ECE202A Notes set 4

2- Port Parameters:

Two-ways of describing device:

Equivalent-Circuit-Model

* physically based
* Includes bias dependence
* Includes frequency dependence
* Includes size dependence - scalability

* Ideal for IC design

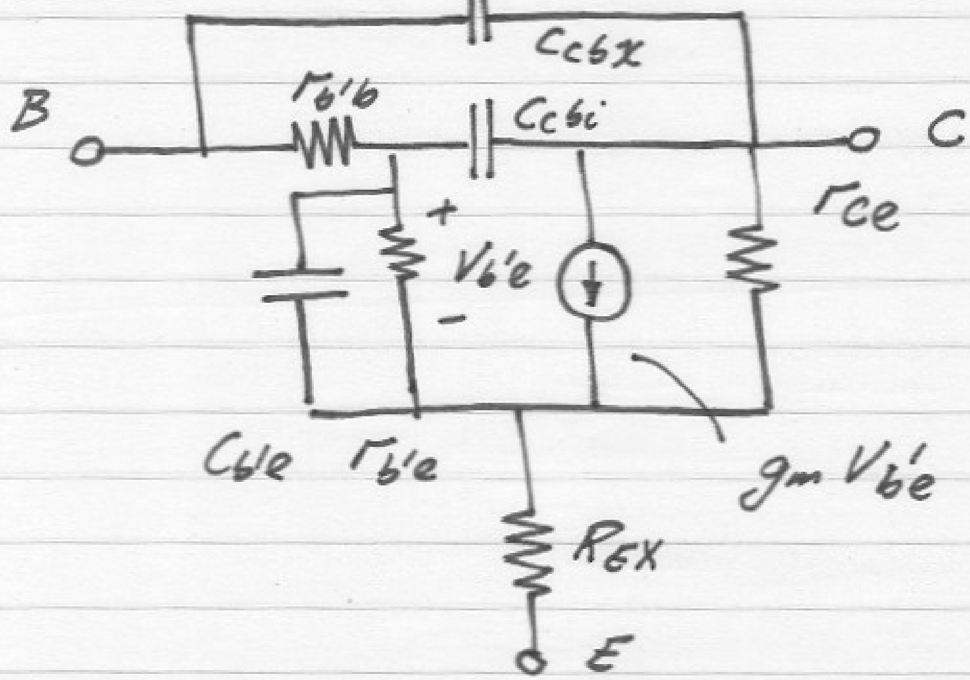
* Weakness: model necessarily simplified
  some errors
  thus weak for highly resonant designs.

2-Port Model

* Matrix of tabular data

* Need one matrix for each bias point & device size
* clumsy - huge data sets required
* Traditional microwave method
* exact

Equivalent Circuit Model of bipolar:



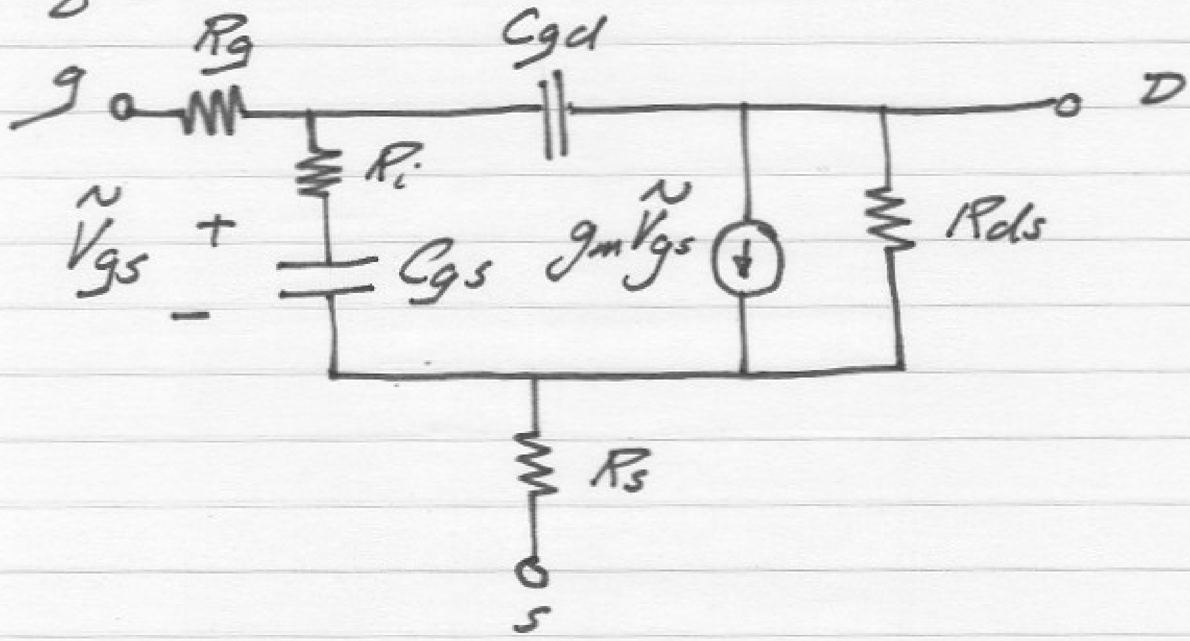
$$C_{b'e} = C_{b'e\,\text{depletion}} + \underbrace{C_{b'e\,\text{diffusion}}}_{g_m(\tau_{base} + \tau_{collector})}$$

$$R_{b'e} = \beta / g_m$$

$$g_m = I_{E,dc} / V_T \quad \text{where } V_T = kT/q$$

$$R_{ce} = V_A / I_{Edc} \quad V_A = \text{early voltage} \simeq 10\text{–}100\,V$$

