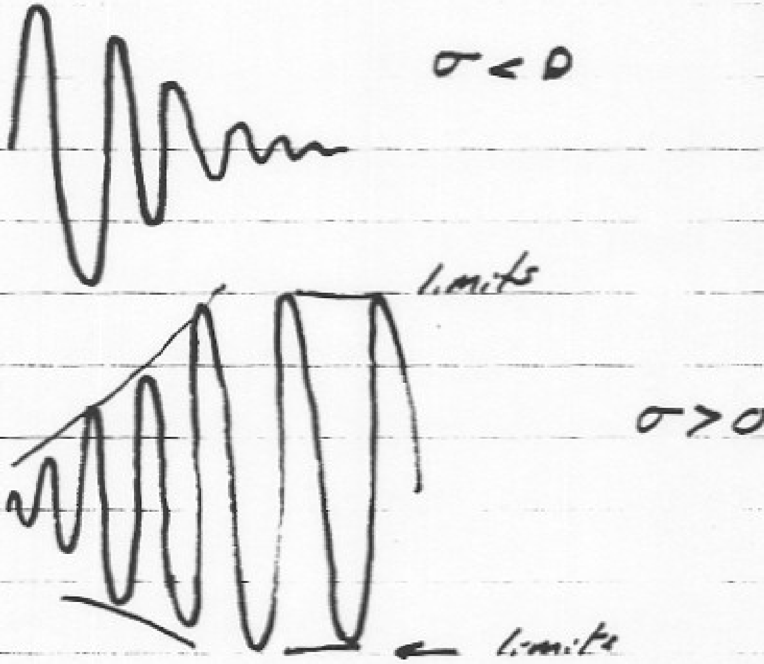
<u>step back for a minute:</u>

Electrical circuits (described by small-signal approx) are described by linear differential equations.

The natural response is always of the form

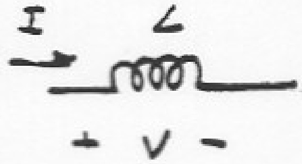
$$k_1 e^{s_1 t} + k_2 e^{s_2 t} + k_3 e^{s_3 t} + \dots$$

s complex $= \sigma + j\omega$



$e^{s_1 t}$ is an eigenfunction and $s_1 = \sigma_1 + j\omega_1$ is an eigenvalue. If there are any with $\sigma \geq 0$ the system is unstable.

Impedances:



$V = V_0 e^{st}$

$I = I_0 e^{st}$

$$V = L\frac{dI}{dt} = sL\,I$$

$$Z = sL$$

similarly for a capacitor $\quad Z = 1/sC$

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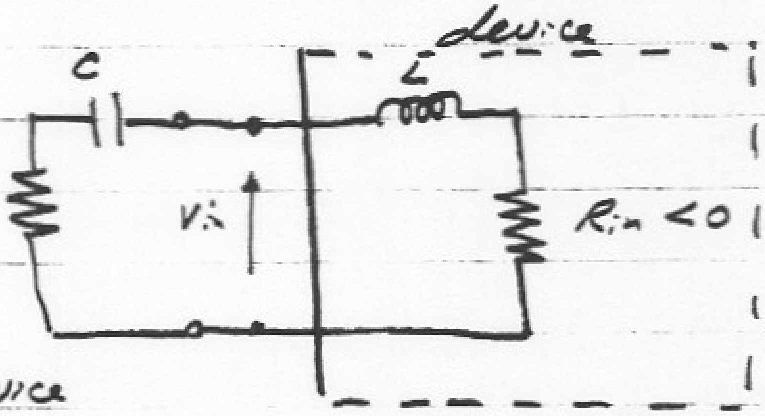
suppose a device has a negative real part (resistive part) to its input impedance at some frequency $(j\omega)$



Impedance connected to device

Nodal analysis:

$$\frac{V_{in}}{R_{in} + sL} + \frac{V_{in}}{R_s + 1/sC} = 0$$

$\boxed{V_{in} = 0}$ (stable) or

$$\frac{1}{R_{in} + sL} + \frac{1}{R_s + 1/sC} = 0$$

$$R_s + 1/sC + R_{in} + sL = 0$$

$$s^2 LC + s(R_{in} + R_s)C + 1 = 0$$

$$s = -\left(\frac{R_{in} + R_s}{2L}\right) \pm \sqrt{\left(\frac{R_{in} + R_s}{2L}\right)^2 - \frac{1}{LC}}$$

If $R_{in} + R_s$ is positive, real part of s will be negative $\Rightarrow$ stable

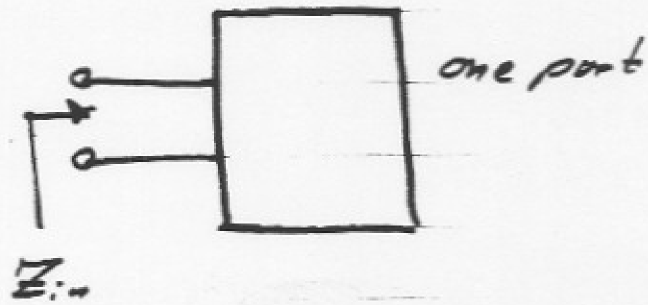
If $R_{in} + R_s$ is negative, real part of s will be positive $\Rightarrow$ unstable.

