ECE 145B / 218B, notes set 7: Distortion and Frequency Conversion

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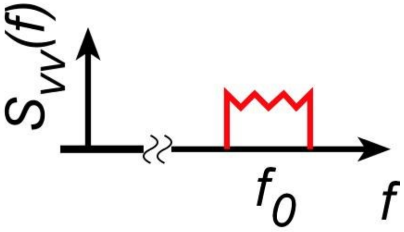
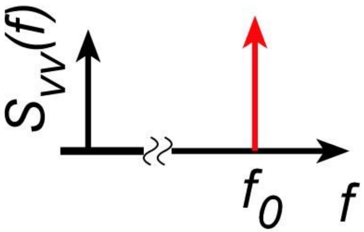
Signal Representation in Drawings

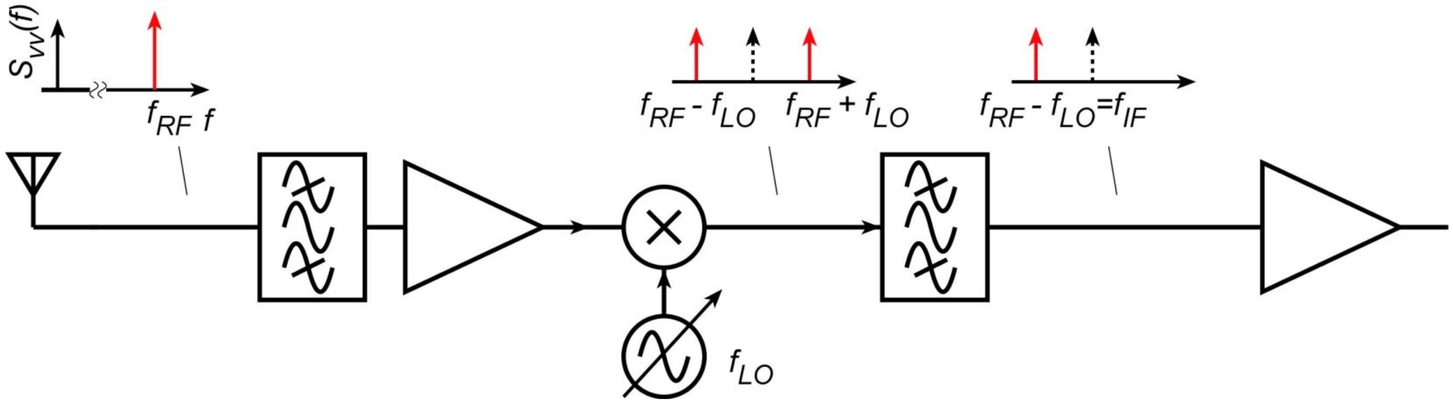
Our signals carry random information, and usually have continuous spectra



But many drawings, and the analysis, will be clearer if we represent these as discrete spectral lines (tones).



