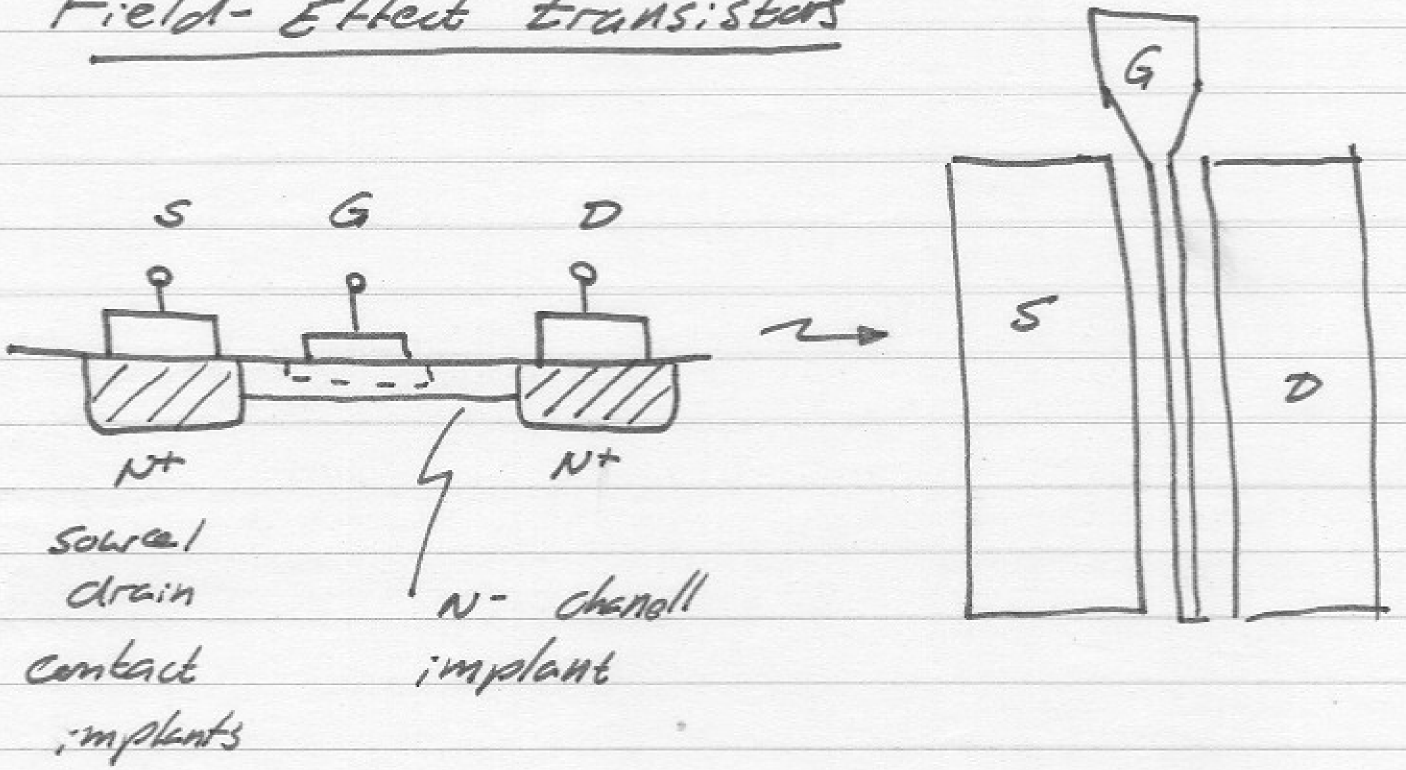
ECE 202a Lecture Notes set 6

Active devices:

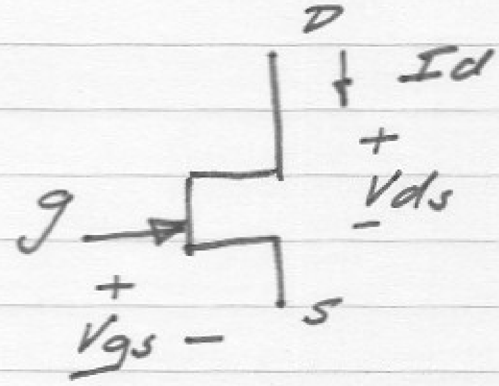
Field-Effect transistors

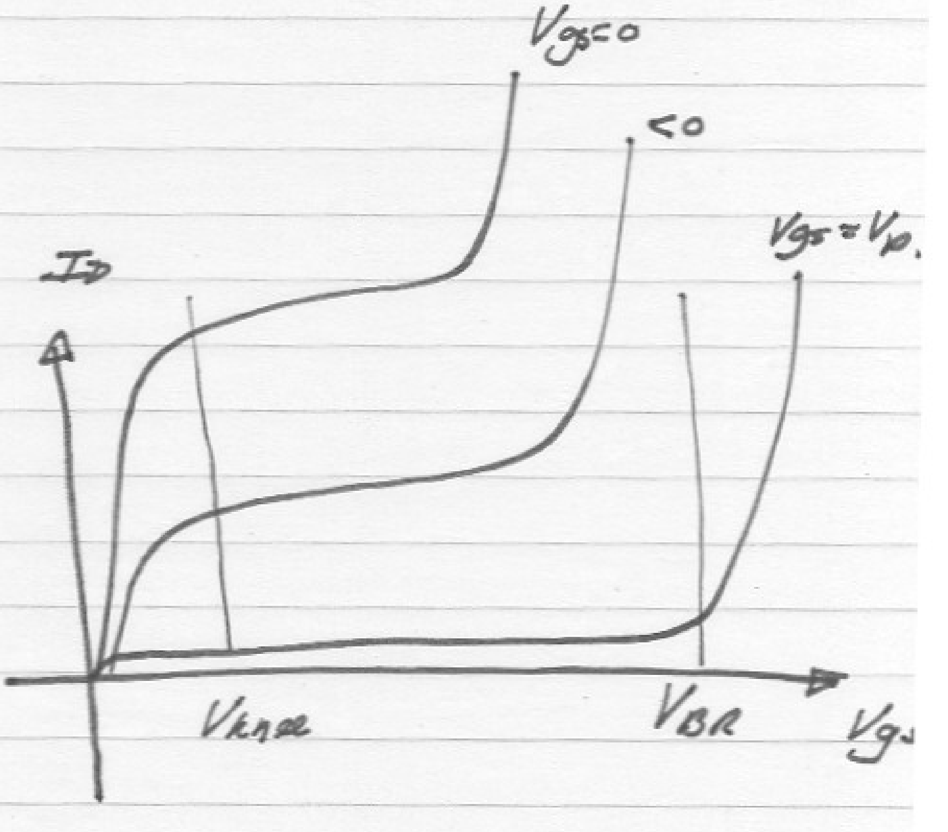


shown above is an ion-implanted mesFet although HEMTs differ somewhat in fabrication and in structure, terminal characteristics are similar...

bias voltage between gate & source sets depletion depth in channel region, restricting current flow between a source and drain.

I-V characteristics:





In elementary texts, for $V_{ds} > V_{knee}$, it is given that

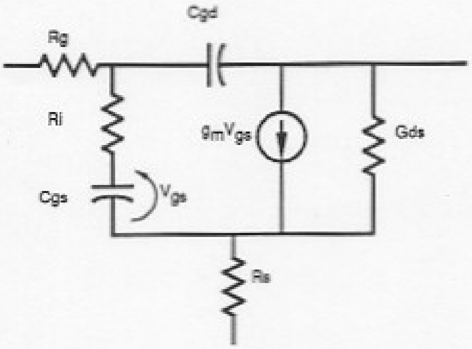
$$I_d \sim I_{dss}\,(1 - V_{gs}/V_p)^2\,(1 + \lambda V_{ds})$$

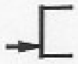
... this assumes the gradual-chanel approximation and ignores velocity saturation. Neither assumption is reliable for high-performance devices. Models tend therefore to be either empirical or based upon finite-element computer analysis.

