ECE145B (undergrad) and ECE218B (graduate)
Final Exam. Tuesday March 21, 2023

Do not open exam until instructed to.
Open notes, open books, etc.
You have 3 hrs.

Use all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***
Hint: Stop and think before doing complicated calculations. ***For some problems, there is an easier way.***

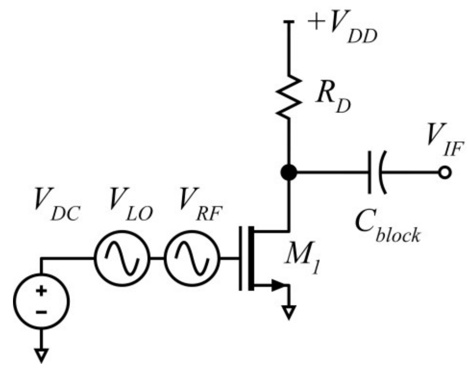
Problem	Points Received	Points Possible (145B)	Points Possible (218B)
1a		10	10
1b		5	5
1c		5	5
1d		10	10
1e		10	10
2a		Do not work	10
2b		Do not work	5
2c		10	10
2d		5	5
2e		10	10
2f		5	5
3a		5	5
3b		5	5
3c		10	10
Total		**90**	**105**

*****Assume T=290 Kelvin for all noise calculations.*****

Name: ______________________

Problem 1, 40 points (218B), 40 points (145B)
mixers and frequency conversion:

part a, 10 points
Basic mixer operation



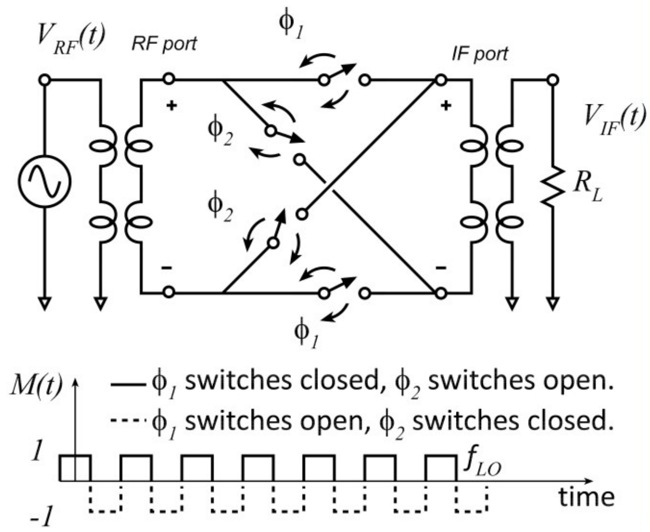
The MOSFET is has $I_D = K_\mu (V_{gs} - V_{th})^2$ with V_{th}=0.3V and K_μ=10 mA/V^2. The gate bias voltage V_{DC}=0.4 V, $V_{LO}(t) = V_{LO}\cos(2\pi f_{LO} t)$, $V_{RF}(t) = V_{RF}\cos(2\pi f_{RF} t)$, with V_{LO}=0.1 V, f_{LO}=1 GHz, V_{RF} = 10 mV, and f_{RF}=1.01 GHz. V_{DD} is sufficiently large for the transistor to be operating in saturation (above the V_{DS} knee voltage), and R_D = 1 kΩ. The blocking capacitor is an AC short-circuit. Find the RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$.

RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$ = _____________

part b, 5 points

Basic mixer operation

The 4-switch mixer is an idealized representation of a passive FET ring mixer. The switches operate at f_{LO} =1 GHz .



$V_{RF}(t) = V_{RF}\cos(2\pi f_{RF} t)$, V_{RF} = 10 mV, and f_{RF} =**1**.01 GHz.

R_L =50 Ω.

Find the RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$.