Frequency Plan in a Radio Receiver

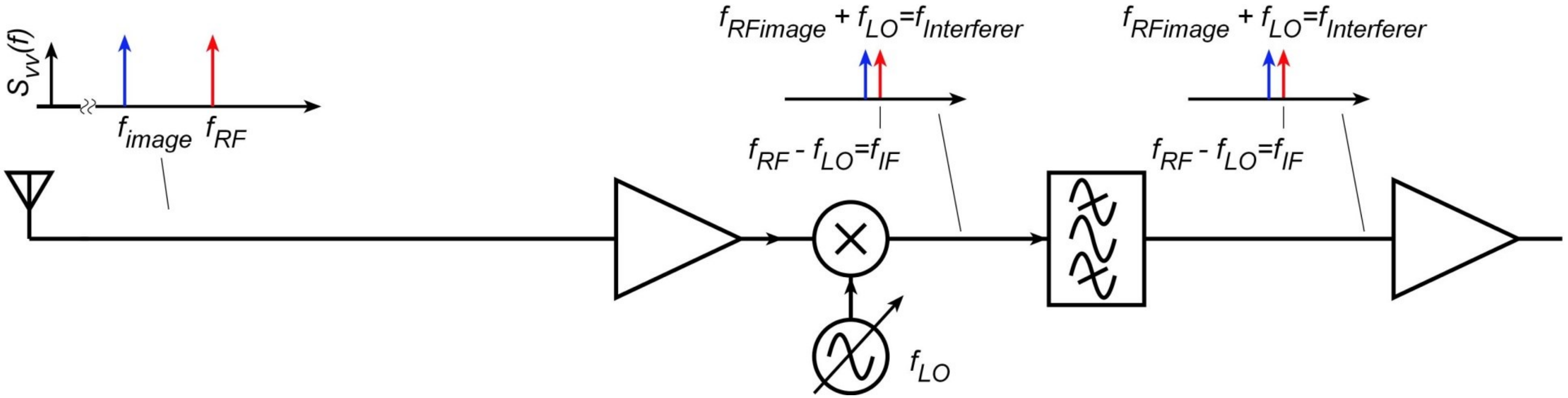


The mixer generates sum and difference frequencies $(f_{RF} - f_{LO})$ and $(f_{RF} + f_{LO})$.

One of these is, by design, the intermediate frequency (IF).

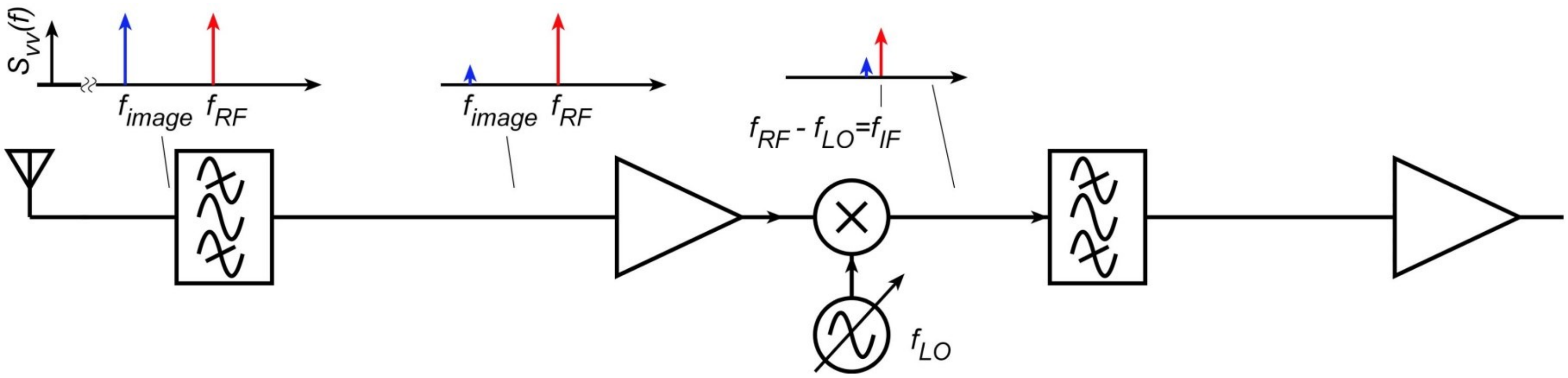
The other is rejected by the IF filter.

Image Response (1)



Assuming that by design $f_{IF} = (f_{RF} - f_{LO})$, hence $f_{RF} = f_{IF} + f_{LO}$
a signal at $f_{\text{Image}} = f_{LO} - f_{IF}$ will also mix to f_{IF}.