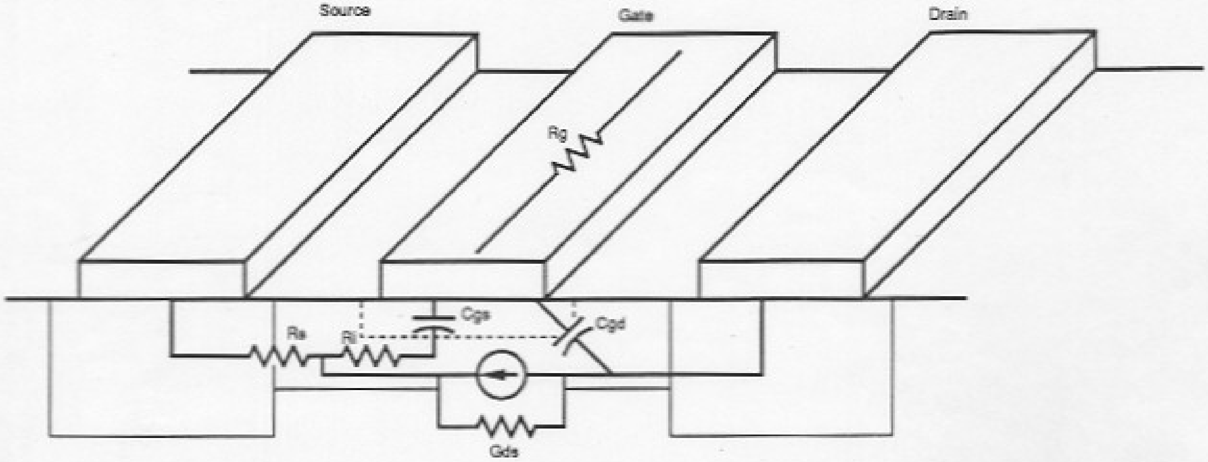
gate will conduct if $V_{gs} > 0.5V$

normal operation: $\begin{cases} V_p < V_{gs} < +0.5V \\ \underbrace{1.5V}_{V_{knee}} < V_{ds} < V_{br} \end{cases}$

FET Model







Here R_i is component of the channel resistance seen by C_{gs}.

R_S is the (bulk + ohmic) parasitic resistance of the source.

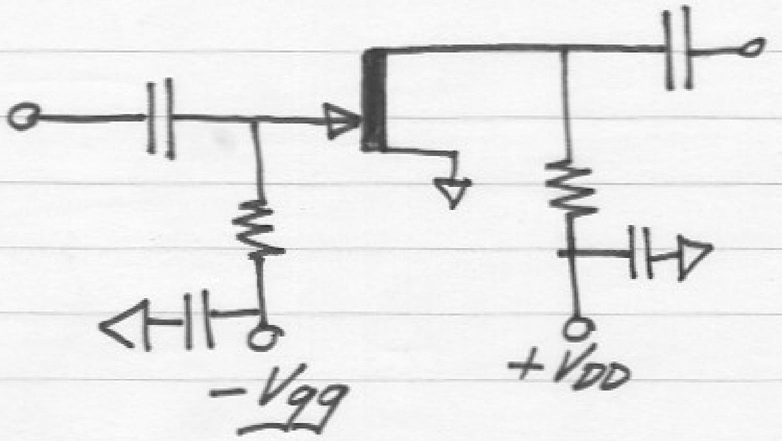
R_g is gate metallization resistance (effective)

note that for velocity-limited transistor, $\frac{v_{sat}}{l_g} = \frac{g_m}{C_{gs}}$.

Biasing an FET

we need to bias the device - e.g. force some desired V_{gs} & V_{DS}

- hence I_D is then set.



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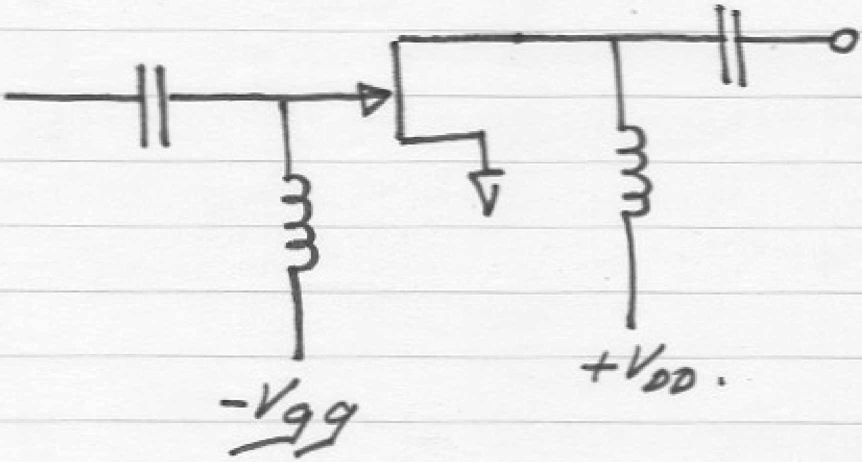

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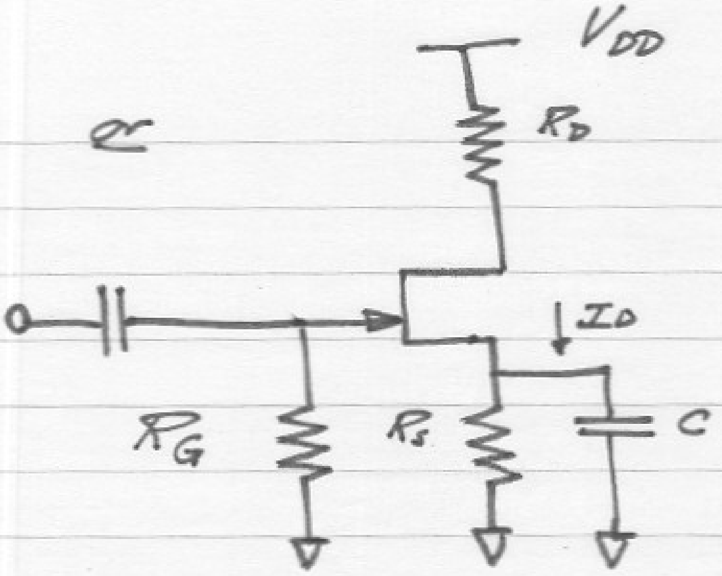
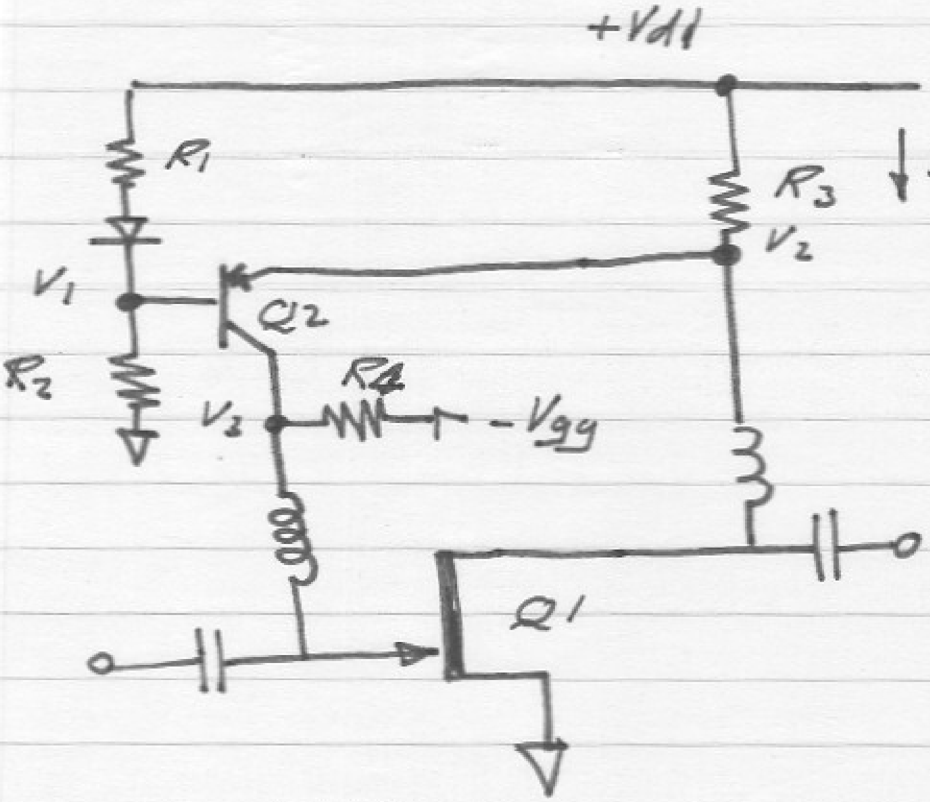


R_S is a "self-bias" resistor which forces $V_{gs} = -I_D R_S$ in order to obtain a negative gate-source voltage without a negative supply.