So in this simple example, the input impedance with the negative resistive component <u>can</u> result in instability <u>if</u> ~~R_s~~ $R_S + R_{in} < 0$

The system is stable under only certain conditions, it is <u>conditionally stable</u>

If R_{in} had been ≥ 0, No value of R_S (other than negative values) would result in instability. The system is then <u>unconditionally stable</u>

<u>So from the impedance point of view:</u>

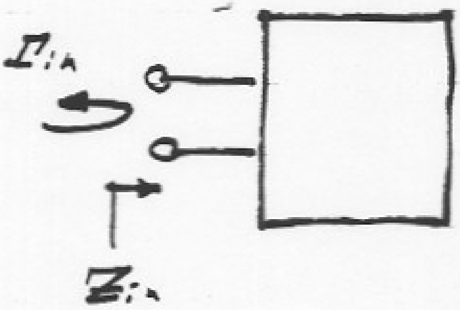


A one port is unconditionally stable iff

$Re\{Z_{in}\} > 0$ <u>at all frequencies</u>

Alternatively, unconditionally stable iff

$Re\{Y_{in}\} > 0$

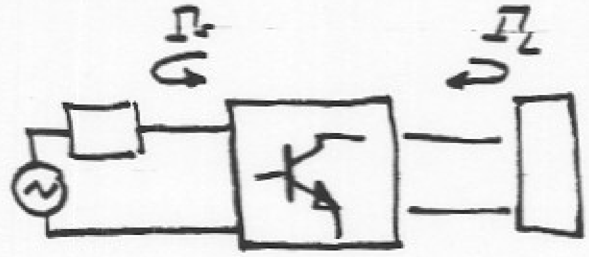


$$\Gamma_{in} = \frac{Z_{in}/Z_0 - 1}{Z_{in}/Z_0 + 1}$$

→ unconditionally stable iff $|\Gamma_{in}| \leq 1$

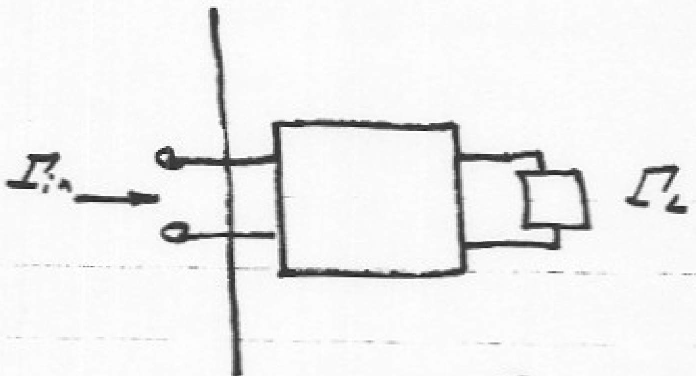
If a device is conditionally stable then some source impedances (or some source reflection coefficients) will result in oscillation.

How about for a two-port?



Two degrees of freedom: Γ_S & Γ_L

Think of it as a one port



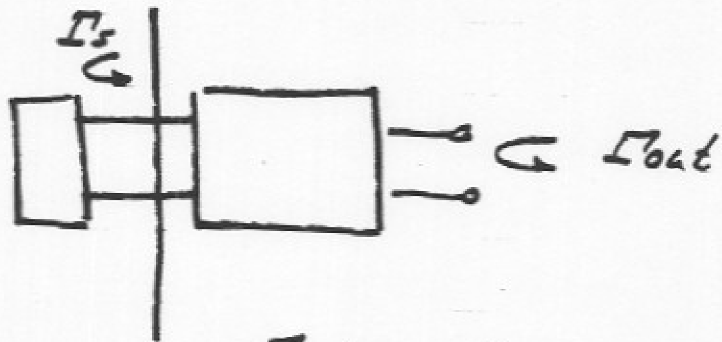
$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

the system will be unconditionally stable ~~for~~

if $|\Gamma_{in}| < 1$ for all load reflection coefficients

Γ_L (worst-case loads consist of $|\Gamma_L| = 1$)