Image Response (2)

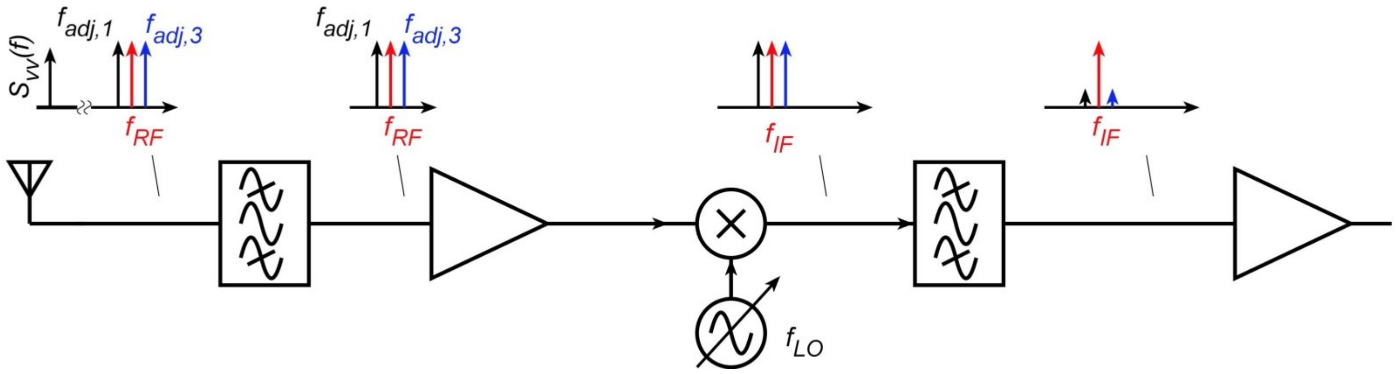


To reject this image response, we choose an RF filter whose bandwidth is (significantly) less than $2 \cdot f_{IF}$.

In a tunable reciver, both the LO and the RF filter must be tuned.

Tracking of the tuning of these 2 elements is made easy if f_{IF} is large.

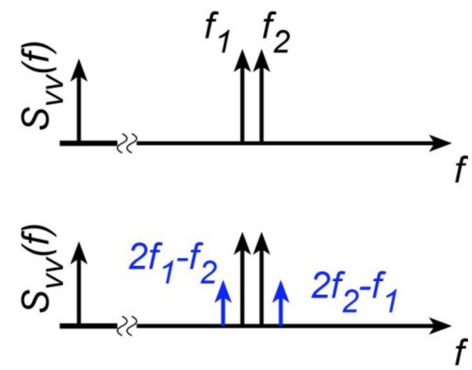
Adjacent Channel Interference



Given the need of the RF filter to be tuned,
it cannot have narrow bandwidth.

Interfering signals in adjacent frequency bands are rejected
by the narrow-bandwidth IF filter.

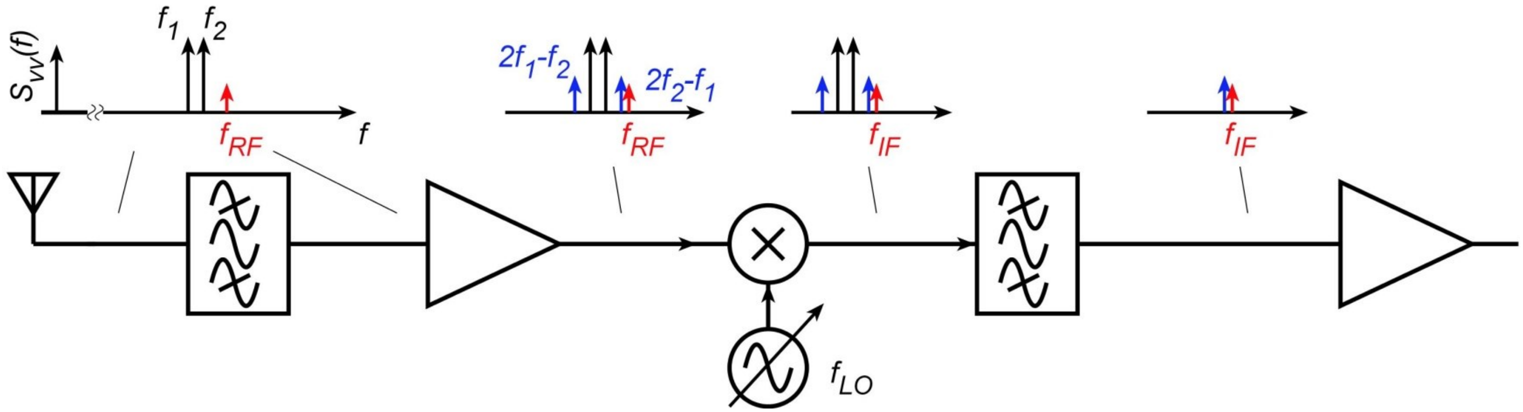
Interference by Intermodulation (1)