The problem: wiring inductance is introduced in the source circuit. This will become significant when $\omega L \sim \frac{1}{10} \cdot \frac{1}{g_m}$ which is generally a very small inductance!

Active biasing is very common in microwave systems for precise bias control:



$V_1 = \frac{R_2}{R_1+R_2}(V_{DD} - V_{be})$ make $\frac{R_2}{R_1+R_2} \geq 0.8$

$V_2 = V_1 + V_{be} = V_{drain}$

$I_x = (V_{dd} - V_2)/R_3$

$I_{c2} = (V_{g1} + V_{gg})/R4$

$I_D = I_x - I_{c2}$ Make $I_{c2} \ll I_x$.

... This can also be done with op-amps etc.

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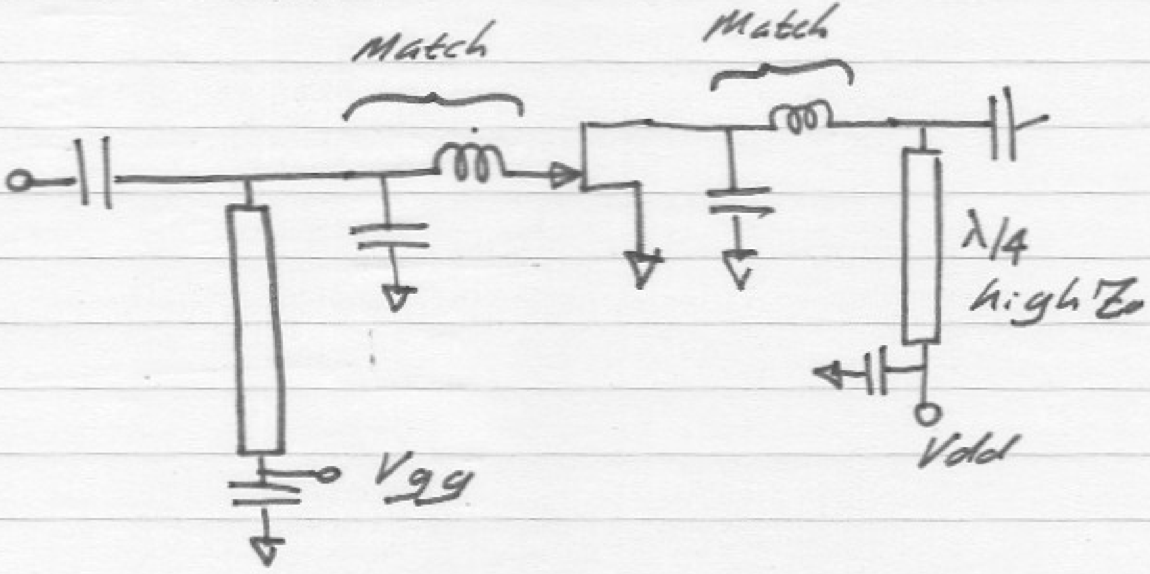
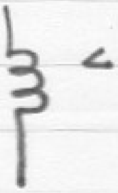
bias tees were shown on the prior page.

where narrow bandwidths are required, we can do this:



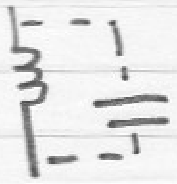
this is necessarily <u>very</u> frequency-selective.

Inductors can be used for bias networks too



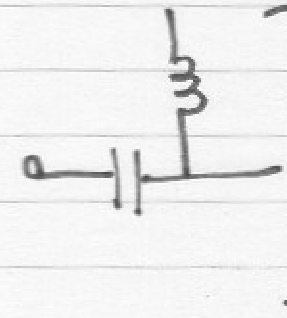
$Z = \omega L$ so isolation is poor at lower frequencies

so we want big L ! (?)



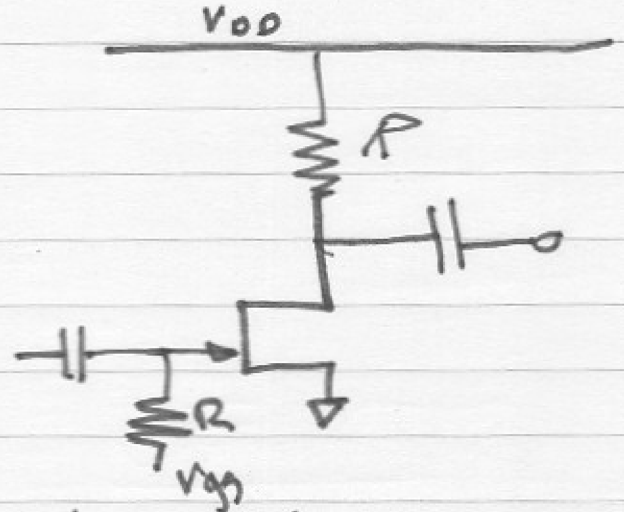
... big inductors have big parasitic winding capacitances $C_{winding}$. The network resonates (→ zero impedance) at $\omega_{par} = (L\,C_{winding})^{-1/2}$

so the problem is getting a wide bandwidth with inductors.



Commercial bias tees have bandwidths of e.g. 45 MHz – 60 GHz.

If we want very-low-frequency response, we must use resistive biasing:



the penalties:

1) DC power is dissipated in R.

2) R loads the high-frequency circuit.

So we have biased the device:

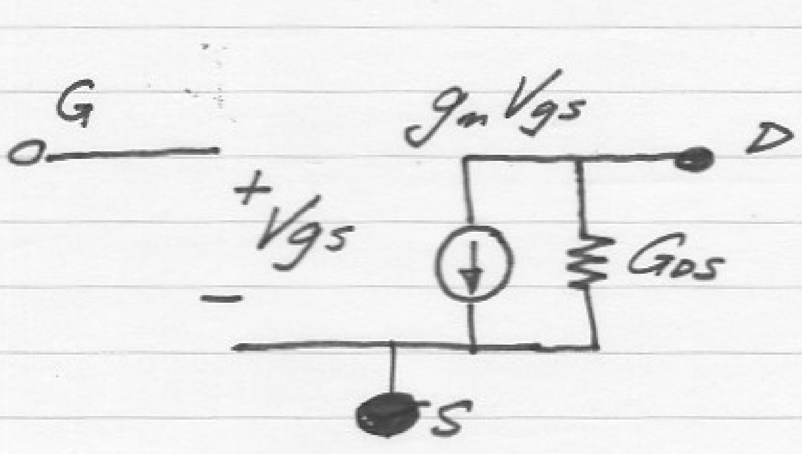
small-signal models quantify the <u>changes</u> in the device terminal currents (with respect to the steady-state bias currents) in response to <u>changes</u> in the applied terminal voltages.

$$\frac{\partial I_{DRAIN}}{\partial V_{gs}} \triangleq g_m = \frac{2 I_{DSS}}{V_p}\left(1 - \frac{V_{gs}}{V_p}\right)(1 + \lambda V_{DS})$$

$$\frac{\partial I_{DRAIN}}{\partial V_{DS}} \triangleq G_{DS} = \frac{I_D}{1 + \lambda V_{DS}} \lambda \cancel{V}_{DS} \simeq I_D \lambda V_{DS}$$

If square-law model

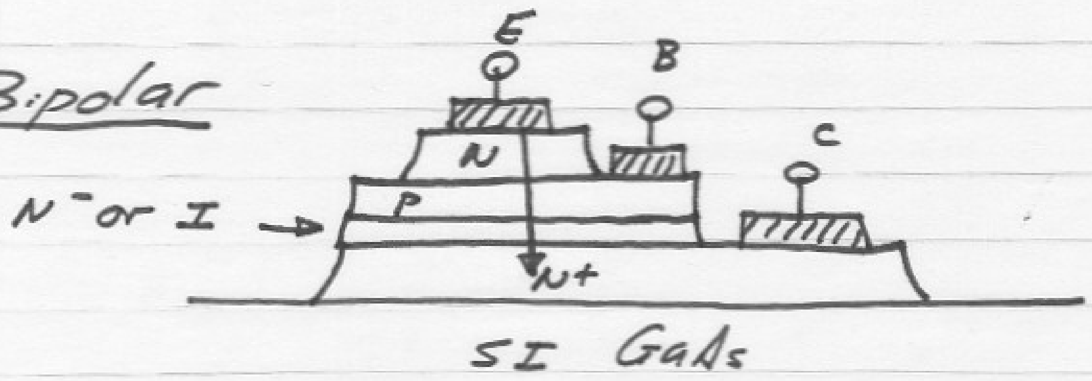
$\Longrightarrow$



The reader is warned of the unreliability of the square-law model.

users tend to refer to an extensive database of experimentally-determined parameters.