Alternatively, look at output port:



$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11}\Gamma_s}$$

the system will be unconditionally stable if $|\Gamma_{out}| < 1$ for all source reflection coefficients Γ_s (worst-case loads consist of $|\Gamma_s| = 1$)

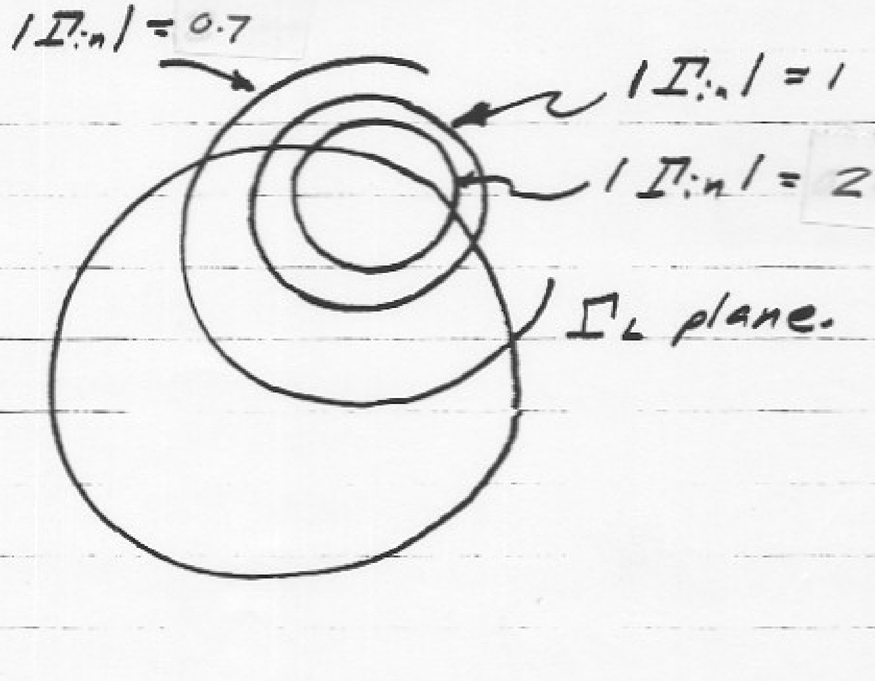
These two criteria are identical, but how can we present graphically?

Stability Circles

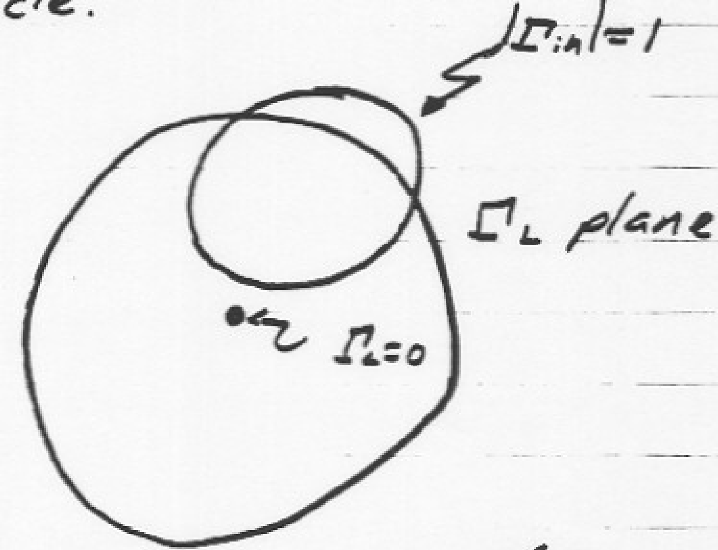
unconditionally stable if $|\Gamma_{in}| < 1$ for all possible values of Γ_L.

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} = \text{function of } \Gamma_L$$

Plot $|\Gamma_{in}|$ in the plane of Γ_L (smith chart,

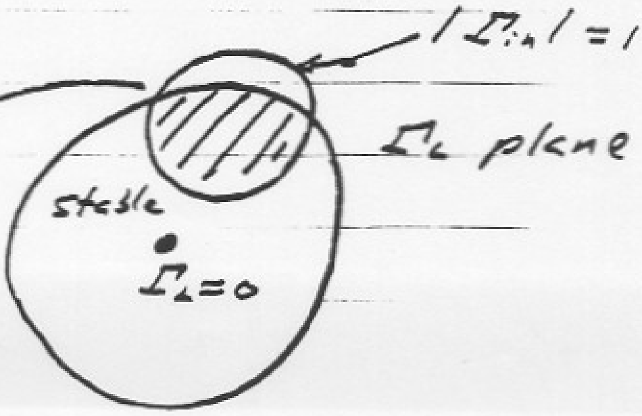


Since we care about whether $|\Gamma_{in}| < 1$, plot only this circle.