Hint, the Fourier series of M(t) is

$$M(t) \ = \frac{4}{\pi}\left[\cos(2\pi f_{LO}t) + \frac{\cos(3\cdot 2\pi f_{LO}t)}{3} + \frac{\cos(5\cdot 2\pi f_{LO}t)}{5} + ...\right]$$

RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$ = ____________

part c, 5 points

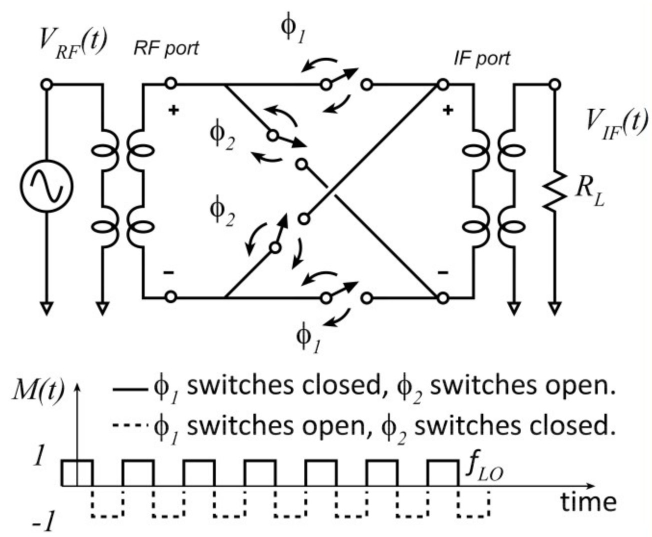
Harmonic mixing

The 4-switch mixer is an idealized representation of a passive FET ring mixer. The switches operate at f_{LO}=1 GHz .

$V_{RF}(t) = V_{RF}\cos(2\pi f_{RF} t)$, V_{RF} = 10 mV, and f_{RF} =**3**.01 GHz.

R_L=50 Ω.

Find the RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$.



RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$ = ____________

part d, 10 points

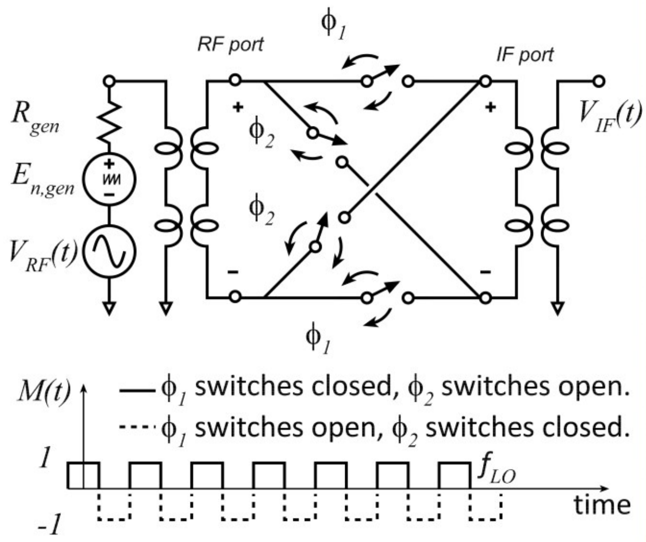
Mixer image noise response

We remove the IF load resistance, but add a generator resistance R_{gen} =50 Ω.

f_{LO} =1 GHz . f_{RF} =**1**.01 GHz.

The spectral density of $E_{n,gen}$ is

$S_{E_{n,gen}E_{n,gen}} = 4kTR_{gen}$



$V_{RF}(t)$ is a random information signal with spectral density $S_{V_{RF}V_{RF}} = 4kTR_{gen} \cdot 10^3$ over 9.9-10.1 MHz, but zero outside with bandwidth. (i.e. the signal/noise ratio is 30 dB in the 9.9-10.1 MHz bandwidth)

1) Writing $V_{IF}(t) = V_{IF,signal}(t) + V_{IF,noise}(t)$, find the spectral densities $S_{V_{IF,signal}V_{IF,signal}}(f)$ and $S_{V_{IFnoise}V_{IF,noise}}(f)$ **at 10 MHz** (not at other frequencies).