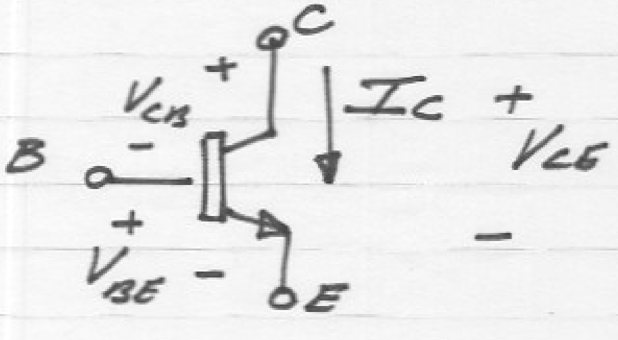
Bipolar

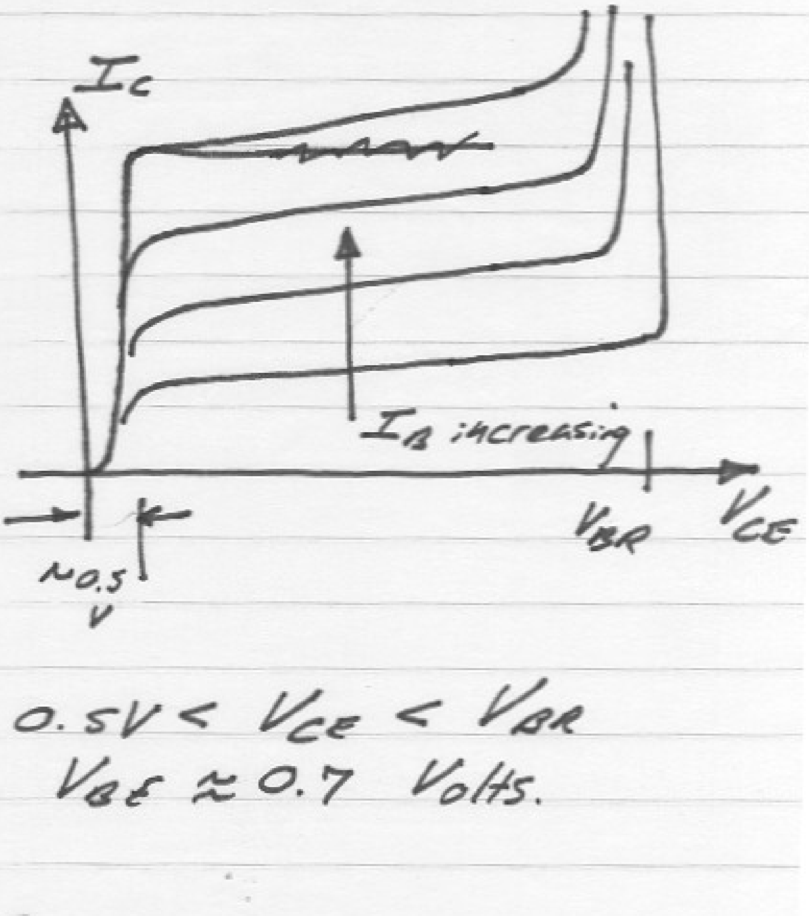


If $V_{cb} > 0$ (cb junction reverse-biased)

then $I_c \simeq I_{SE}\left[\exp\{qV_{BE}/kT\}\right](1+\lambda V_{CE})$

$I_B \simeq I_c/\beta$





Normal operation: $0.5V < V_{CE} < V_{BR}$
& $V_{BE} \approx 0.7$ Volts.

Bias circuit must force (set) V_{CE} and I_c

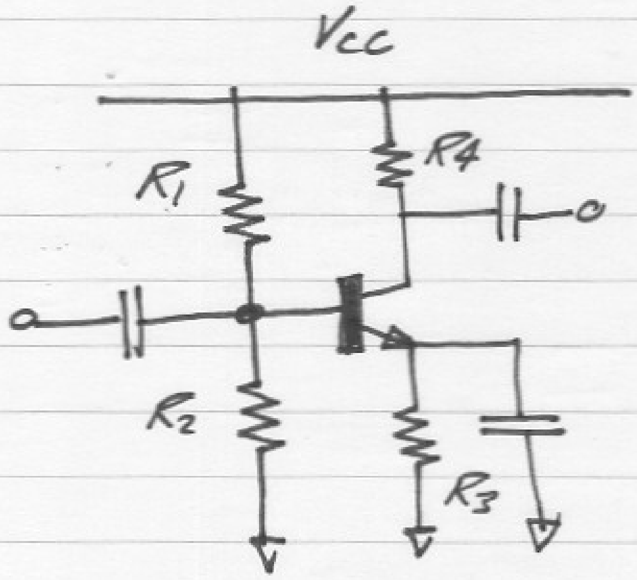
__not__ V_{CE} and V_{BE}.

Bipolar Bias examples

Must force V_{CE} & I_C

Never V_{BE}: $I_C = I_{SE}\, e^{V_{BE}/V_T}$ but I_{SE} varies exponentially with temperature.

Never I_B: $I_C = \beta I_B$ but β varies tremendously from device to device & also varies with temperature



$$V_{base} = V_{CC} \cdot \frac{R_2}{R_1 + R_2}$$

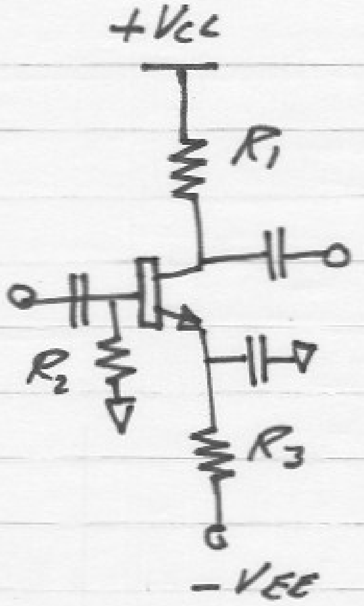
if I_{base} is small.

$$V_{emitter} = V_{base} - V_{be}$$

$$I_E = V_{emitter} / R_3$$
$$\simeq I_C$$

$$V_{collector} = V_{CC} - I_C R_4$$

or:



$V_{base} \cong 0$ if $I_B \cdot R_2$ small

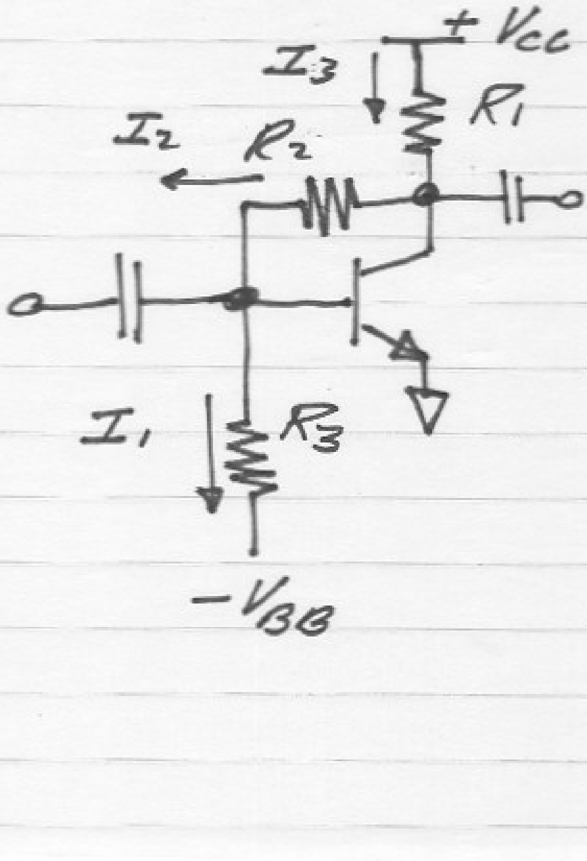
$V_{emitter} = -V_{BE}$

$$I_E = (V_{EE} - V_{be})/R_3$$
$$\cong I_c$$

$$V_c = V_{cc} - I_c R_1$$

(the Possibilities are endless)