Equivalent Circuit Model of FET

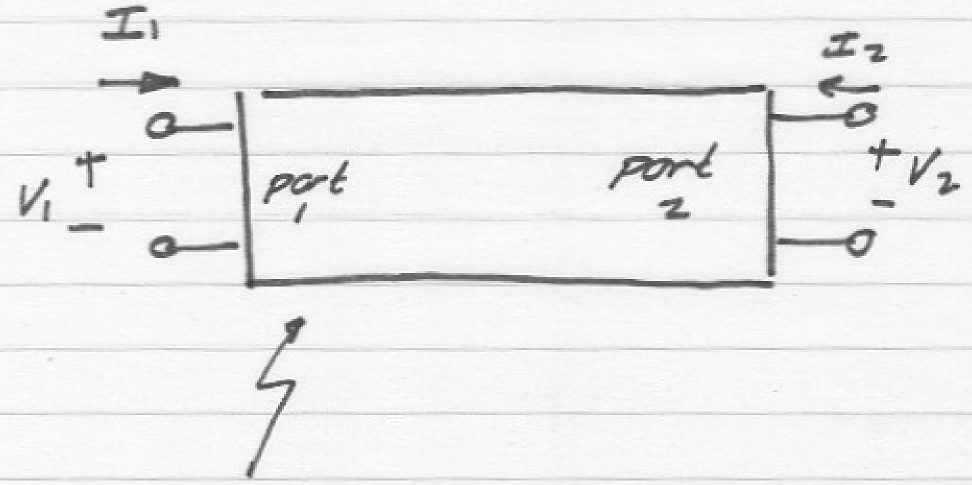


The above are physically-derived models.

More about them later.

2 Port descriptions

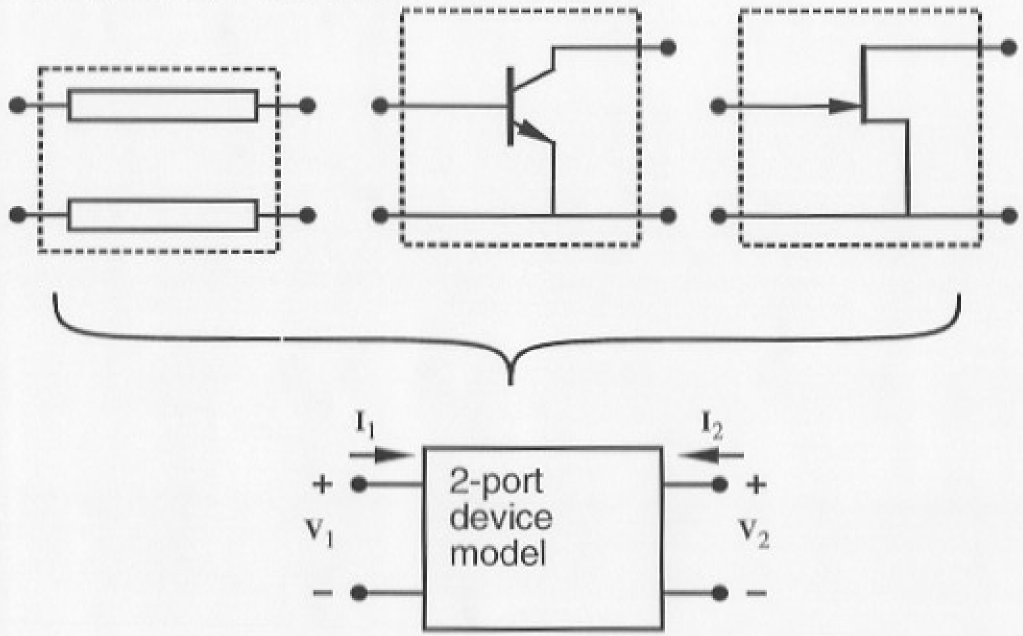
these are black-box (mathematical) descriptions.



inside might be a transistor, a FET, a transmission line, or just about anything.

The terminal characteristics are V_1 V_2 I_1 & I_2 — there are 2 degrees of freedom.

General device model:



$$\begin{bmatrix} I_1(\omega) \\ I_2(\omega) \end{bmatrix} = \begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) \\ Y_{21}(\omega) & Y_{22}(\omega) \end{bmatrix} \begin{bmatrix} V_1(\omega) \\ V_2(\omega) \end{bmatrix}$$

Frequency-domain description:

$v_1(t) = V_1(\omega)e^{j\omega t}$,

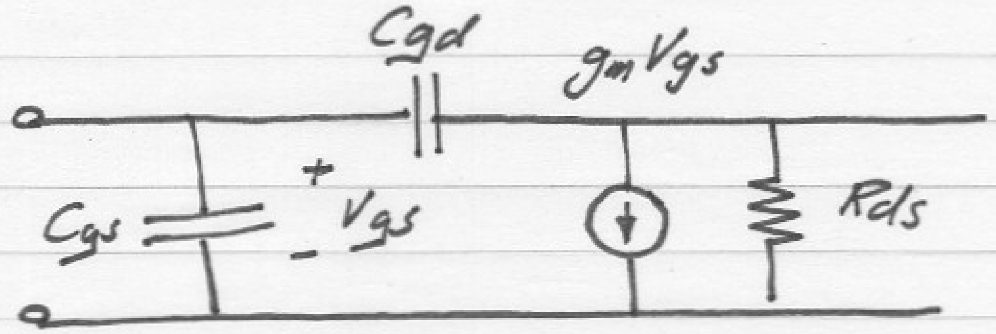
$i_1(t) = I_1(\omega)e^{j\omega t}$, etc.