2) From this, calculate the mixer noise figure including all mixer noise image responses.
Hint: $(1/1)^2 + (1/3)^2 + (1/5)^2 + (1/7)^2 + ... = \pi^2 / 8$

$S_{V_{IF,signal}V_{IF,signal}}(f)$ at 10 MHz = ____________________ (V^2/Hz)

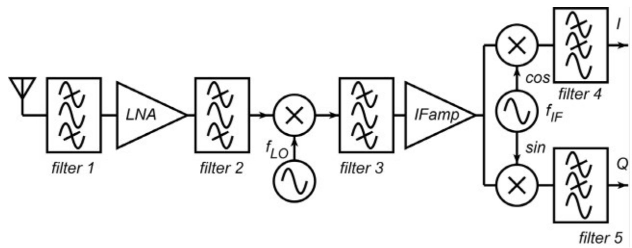
$S_{V_{IFnoise}V_{IF,noise}}(f)$ at 10 MHz = ____________________ (V^2/Hz)

Noise figure = ____________(linear) = ____________ (dB)

part e, 10 points

Image responses in receivers.

f_{LO} =190 GHz , $f_{RF} = 200 \pm 1$ GHz (i.e. has 2 GHz modulation bandwidth centered at 200 GHz), f_{IF} =10 GHz. T=290 Kelvin. The LNA has $F = 10$ dB; all other components have $F = 0$ dB.



Filter 1 has a 200 ± 30 GHz passband,
filter 3 has a 10 ± 1 GHz passband,
filters 4,5 have a 0-1 GHz passband

1) If filter 2 has a 200 ± 30 GHz passband, what is the receiver noise figure ?

noise figure = __________ (dB)

2) If filter 2 has a 200 ± 10 GHz passband, what is the receiver noise figure ?

noise figure = __________ (dB)

Problem 2, 45 points (218B), 30 points (145B)
nonlinearities, harmonic generation and intermodulation generation. :

<u>part a, 10 points **ECE218B only**</u>
Circuit nonlinearity analysis (somewhat difficult)
With ideal transistors,

$$V_{out} = 2I_0R_L \frac{\exp(V_{in}/V_T)-1}{\exp(V_{in}/V_T)+1}$$ where

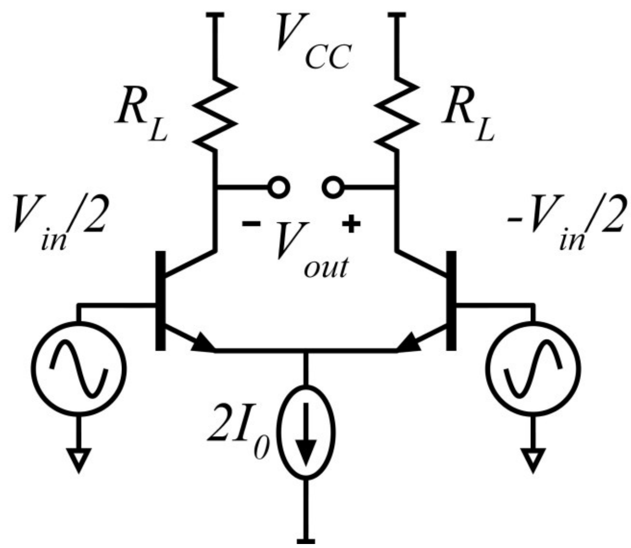
$V_t = kT/q$

To third order

$e^x = 1 + x + x^2/2 + x^3/6 + O(x^4)$ and

$\frac{1}{1+y} = 1 - y + y^2 - y^3 + O(y^4)$ and

$V_{out} = a_1V_{in} + a_3V_{in}^3 + O(V_{in}^5)$



Can you find a_1 and a_3 ?

Hint: to keep the math tractable, at each calculation, drop any term higher in order than V_{in}^3.