feedback dc biasing - Very handy with microwave feedback amplifiers:



$$I_1 \simeq (V_{be} + V_{BB})/R_3$$

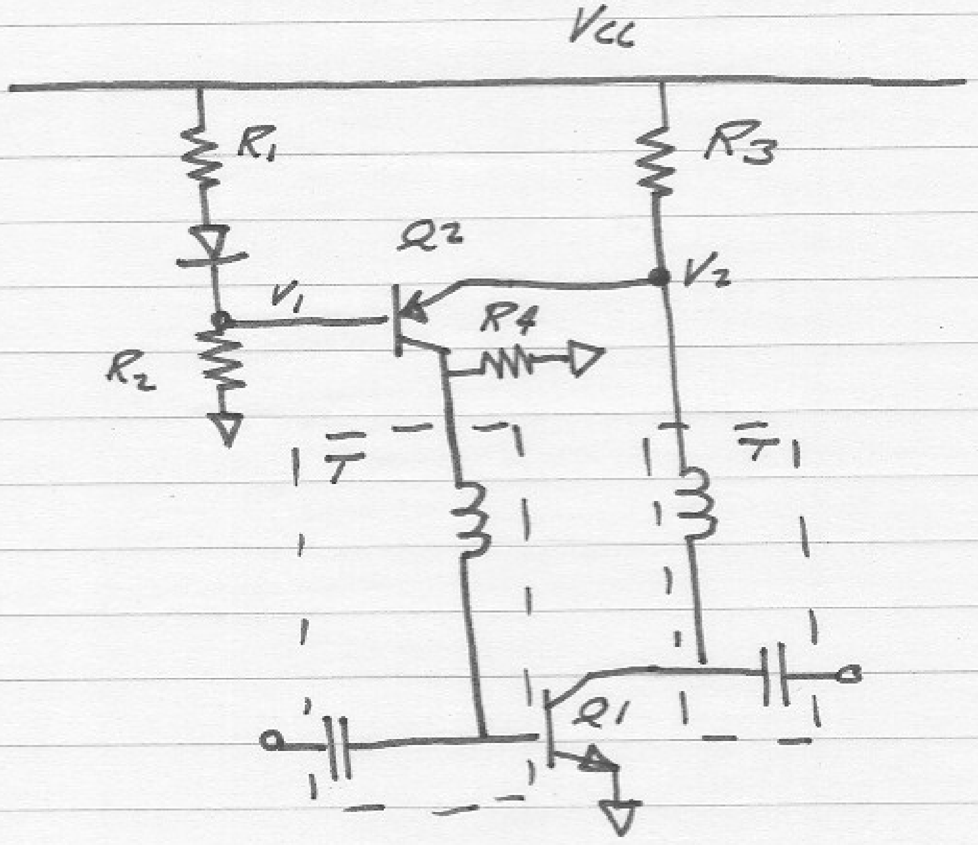
$I_2 \cong I_1$ if I_B is small

$$\therefore V_c = V_{be} + I_2 R_2$$

$$I_3 = (V_{cc} - V_c)/R_1$$

$$I_c = I_3 - I_2$$

or we can use feedback:



$V_1 = \frac{R_1}{R_1+R_2}(V_{cc} - V_{be})$ if I_b small

$V_2 = V_1 + V_{be} = V_{c1}$

$I_{c2} = V_{be}/R_4 + I_{B1}$

$I_{c1} = \frac{V_{cc} - V_2}{R_3} - I_{c2}$

Feedback loop must be designed <u>not</u> <u>to</u> <u>oscillate</u>!

Bipolar small-signal Parameters:

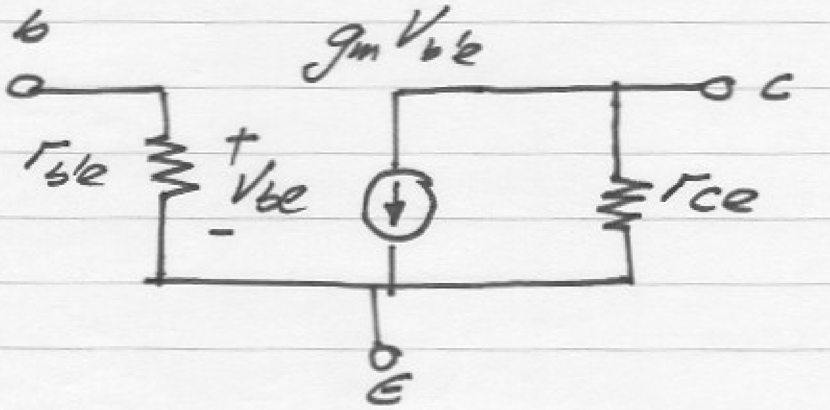
$$\frac{\partial I_c}{\partial V_{be}} = \frac{\partial}{\partial V_{be}}\left[I_{SE}\, e^{V_{be}/V_T}\right] = I_c/V_T = g_m$$

$$\frac{\partial I_b}{\partial V_{be}} = \frac{\partial}{\partial V_{be}}\left[\frac{I_{se}}{\beta}\, e^{V_{be}/V_T}\right] = \frac{g_m}{\beta} = \frac{1}{r_{b'e}}$$

where $V_T = kT/q = 26\,mV$ @ 300°K

$$\frac{\partial I_c}{\partial V_{CE}} \cong I_c/\underset{\approx 25-100V}{V_A} = 1/r_{ce}$$

$\Longrightarrow$



Origin of parasitics :

$$\tau_{base} \overset{\Delta}{=} \frac{Q_{base\,(stored)}}{I_c} \cong \frac{W_{base}^2}{2D_n}$$

electron diffusivity in base

$\tau_{collector}$ = collector depletion layer transit time

$$= \frac{\text{collector depletion width}}{2 \times \text{electron velocity}}$$

$$= W_{DC}/2v_{sat} \leftarrow \quad v_{sat} \sim 10^7 \text{ cm/sec}$$

(0.1 μm/ps)

C_{cb} = base-collector junction capacitance

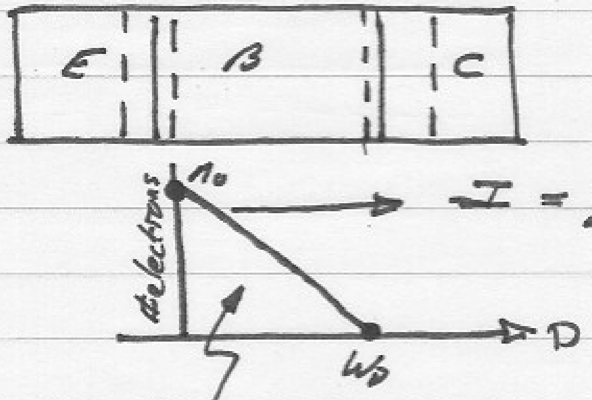
$C_{be}(depl)$ = emitter-base depletion capacitance.

R_{ex} = emitter parasitic resistance.

$R_{b'b}$ = base parasitic series resistance.