Two-port *Admittance* parameters

Admittance Parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

example: simple FET Model



by inspection:

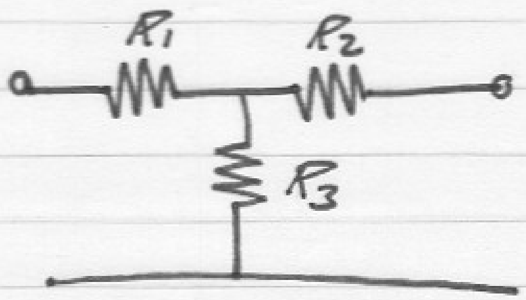
$$Y = \begin{bmatrix} j\omega C_{gs} + j\omega C_{gd} & -j\omega C_{gd} \\ g_m - j\omega C_{gd} & G_{ds} + j\omega C_{gd} \end{bmatrix}$$

easy!

Impedance Parameters

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

example



by inspection

$$\mathbb{Z} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}$$

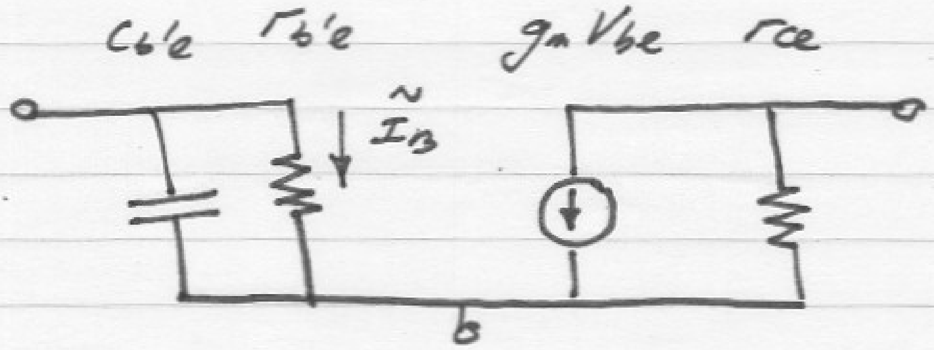
easy

Hybrid Parameters – old and obscure

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

I have never understood *why* you'd want this...

Example



$$g_m V_{be} = \beta V_{be} / r_{be} = \beta \tilde{I}_B$$

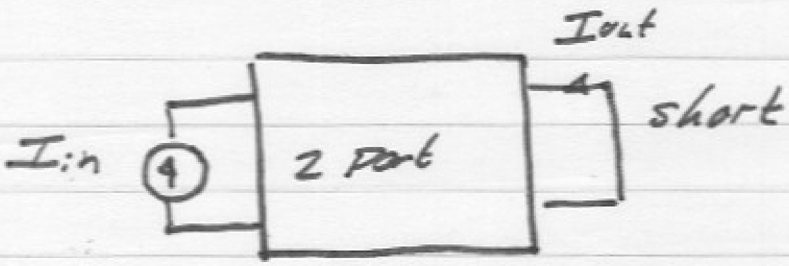
$$H = \begin{bmatrix} \dfrac{1}{1/r_{be} + j\omega C_{b'e}} & 0 \\ \dfrac{\beta}{1 + j\omega r_{b'e} C_{be}} & \dfrac{1}{r_{ce}} \end{bmatrix}$$

the *only* reason to introduce this is

short-circuit current gain.

Short-Circuit Current gain:

$$\text{s.c.c.g.} = \frac{I_2}{I_1}\bigg|_{v_2=0} = \frac{I_2}{I_1}\bigg|_{\text{output shorted.}}$$



$$\frac{I_{out}}{I_{in}} = h_{21}$$

f_T = frequency at which $\| H_{21} \| = 1$

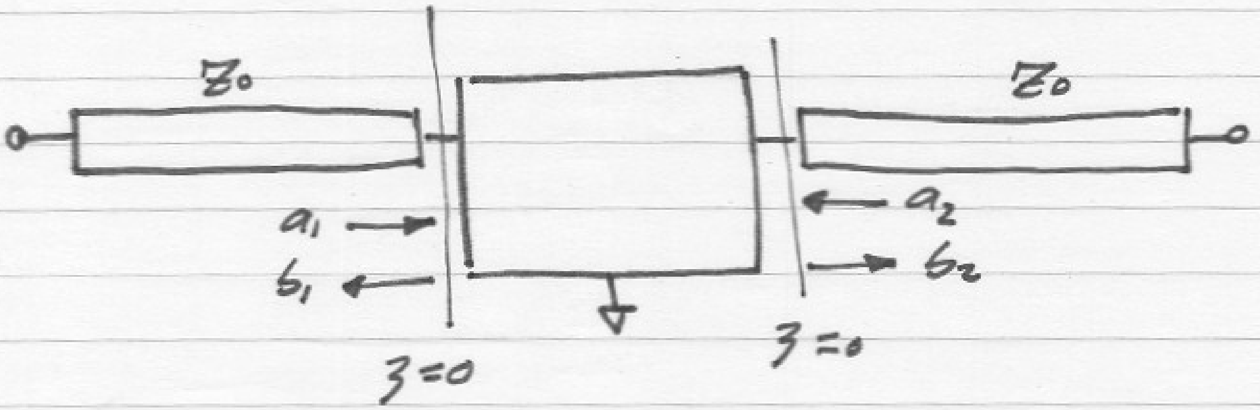
= "short-circuit current gain cutoff frequency"