For $\Gamma_L = 0$, $\Gamma_{in} = S_{11}$, so if $|S_{11}| < 1$ then system is stable for $\Gamma_L = 0$

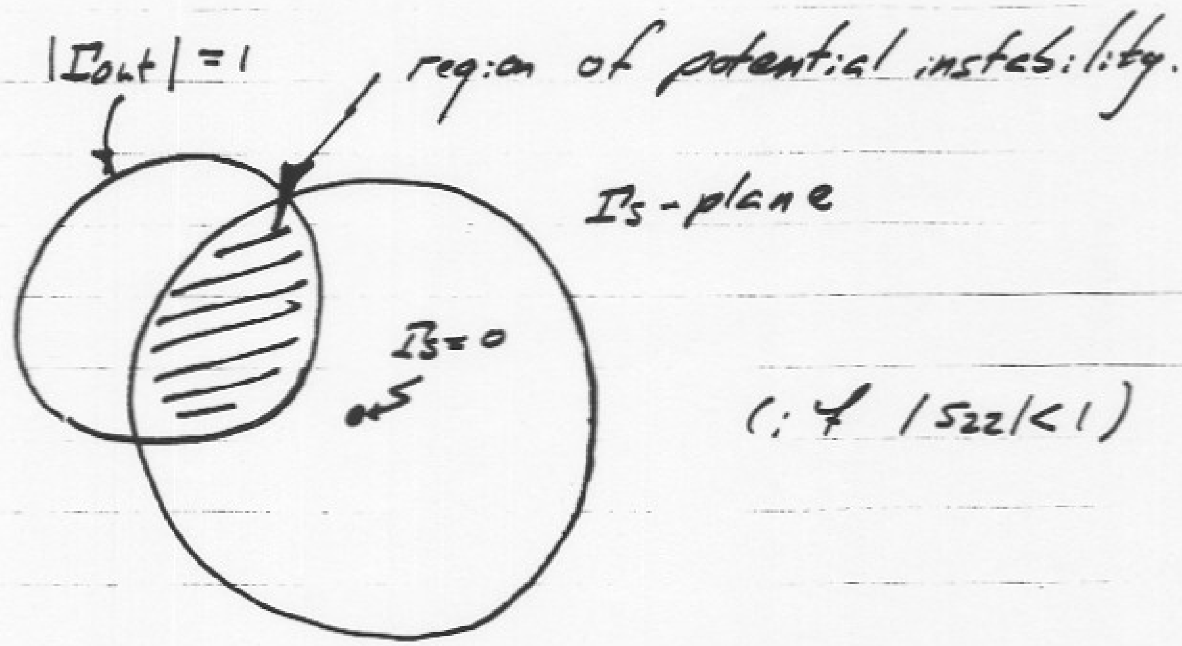
→ if $|S_{11}| < 1$ then potentially unstable region is the region in the Γ_L-plane separated from $\Gamma_L = 0$ by the $|\Gamma_{in}| = 1$ circle



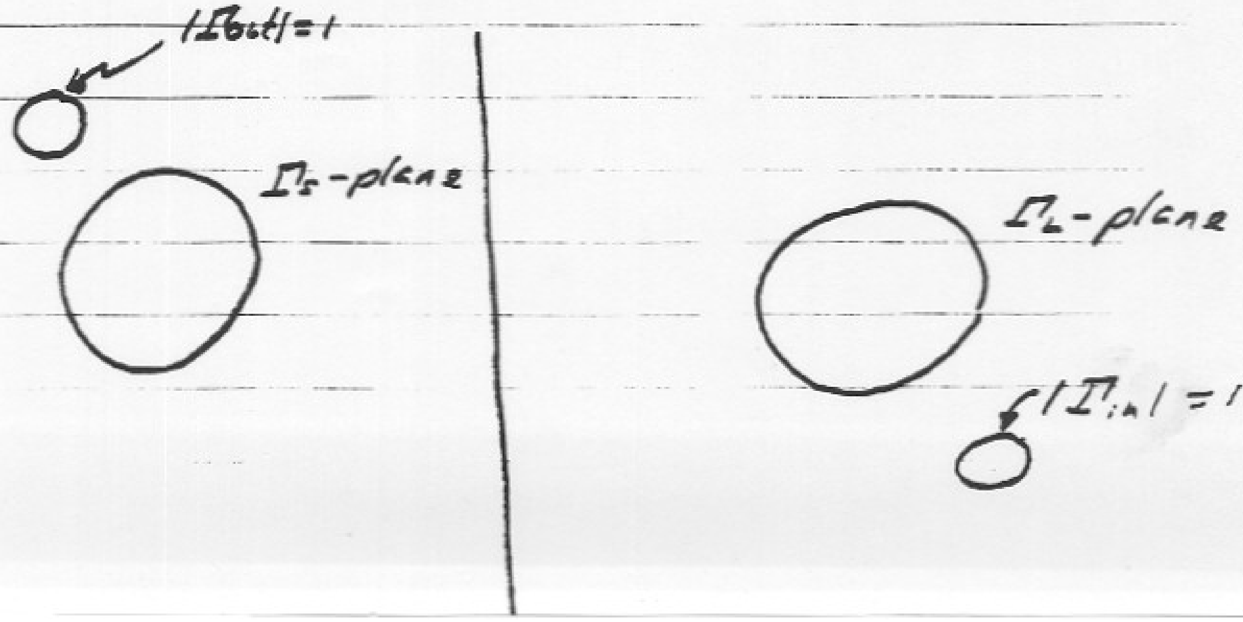
potentially unstable; these values of Γ_L will result in $|\Gamma_{in}| > 1$