(draw pictures of these)

τ_{base}



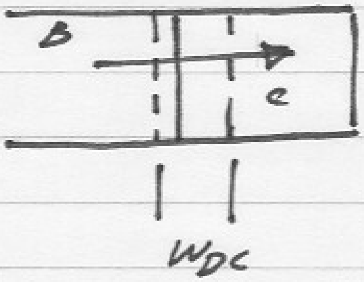
$$I = qD_nA\frac{\partial n}{\partial x} = qD_nA\frac{n_0}{W_b}$$

$$Q_s = qAn_0W_b/2$$

$$\Rightarrow \tau_b \triangleq Q_s/I = W_b^2/2D_n + W_b/v_{sat}$$

~~Suprisingly, the W_b/v_{sat} term is dominant in modern HBTs.~~

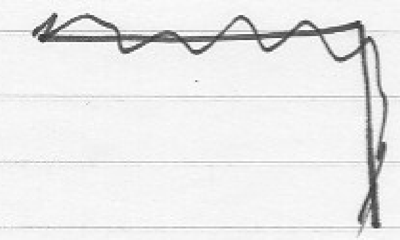
τ_c : analysis is a little harder (we will omit)

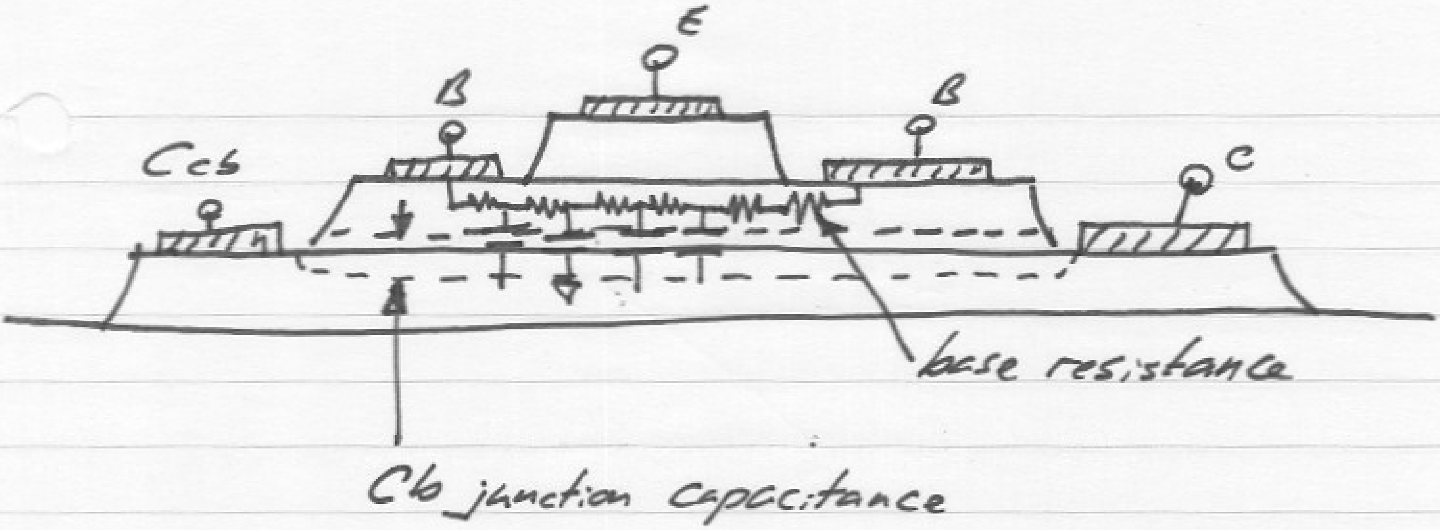


detailed analysis is necessary to get the (1/2) factor.

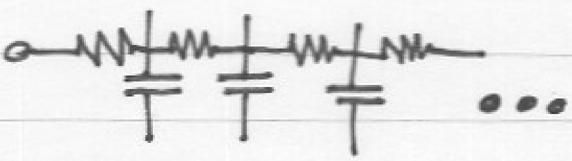
$$\frac{Q_{stored}}{I} = W_{DC}/v_{sat}$$

~~[illegible]~~

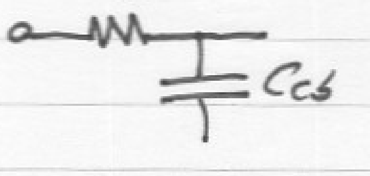




* note that base resistance & C_{cb} are a distributed network, thus:



* for simplicity, this is replaced by a lumped R-C network with 2-4 elements



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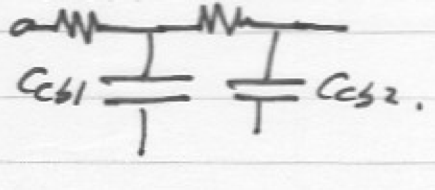
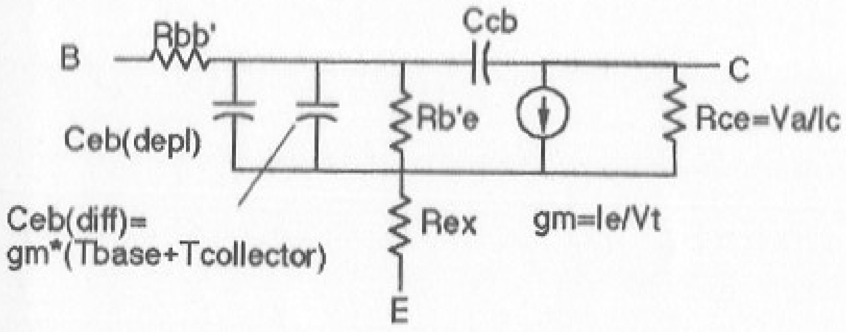
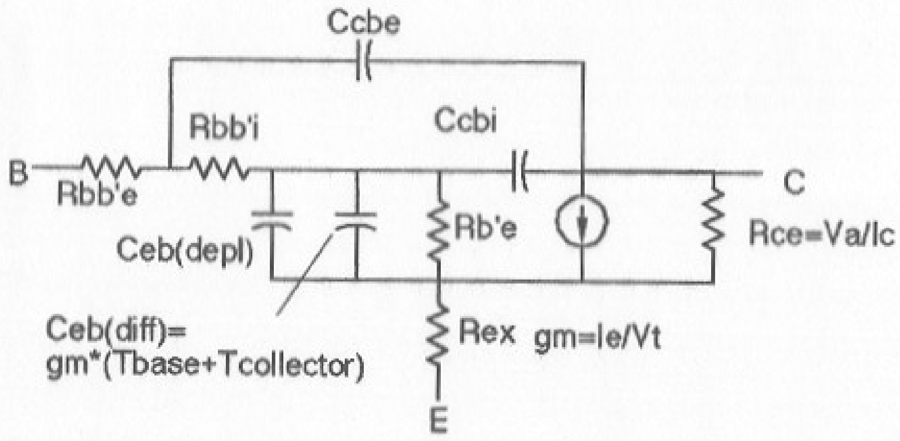
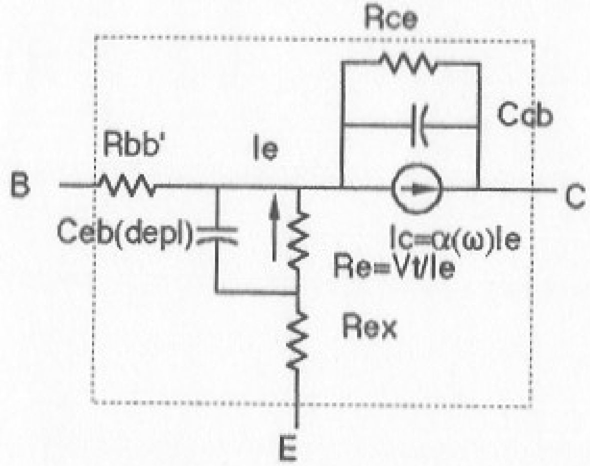


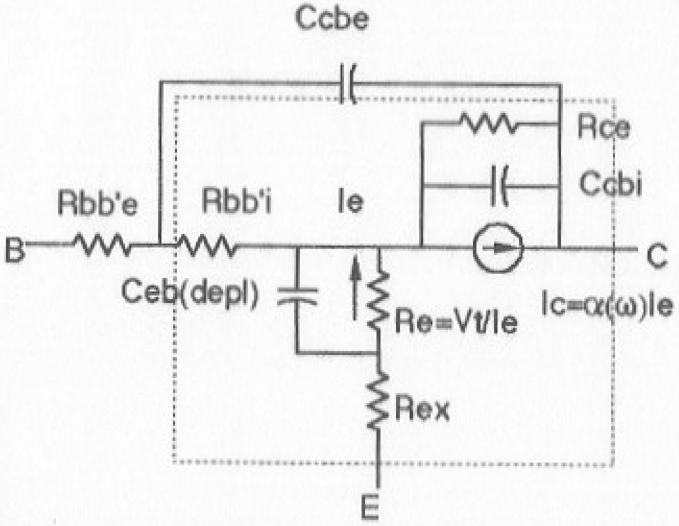
Hybrid-π Models (derived)