For the simplified model of the prior page, if $\beta >> 1$,

$$f_T = \frac{g_m}{2\pi C_{b'e}}$$

... more on f_T later ...

S-parameters



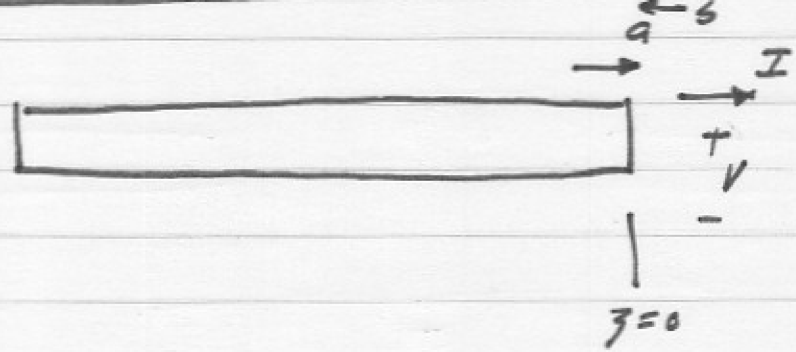
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

note that Z_0 must be defined

we dont really need transmission lines

our objective now is to de-mystify S-parameters – they are easy!

recall voltages & currents on lines:



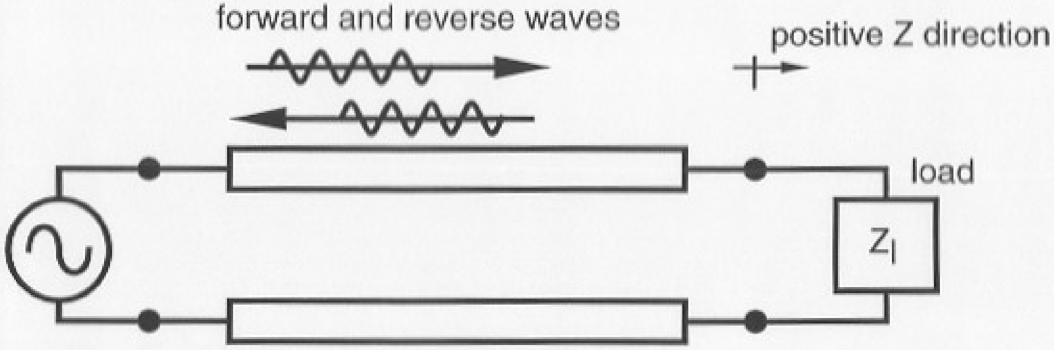
a & b have the usual $\sqrt{Z_0}$ normalization:

$$\begin{cases} V = V^+ + V^- = \sqrt{Z_0}\, a + \sqrt{Z_0}\, b \\ I = (V^+ - V^-)/Z_0 = a/\sqrt{Z_0} - b/\sqrt{Z_0} \end{cases}$$

$$\begin{cases} a = \dfrac{V/\sqrt{Z_0} + \sqrt{Z_0}\, I}{2} \\ b = \dfrac{V/\sqrt{Z_0} - \sqrt{Z_0}\, I}{2} \end{cases}$$

So If we know the relationship between the I's and the V's (Y, Z, or H parameters) we can calculate the relationship between the a's & the b's (S-parameters)

Waves on transmission lines:



a,b: forward and reverse waves

Forward, reverse power: $|a|^2$, $|b|^2$

forward wave reverse wave

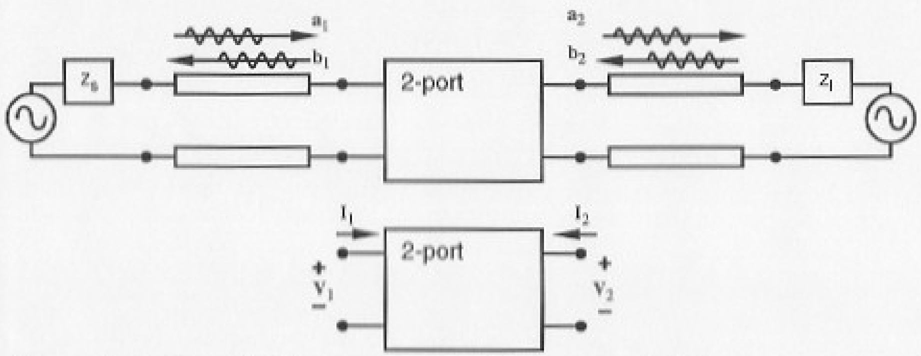
$$v(z,t) = a(t - z / velocity)\sqrt{Z_0} + b(t + z / velocity)\sqrt{Z_0}$$ Voltage

$$i(z,t) = \frac{a(t - z / velocity)}{\sqrt{Z_0}} - \frac{b(t + z / velocity)}{\sqrt{Z_0}}$$ Current

Voltage at any point: $v(z,t) = a(z,t)\sqrt{Z_0} + b(z,t)\sqrt{Z_0}$

Current at any point: $i(z,t) = a(z,t)/\sqrt{Z_0} - b(z,t)/\sqrt{Z_0}$

Two-port parameters described in terms of incident and emanating waves from the device (when connected to transmission lines)



Equivalent S-parameter and Y-parameter models of a 2-port.