$a_1 =$ ________________________ $a_3 =$________________________

part b, 5 points **ECE218B only**

Circuit nonlinearity analysis

We can also write (with $V_t = kT/q$)

$V_{in} = V_t \ln(1 + V_{out}/2I_0R_L)$

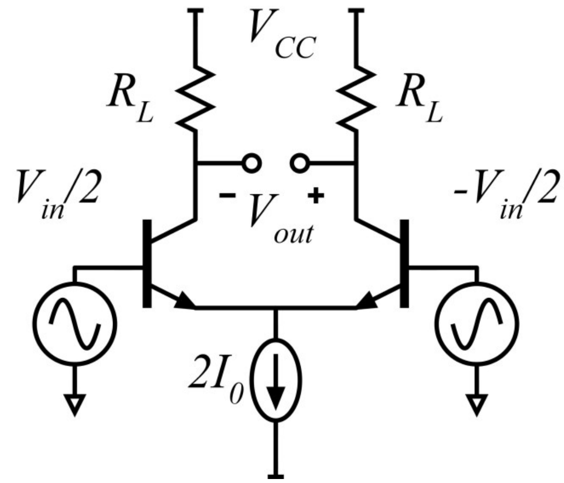
$\quad - V_t \ln(1 - V_{out}/2I_0R_L)$

hence $V_{in} = b_1 V_{out} + b_3 V_{out}^3 + O(V_{out}^5)$.

Use the expansion

$\ln(1+x) = x - x^2/2 + x^3/3 + O(x^4)$

to find b_1 and b_2.



If $V_{in} = b_1 V_{out} + b_3 V_{out}^3 + O(V_{out}^5)$ then

$b_3(V_{in}/b_1)^3 \simeq \left(V_{out} + b_3 V_{out}^3/b_1\right)^3$

$\quad = b_3 V_{out}^3 + O(V_{out}^4)$

so $V_{in} - b_3(V_{in}/b_1)^3 = b_1 V_{out}$, hence $V_{out} = V_{in}/b_1 - b_3 V_{in}^3/b_1^4$

Comparing this to: $V_{out} = a_1 V_{in} + a_3 V_{in}^3 + O(V_{in}^5)$

we have: $a_1 = 1/b_1$ and $a_3 = -b_3/b_1^4$

Again, please find a_1 and a_3.

$a_1 =$ ________________________ $a_3 =$________________________

part c, 10 points

nonlinear amplification of sine waves

$V_{in1}(t) = 2^{0.5} V_{RMS1} \cos(2\pi f_1 t)$

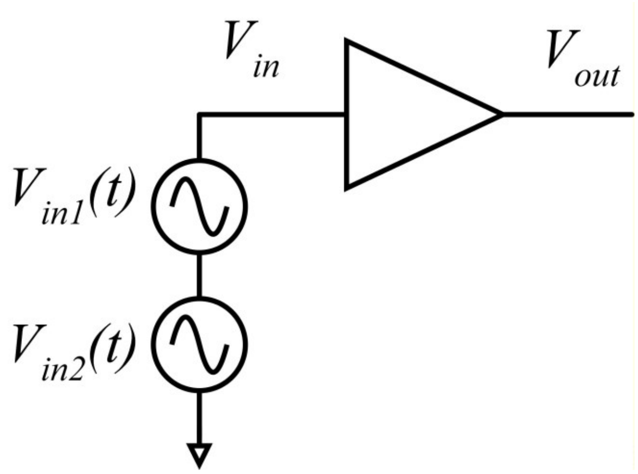
$V_{in2}(t) = 2^{0.5} V_{RMS2} \cos(2\pi f_2 t)$