similarly, we can look at the criteria

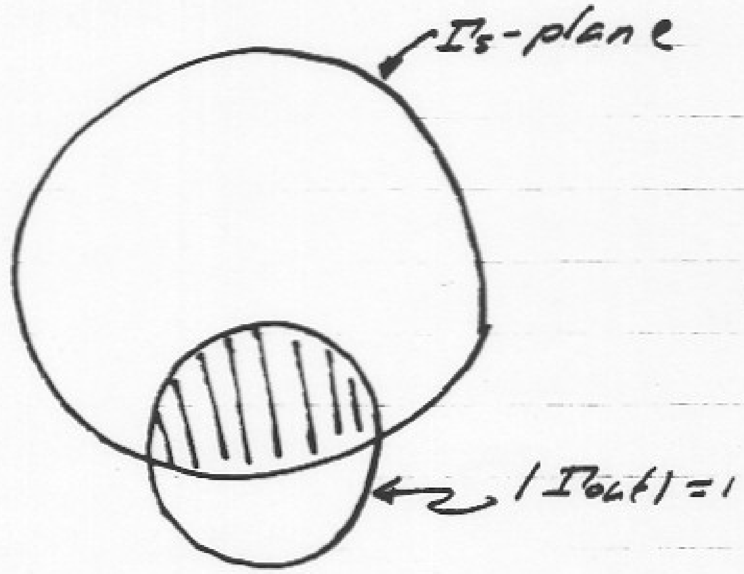
$|\Gamma_{out}| < 1$ for all Γ_s if unconditionally stable



so for unconditional stability:



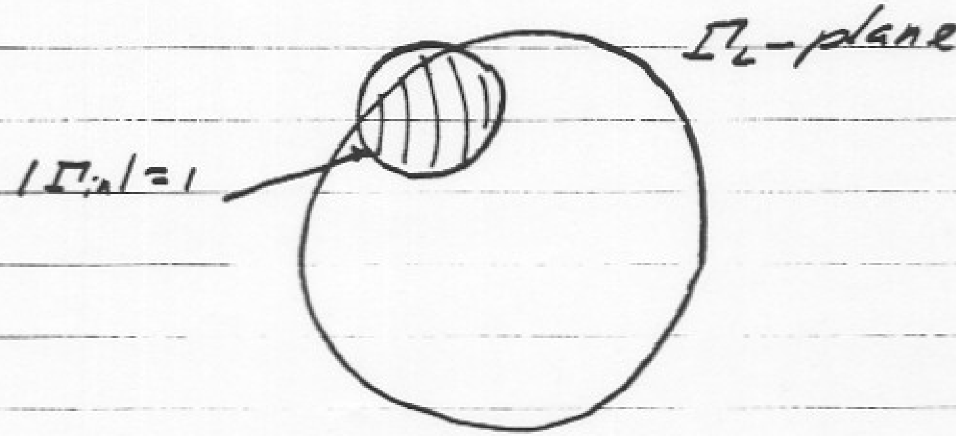
So, suppose we have this:



$|S_{22}|<1$

what values of Γ_S are "safe"?

How about this?



The stability factor: A two port is unconditionally stable if $K > 1$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\det S|^2}{2|S_{12} S_{21}|}$$

and $|\det S| < 1$

stability factor is a quick summary; the stability circles will show which source and load reflection coefficients are dangerous.