Scattering parameter model

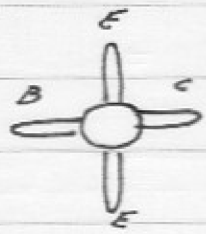
$$\begin{bmatrix} b_1(\omega) \\ b_2(\omega) \end{bmatrix} = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) \end{bmatrix} \begin{bmatrix} a_1(\omega) \\ a_2(\omega) \end{bmatrix}$$

Admittance parameter model

$$\begin{bmatrix} I_1(\omega) \\ I_2(\omega) \end{bmatrix} = \begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) \\ Y_{21}(\omega) & Y_{22}(\omega) \end{bmatrix} \begin{bmatrix} V_1(\omega) \\ V_2(\omega) \end{bmatrix}$$

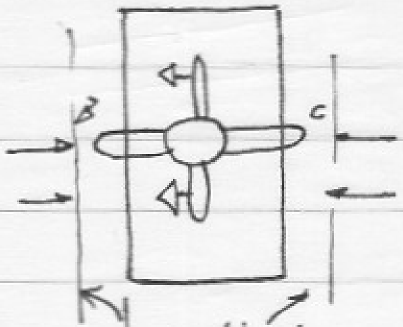
SInce $v = a\sqrt{Z_0} + b\sqrt{Z_0}$ and $i = a/\sqrt{Z_0} - b/\sqrt{Z_0}$, the scattering (S) parameters can be directly computed from the admittance (Y) parameters.

Reference planes



Microwave transistor in package.

on board:



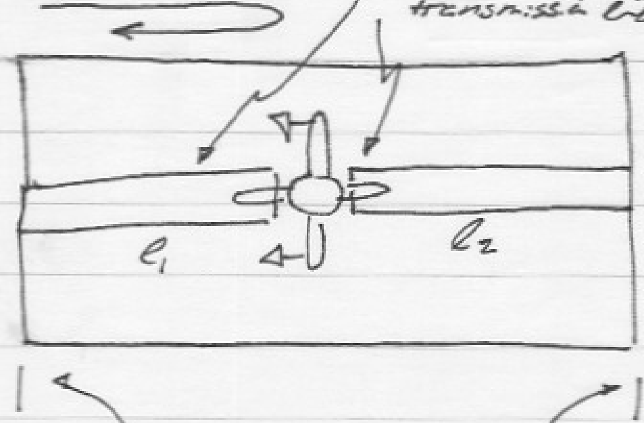
$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

connection to
instruments here
defining $z=0$ here.

defining the reference planes differently
changes the s-parameters.

on bigger board:
phase shifts!

50 Ω microstrip
transmission lines



$$\tilde{S} = \begin{bmatrix} S_{11} e^{j2\theta_1} & S_{12} e^{j(\theta_1+\theta_2)} \\ S_{21} e^{j(\theta_1+\theta_2)} & S_{22} e^{j2\theta_2} \end{bmatrix}$$

$$\theta_1 = 2\pi \frac{l_1}{\lambda}$$

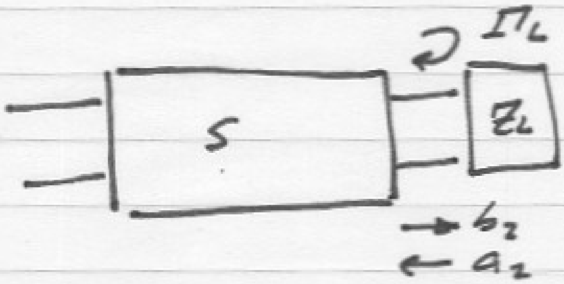
$$\theta_2 = 2\pi \frac{l_2}{\lambda}$$

Connections to instruments
here

How to Calculate s-parameters Quickly

First Comment

$$S_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0}$$



if $Z_L = Z_0$, then Γ_L is zero

and so $a_2 = \Gamma_L b_2 = 0$

so $$\boxed{S_{11} = \frac{b_1}{a_1}\bigg|_{Z_L=Z_0}}$$

so If we say that

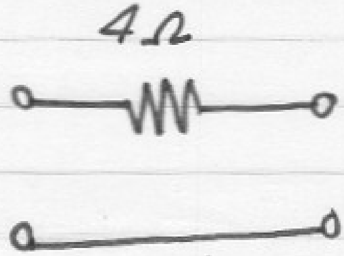
$Z_{in}\big|_{Z_L=Z_0}$ is the input impedance with $Z_0 = Z_L$

then $$S_{11} = \frac{Z_{in}\big|_{Z_L=Z_0} - Z_0}{Z_{in}\big|_{Z_L=Z_0} + Z_0}$$

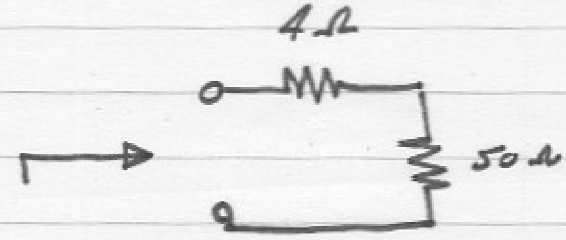
The same comment clearly applies for

S_{22}

Example:



Given $Z_0 = 50\Omega$, what is S_{11}?