$V_{in}(t) = V_{in1}(t) + V_{in2}(t)$

$V_{RMS1} = V_{RMS2} = V_{RMS}$

$V_{out} = a_1 V_{in} + a_2 V_{in}^2$

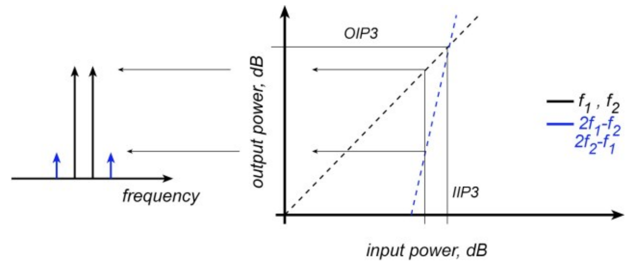
find the *second order* intercept, i.e. the value of V_{RMS} at which the Fourier component of V_{out} at $(f_1 + f_2)$ is equal to the Fourier component of V_{out} at f_1.



part d, 5 points

Third-order distortion levels

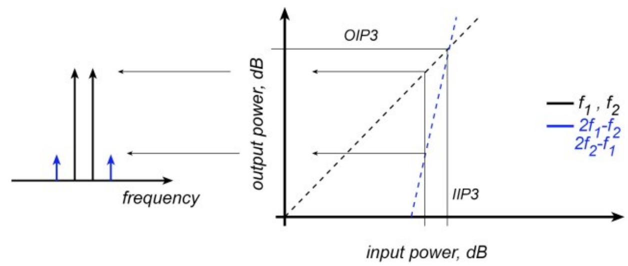
An amplifier has 10 dB gain and a +10 dBm third-order intercept. Two input signals at f_1 and f_2 are applied, both with -20 dBm power. Find the output powers at f_1, f_2, $(2f_1 - f_2)$ and $(2f_2 - f_1)$



part e, 10 points

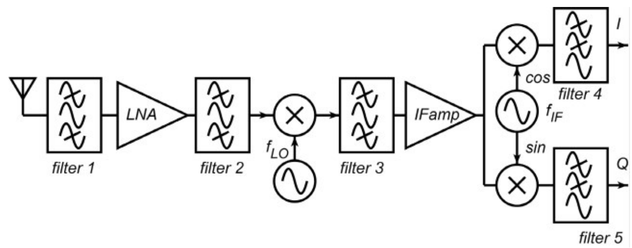
Third-order distortion levels

An amplifier has 10 dB gain and a +10 dBm third-order intercept. Two input signals at f_1 and f_2 are applied, with power levels -10 dBm and -20 dBm respectively. Find the output powers at f_1, f_2, $(2f_1 - f_2)$ and $(2f_2 - f_1)$



part f, 5 points

Receiver dynamic range calculation. A receiver is designed for 2 GHz RF signal frequency. The RF signal power, at sensitivity, is -60 dBm, i.e. 10^{-9} W. The receiver has a -20 dBm input-referred third order intercept. There are two interfering radio stations, one at 1.8 GHz, one at 1.9 GHz.



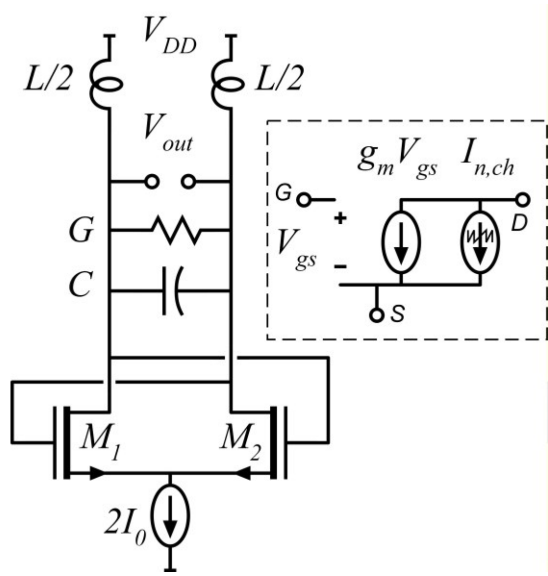
What RF signal power for the interfering signals would result in a 30 dB carrier-to-interference ratio, i.e. the 2 GHz IM3 product from the 2 interferers is 30 dB below the desired RF signal ?