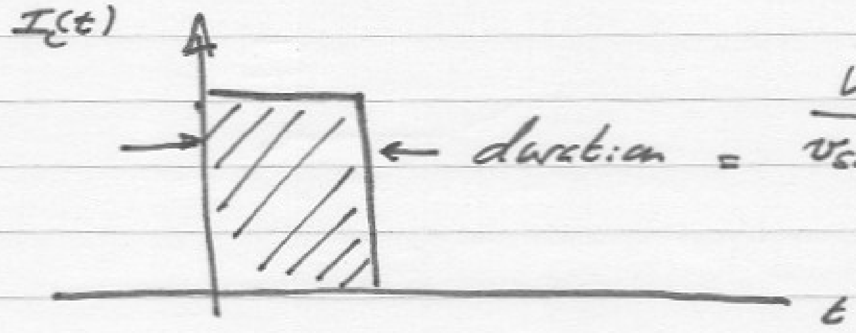
T-Models : physically-derived.





$$\alpha(\omega) \cong \alpha_0 \cdot \frac{1}{1+j\omega\tau_b} \cdot e^{-j\omega\tau_c} \cdot \frac{\sin(2\omega\tau_c)}{2\omega\tau_c}$$

If an impulse of charge enters the collector-space-charge region from the base, the collector terminal current is

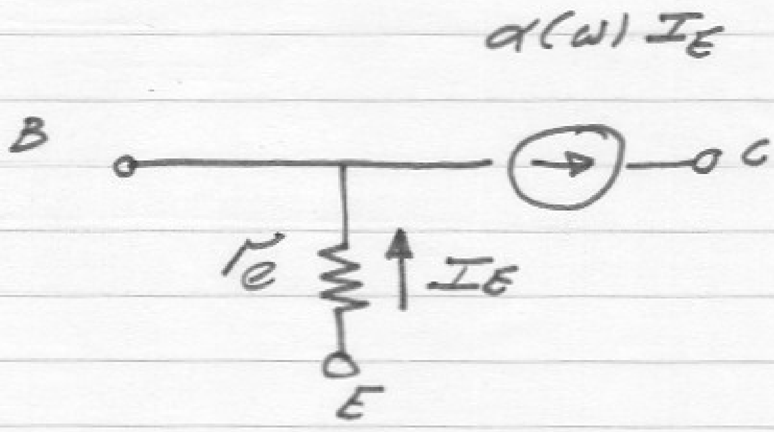


← duration $= \frac{W_c}{v_{sat}} = 2\tau_c$

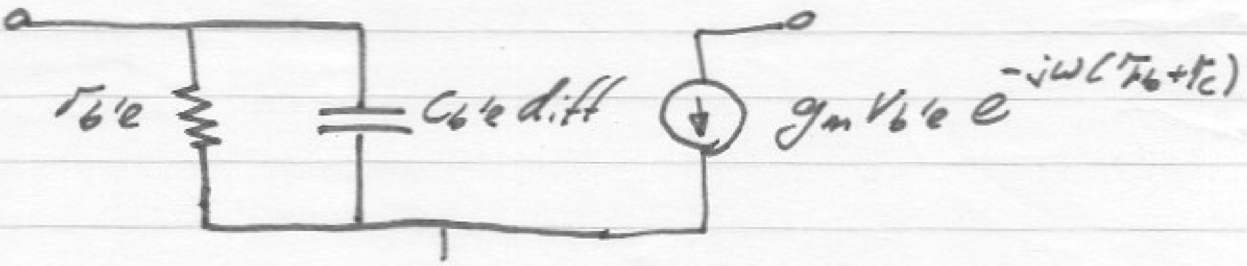
the Fourier transform of this is

$$e^{-j\omega\tau_c} \cdot \frac{\sin(2\omega\tau_c)}{2\omega\tau_c}$$

How do we arrive at the <u>hybrid-π</u> model? We start with the T-model, and separate out the network below:



We compute the Y-parameters of this network, approximate with a Taylor Series to first order in $\omega(\tau_{base} + \tau_{collector})$ and obtain

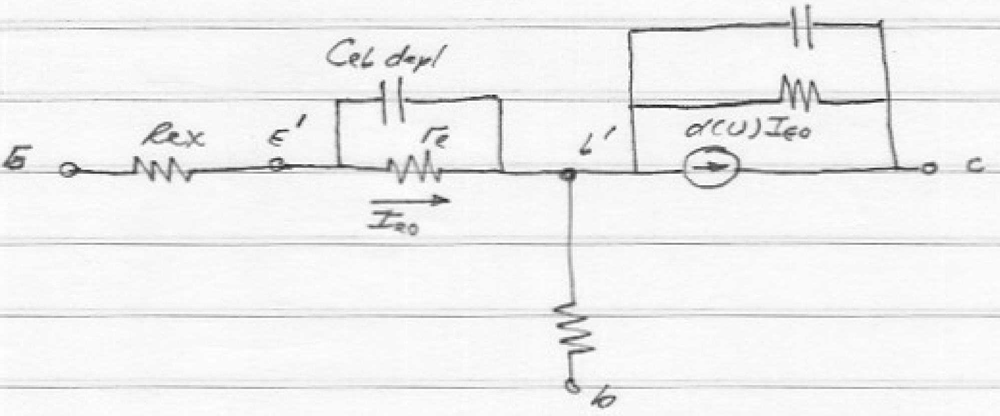


and add back the remaining elements.

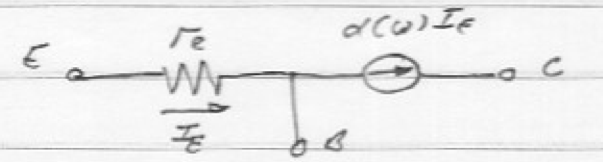
This is illustrated on the (optional) following pages.

Going between common-base T-model & hybrid-π

<u>full model:</u>



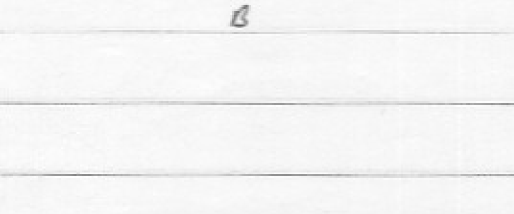
first work only with E'B'C transistor, and put aside $C_{cb'}$ & $C_{Eb'\,depl}$ for the time being.



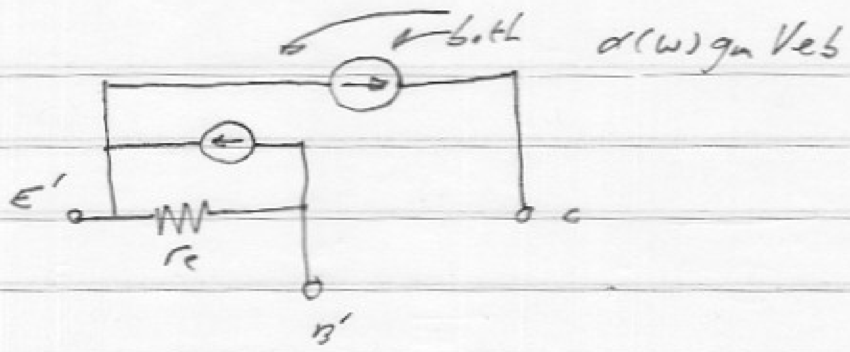
$$\boxed{r_e = V_T / I_{E\,dc}}$$

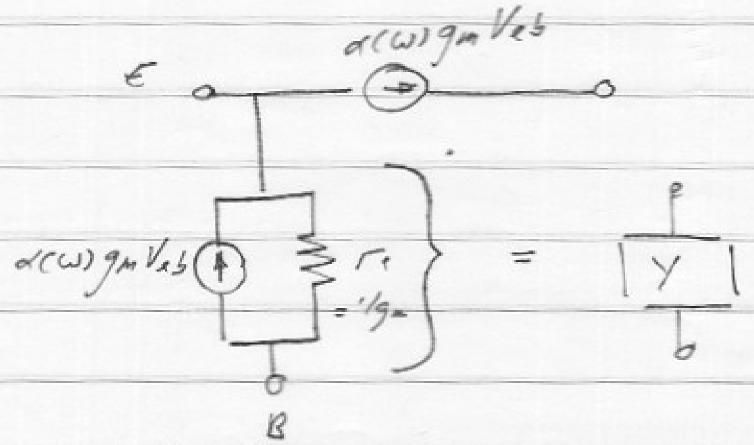
$$\alpha(\omega) = \frac{\alpha_0}{1 + j\omega\tau_{base}} e^{-j\omega\tau_{collector}} \times \frac{\sin \omega 2\tau_{collector}}{2\omega\tau_{collector}}$$

$$\cong \frac{\alpha_0}{1 + j\omega\underbrace{(\tau_{base} + \tau_{collector})}_{\tau_f}}$$