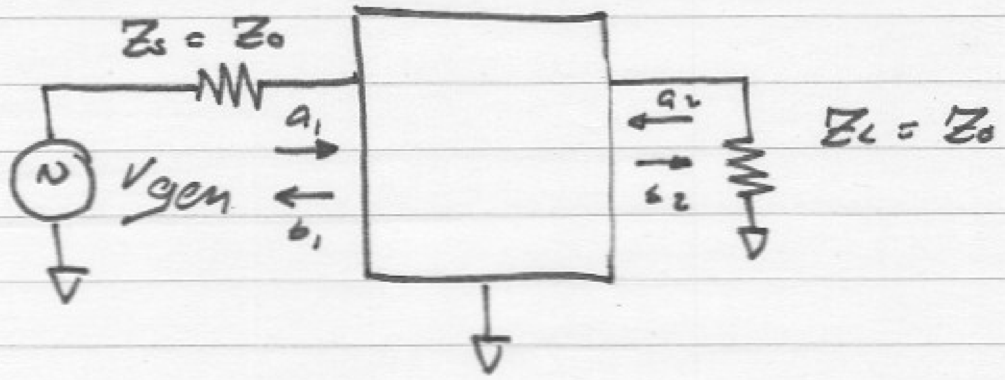


$$Z_{in}\Big|_{Z_L = Z_0} = 54\ \Omega$$

$$S_{11} = \frac{54 - 50}{54 + 50} = \frac{4}{104}$$

similar arguments give $S_{22} = \dfrac{4}{104}$

Now construct the following



Consider the generator:

$$a_1 = T_s V_{gen} + \Gamma_s b_1$$

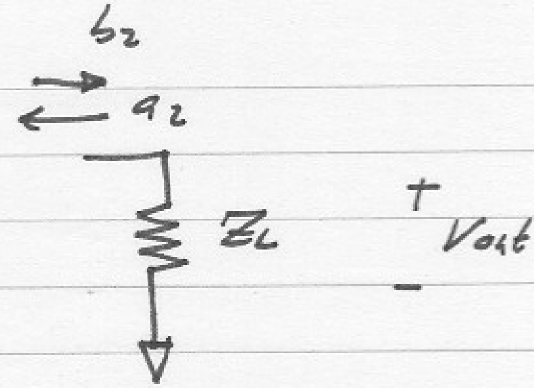
where $\Gamma_s = \dfrac{Z_s - Z_0}{Z_s + Z_0}$

and $T_s = \dfrac{1}{\sqrt{Z_0}} \dfrac{Z_0}{Z_0 + Z_s}$

since $Z_s = Z_0$, $T_s = 1/\sqrt{Z_0}$ and $\Gamma_s = 0$

So $\boxed{a_1 = V_{gen} / 2\sqrt{Z_0}}$

Consider the load:



$$a_2 = \Gamma_L b_2$$

$$= 0 \quad \text{because} \quad Z_L = Z_0$$

$$V_{out} = V^+ + V^- = \sqrt{Z_0} \cdot a_2 + \sqrt{Z_0} \cdot b_2$$

$$= \sqrt{Z_0} \cdot b_2 \quad \text{because} \quad a_2 = 0.$$

Now calculate V_{out} / V_{gen}

$$V_{out} = \sqrt{Z_0}\, b_2$$

$$= \sqrt{Z_0}\left[S_{21}\, a_1 + S_{22}\, \overset{0}{\cancel{a_2}} \right]$$

$$= \sqrt{Z_0}\, S_{21}\, a_1$$

but

$$a_1 = V_{gen} / 2\sqrt{Z_0}$$

so

$$\frac{V_{out}}{V_{gen}} = \sqrt{Z_0}\, S_{21} \cdot \frac{1}{2\sqrt{Z_0}}$$

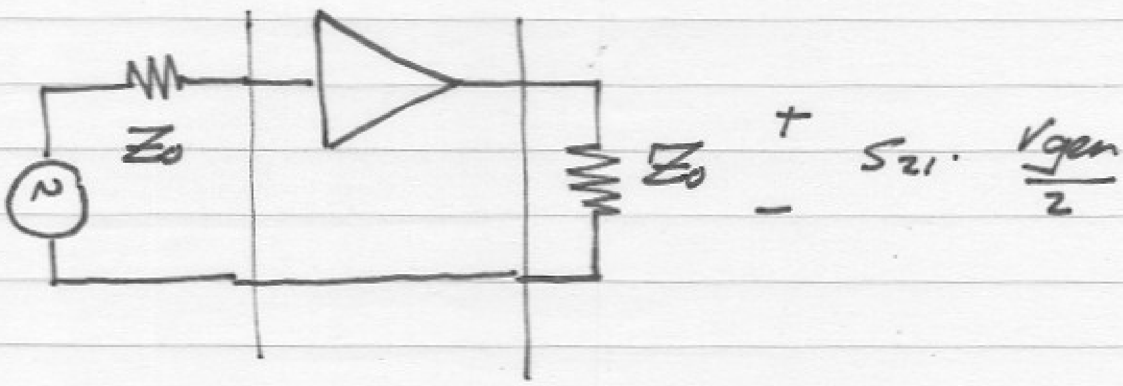
$$\Rightarrow \quad \boxed{S_{21} = \frac{2\, V_{out}}{V_{gen}}} \quad \Leftarrow \quad !$$