Problem 3, 20 points (218B), 20 points (145B)
Oscillators and phase noise:

part a, 5 points
Oscillator design principles.

The MOSFETs have $I_D = K_\mu (V_{gs} - V_{th})^2$ with V_{th}=0.3 V and K_μ =8 mA/V^2. There are no parasitic capacitances or resistances. I_0= 1 mA. Note that the transistors operate with a DC bias of V_{GD} = 0, hence $V_{DS} = V_{GS}$, whereas the knee (saturation) voltage is $V_{DS,\text{"knee"}} = V_{DS,sat} = V_{GS} - V_{th}$, consequently the maximum peak-peak oscillator output voltage is $V_{LO}(t) = V_{LO,\max}\cos(2\pi f_{LO} t)$ with $V_{LO,\max} = V_{th}/2$.

Set $Z_r = L^{1/2}/C^{1/2}$=100Ω, and $f_{LO} = 1/2\pi L^{1/2} C^{1/2}$ = 10 GHz.



1) What is the FET transconductance ?

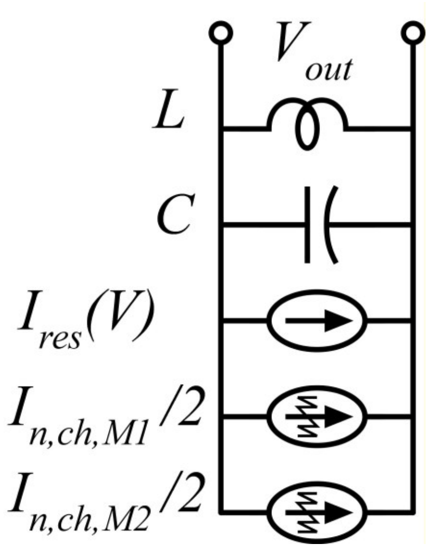
2) What is the negative conductance $-a_1$ presented by the FETs to the resonator ?

3) What is the maximum (positive) load conductance G that will maintain oscillation ?

part b, 5 points

Oscillator phase noise.

The circuit is modelled as is shown to the right, with $I_{res}(V) = a_1 V + a_3 V^3$ and with channel noise generators $I_{n,ch,M1}$ and $I_{n,ch,M2}$ having spectral densities $S_{I_{n,ch1}I_{n,ch1}} = 4kT\Gamma g_{m1}$ and $S_{I_{n,ch2}I_{n,ch2}} = 4kT\Gamma g_{m2}$, where Γ=2/3 Note the factors of (1/2) in the circuit diagram.



From the expression

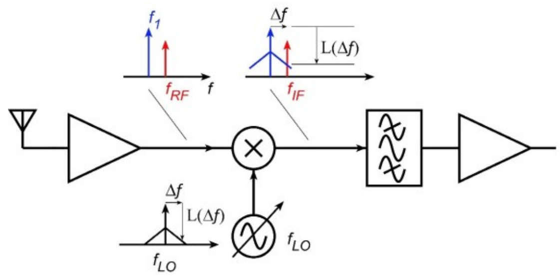
$$L(\Delta f) = \left(\frac{Z_r}{4V_0}\right)^2 \left(\frac{f_{LO}}{\Delta f}\right)^2 S_{I_n I_n}(f)$$

find the phase noise spectral density at 100 MHz offset from carrier, expressed as xxx dBc (1 Hz). Note that $V_0 = V_{th}/2$ =0.15 V (peak, not RMS)

part c, 10 points

Impact of phase noise

A receiver is designed to receive signals in the 2 GHz ± 1 MHz RF frequency range. The receiver noise figure is 4 dB. The receiver's local oscillator has -120 dBc (1 Hz) phase noise at 100 MHz offset from carrier. There is an interfering radio station at 1.9 GHz.



What RF power for this 1.9 GHz interfering signal would result in 10 dB sensitivity degradation in receiving the 2 GHz ± 1 MHz signals of interest ?