We will soon show that, given *odd-order distortion*, strong input signals at frequencies f_1 and f_2 will generate distortion tones at frequencies $(2f_1 - f_2)$ and $(2f_2 - f_1)$.

These distortion tones will also cause interference.

Interference by Intermodulation (2)

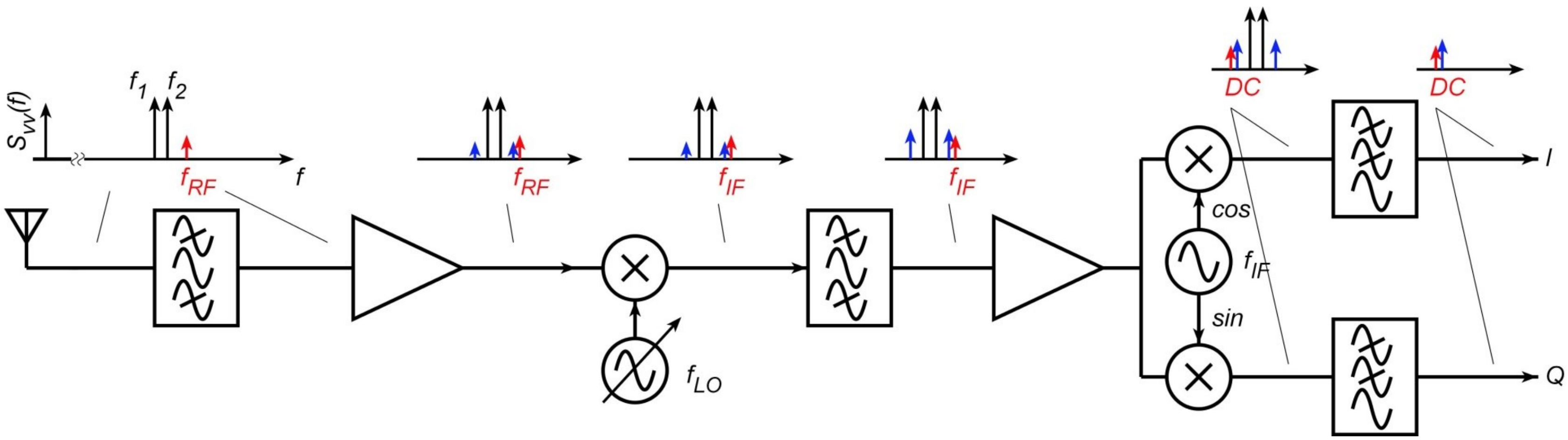


If the tones at f_1 and f_2 lie outside the passband of the RF filter, they will be rejected, and the receiver will suffer no interference.

If the tones at f_1 and f_2 lie within the RF filter the passband, then tones at $(2f_1 - f_2)$ and $(2f_2 - f_1)$ will be produced by the RF stage and mixer.

After the mixer, the original interfering tones (f_1, f_2) may be translated to frequencies outside the IF filter passband, yet either $(2f_1 - f_2)$ or $(2f_2 - f_1)$ may lie within the IF filter passband, and will interfere with the desired signal.