Explicity:

$$\boxed{S_{21} = 2 \cdot \left.\frac{V_{out}}{V_{gen}}\right|_{Z_L = Z_S = Z_0}}$$

why the factor of 2 ?

now insert an amplifier:



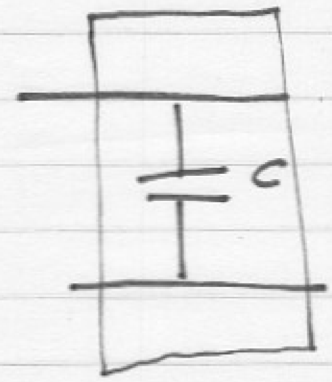
The signal has been increased by an amount S_{21}

so S_{21} is the forward insertion gain

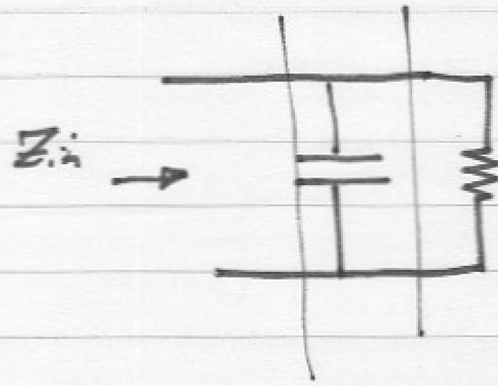
in a system of impedance Z_0

We can similarly find S_{21} & S_{12}.

Examples



S_{11}:



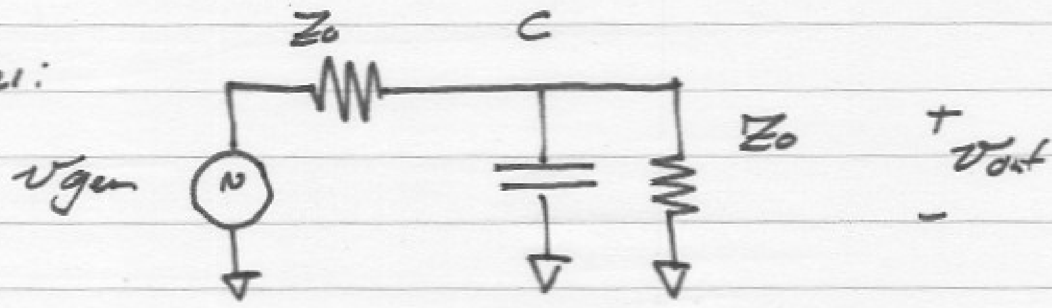
$$Z_{in}\Big|_{Z_L = Z_0} = (sC + 1/Z_0)^{-1} = X$$

$$S_{11} = \frac{X - Z_0}{X + Z_0} = \frac{1/Z_0 - 1/X}{1/Z_0 + 1/X}$$

$$= \frac{Y_0 - (sC + Y_0)}{Y_0 + (sC + Y_0)} = \frac{-sC}{2Y_0 + sC} = \frac{-sC/2Y_0}{1 + sC/2Y_0}$$

$$= \frac{-j\omega C Z_0/2}{1 + j\omega C Z_0/2}$$

S_{21}:



$$\frac{v_{out}}{v_{gen}} = \frac{1}{2} \quad \frac{1}{1 + s\,C\,(Z_0/2)}$$ by inspection.

$$S_{21} = 2\frac{v_{out}}{v_{gen}} = \frac{1}{1 + j\omega C\,(Z_0/2)}$$

$$S = \begin{bmatrix} \dfrac{-j\omega C\,Z_0/2}{1 + j\omega C\,Z_0/2} & \dfrac{1}{1 + j\omega C\,Z_0/2} \\ \\ \dfrac{1}{1 + j\omega C\,Z_0/2} & \dfrac{-j\omega C\,Z_0/2}{1 + j\omega C\,Z_0/2} \end{bmatrix}$$

EASY!

Network Analysis

- Measures linear 2-port stimulus-response characteristics of a device

- Data usually presented as admittance or wave scattering parameters as a function of frequency.