$g_m = 1/r_e$





$$Y = g_m \left[1 - \alpha(\omega) \right] \cong g_m \left[\frac{1 + j\omega\tau_f - \alpha_0}{1 + j\omega\tau_f} \right]$$

$$= g_m(1 - \alpha_0) \quad \frac{1 + j\omega\tau_f/(1-\alpha_0)}{1 + j\omega\tau_f}$$

$$\frac{1}{1-\alpha_0} = \beta_0$$

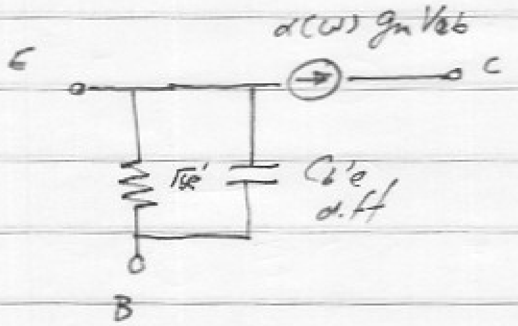
$$= \frac{g_m}{\beta} \quad \frac{1 + j\omega\beta\tau_f}{1 + j\omega\tau_f}$$

$$\cong \frac{g_m}{\beta} + j\omega g_m \tau_f \quad \text{for } f < f_T$$

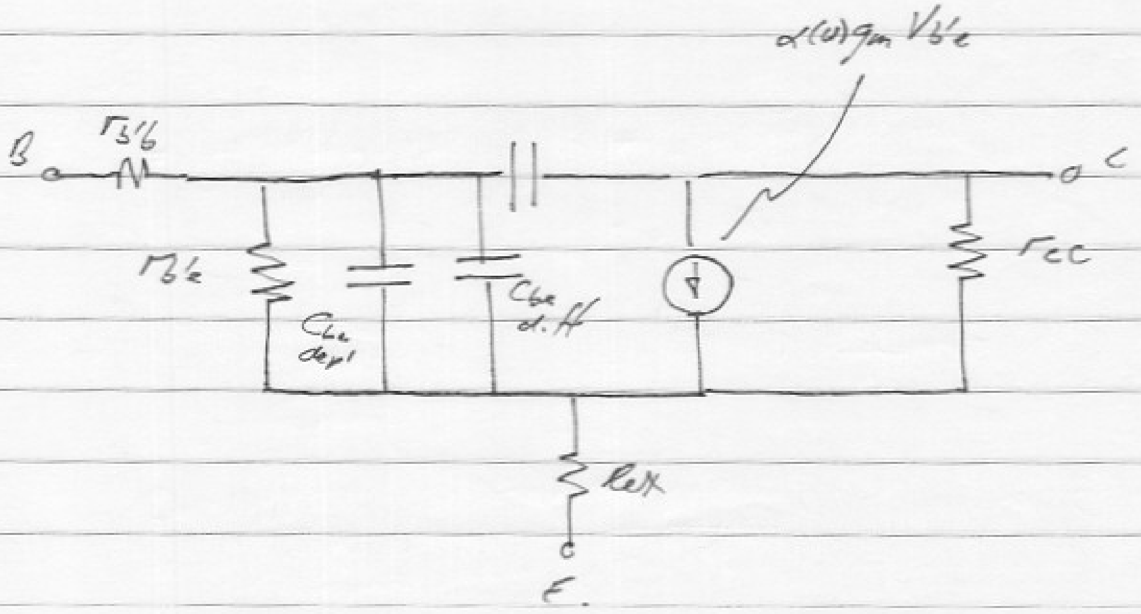
$$Y = \frac{1}{r_{b'e}} + j\omega C_{b'e_{diff}}$$

$$C_{b'e\,diff} = g_m \tau_f$$

$$r_{b'e} = \beta / g_m$$



redraw, and re-add other parasitics:



Device figures-of-merit

short-circuit current gain cutoff frequency, f_τ

Rigorous definition: $\| H_{21}(f_\tau) \| = 1$

FET: $$f_\tau \approx \frac{g_m}{2\pi(C_{gs}+C_{gd})} \cong \frac{g_m}{2\pi C_{gs}} \simeq \frac{v_{sat}}{l_{gate}}$$

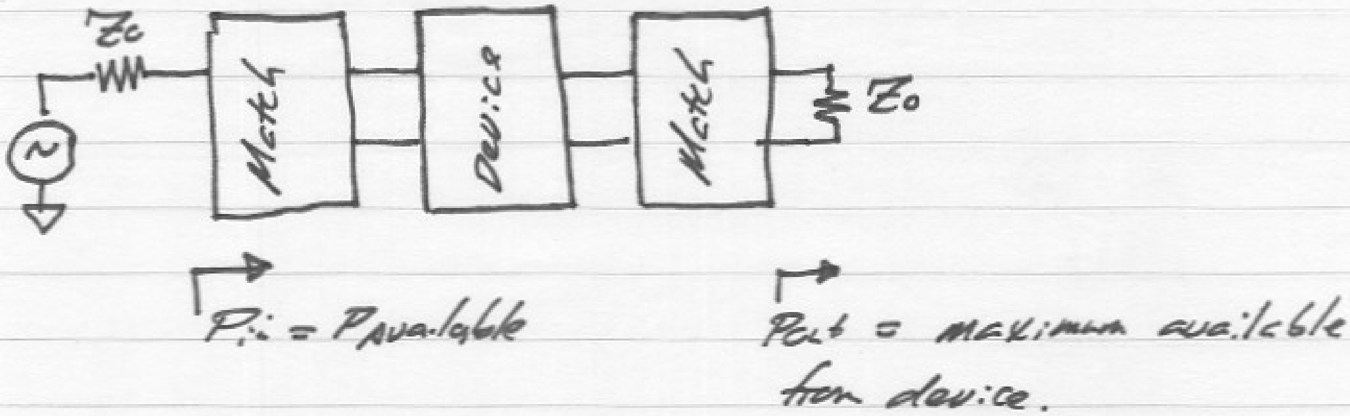
Bipolar: $$f_\tau \approx \frac{g_m}{2\pi(C_{be\,depl}+C_{be\,diff}+C_{bc})}$$

$$= \frac{1/2\pi}{\underbrace{r_e C_{be\,depl}}_{\text{"emitter charging time"}} + \underbrace{r_e C_{bc}}_{\text{"collector charging time"}} + \tau_{base} + \tau_{collector}}$$

($r_e = 1/g_m$)

expressions are approximate & assume "typical" magnitudes of various parasitics.

Maximum available gain (if it exists!)



we will discuss this much more carefully later.

for now, $G_{max} = P_{out}/P_{in}$ given Matching on both input and output. G_{max} (if it exists) is the maximum power gain we can obtain.

G_{max} decreases with frequency.

when $G_{max} \geq 1$, we can use the device to build an amplifier or oscillator. If $G_{max} < 1$, the device is useless. f_{max} is the frequency at which $G_{max} = 1$. Hence:

$f < f_{max}$ device is useful.
$f > f_{max}$ device is useless.

Again, for "typical" values, such that secondary parasitic effects are negligible, we have:

for FET's

$$f_{max} \simeq \frac{f_T}{2\sqrt{(R_i + R_g + R_s)G_{ds} + 2\pi f_T R_g C_{gd}}}$$

for Bipolars ($\beta >> 1$)

$$f_{max} \simeq \frac{f_T}{2\sqrt{\underbrace{(R_{ex} + R_{b'b})/R_{CE}}_{\text{usually negligible}} + 2\pi f_T r_{b'b} C_{bc}}}$$

$$= \sqrt{\frac{f_T}{8\pi \underbrace{r_{b'b} C_{bc}}_{\text{"base-collector time constant"}}}}$$