Interference by Intermodulation (3)

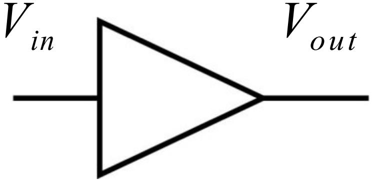


In a dual-conversion receiver, the strong interferers at f_1 and f_2 may lie inside the passbands of both the RF *and* IF filters.

The IF amplifiers have very high gain; in this case, it is in the IF stages that strong intermodulation may occur.

The intermodulation products may lie within the baseband filter's passband, even the interfering signals themseleves lie outside it.

Distortion and Intermodulation



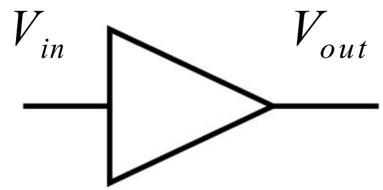
Consider now an amplifier with $V_{out} = f(V_{in})$.

We can now write a Taylor series for V_{out}.

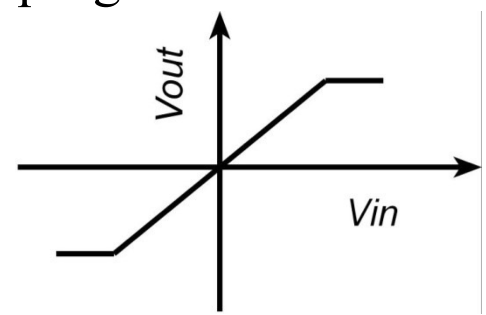
$$V_{out}(V_{in,DC} + \delta V_{in}) = V_{out,DC} + \frac{d\ V_{out}}{dV_{in}} \cdot (\delta V_{in}) + \frac{1}{2}\frac{d^2 V_{out}}{dV_{in}^2} \cdot (\delta V_{in})^2 + \frac{1}{6}\frac{d^3 V_{out}}{dV_{in}^3} \cdot (\delta V_{in})^3 + \ldots$$

$$V_{out}(V_{in} + \delta V_{in}) = a_0 + a_1 \cdot (\delta V_{in}) + a_2 \cdot (\delta V_{in})^2 + a_3 \cdot (\delta V_{in})^3 + \ldots$$

Limits to the Taylor Series Model



1) Taylor series are power series. Power series have a *radius of convergence* outside which they do not converge. Taylor series of hard clipping characteristics need many terms in the expansion and converge only slowly.

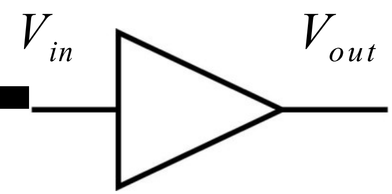


2) For circuits whose gain varies with frequency (all circuits in fact),

$$V_{out}(t) = f(V_{in}, dV_{in}/dt, d^2V_{in}/dt^2, \ldots, \int V_{in}(t)dt, \iint V_{in}(t)dt^2, \ldots)$$

In such cases the Taylor series must be replace with a Volterra series.
This is much more complex.

Linear Response.



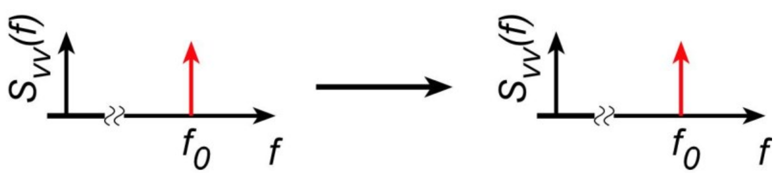
Consider now an amplifier with $V_{in}(t) = V_0\cos(\omega_0 t)$

First write $V_0\cos(\omega_0 t) = (V_0/2)\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) = (V_0/2)\left(z + z^{-1}\right)$

$V_{out} = a_0 + a_1\cdot(\delta V_{in}) + a_2\cdot(\delta V_{in})^2 + a_3\cdot(\delta V_{in})^3 + \ldots$