- 2 Purposes:

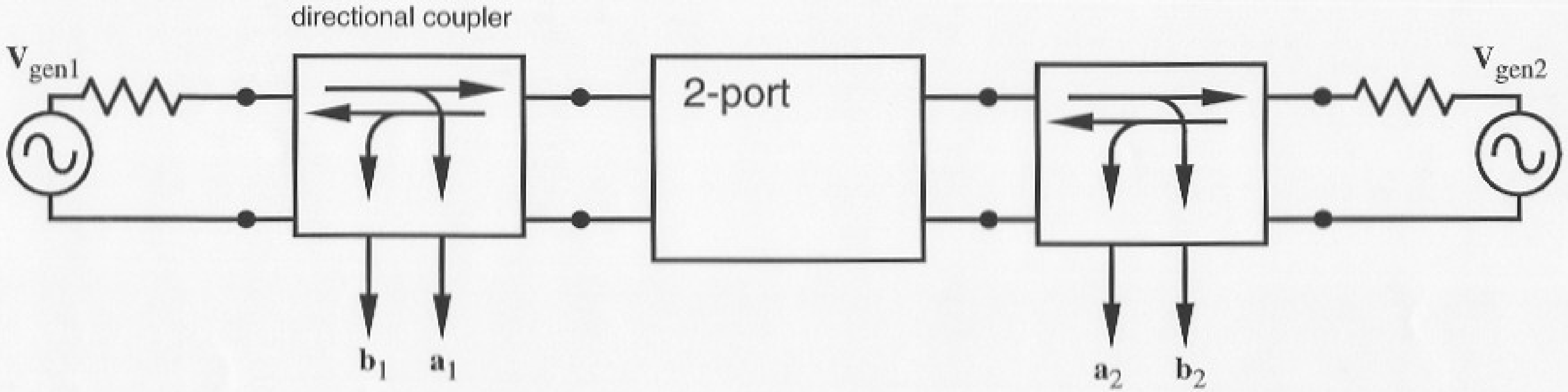
Functional measurements of a component (gain-frequency curve, etc.)

Device characterization and modelling

20

The Microwave Network Analyzer

Measurement of (linear / small signal) 2-port network parameters in the frequency domain.



Swept-frequency sources (V_{gen1} and V_{gen2}) are alternately applied to the 2-port input and output, and the incident and emanating waves measured with directional couplers.

Calibration: amplitude/phase contributions of cabling (etc.) between the instrument and the d.u.t. are corrected for by first measuring a series of devices of known characteristics in place of the d.u.t. , either 50Ω load, open, short, and through line, or a series of through lines of differing lengths ("LRL")

(12)

Performance of modern network analyzers:

After Calibration: DC-62.5 GHz instrument
(Coaxial-based system, using coplanar microwave wafer probes)

Amplitude accuracy, 0 dB signal: $\approx\pm 0.05$ dB

Phase accuracy, 0 dB signal: $\approx\pm 3°$

Directivity*: $\approx$ -40 dB
*Measured reflection magnitude for a zero-reflection device

Given accurate calibration standards, network analyzers can provide very precise device models. Competing optical techniques offering wider bandwidth must attain competitive accuracy. This places stringent demands on laser intensity stability (and often laser pulse timing stability).

22

Measuring s-parameters

Calibration

Errors due to

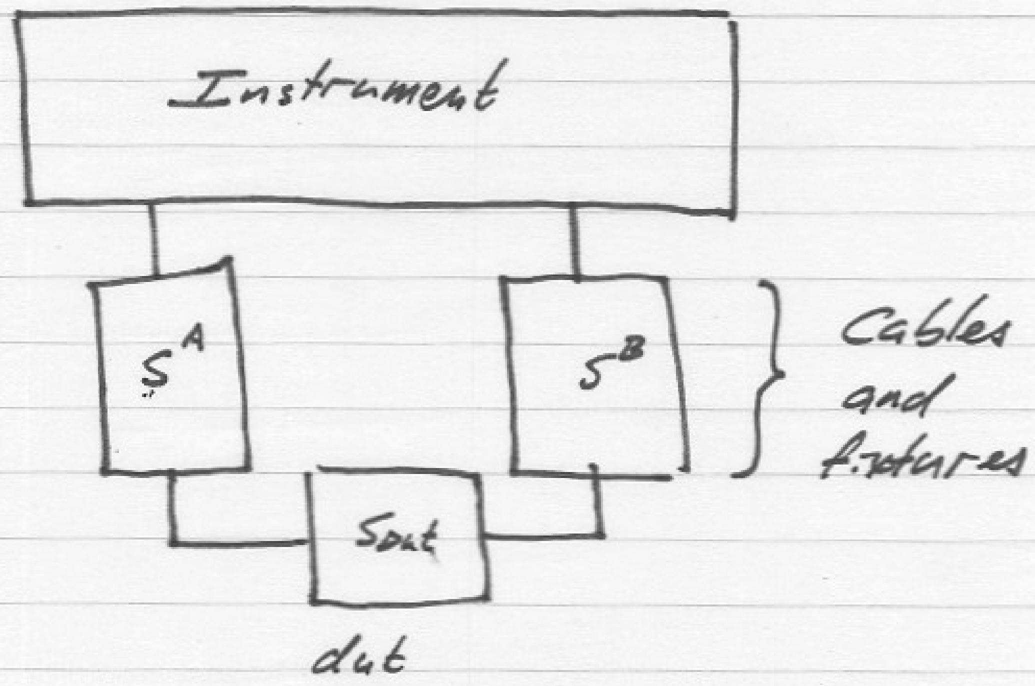
phase shift in Cables

attenuation.

reflections from cables

and many others

systematic Picture:



Instrument measures a cascade of 3 2-port networks: the dut plus 2 error 2 ports representing cables and fixtures.

There are seemingly 8 unknowns - in reality six, because the error two ports are reciprocal ($S_{21} = S_{12}$)

Instrument is calibrated by measuring a set of known standards. Various calibration methodolies are used.

open - short - load - through (OSTL)

Line - Reflect - Line (LRL)

Line - reflect - Match (LRM)