$a_1\cdot(\delta V_{in}) = a_1\cdot V_0\cos(\omega_0 t).$

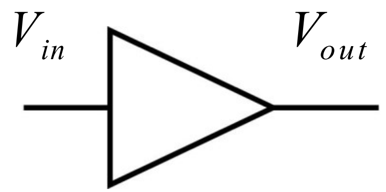
This is the linear response, with output frequencies equal to input frequencies



Quadratic Response and 2nd Harmonic Distortion

$$V_{in}(t) = V_0\cos(\omega_0 t) = (V_0/2)\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) = (V_0/2)\left(z + z^{-1}\right)$$

$$V_{out} = a_0 + a_1\cdot(\delta V_{in}) + a_2\cdot(\delta V_{in})^2 + a_3\cdot(\delta V_{in})^3 + \ldots$$



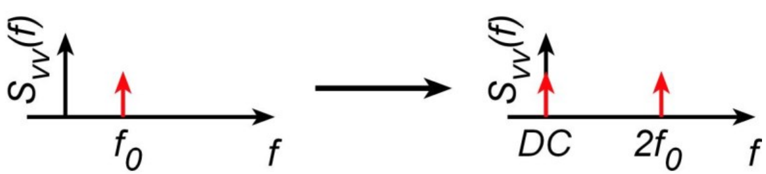
$$(\delta V_{in})^2 = (V_0/2)^2\left(z + z^{-1}\right)^2 = (V_0/2)^2\left(z^2 z^{-0} + 2z^1 z^{-1} + z^0 z^{-2}\right)$$

$$(\delta V_{in})^2 = (V_0/2)^2\left(z^2 + z^{-2}\right) + (V_0/2)^2(2)$$

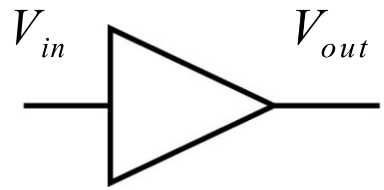
$$(\delta V_{in})^2 = (V_0/2)^2\cdot 2\cos(2\omega_0 t) + V_0^2/2$$

Quadratic response generates DC and the 2nd harmonic

Output amplitudes growth in proportion to the square of the input amplitude.



Cubic Response and 3rd Harmonic Distortion



$$V_{in}(t) = V_0 \cos(\omega_0 t) = (V_0/2)\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) = (V_0/2)\left(z + z^{-1}\right)$$

$$V_{out} = a_0 + a_1 \cdot (\delta V_{in}) + a_2 \cdot (\delta V_{in})^2 + a_3 \cdot (\delta V_{in})^3 + \ldots$$

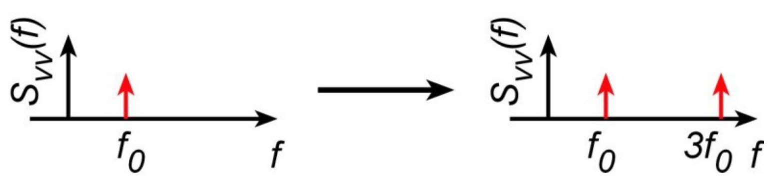
$$(\delta V_{in})^3 = (V_0/2)^3 \left(z + z^{-1}\right)^3 = (V_0/2)^3 \left(z^3 z^{-0} + 3z^2 z^{-1} + 3z^{-2} z^1 + z^0 z^{-3}\right)$$

$$(\delta V_{in})^3 = (V_0/2)^3 \left(z^3 + z^{-3}\right) + 3(V_0/2)^3 \left(z + z^{-1}\right)$$

$$(\delta V_{in})^3 = (V_0/2)^3 \cdot 2\cos(3\omega_0 t) + 3(V_0/2)^3 \cdot 2\cos(\omega_0 t)$$

Cubic repsonse generates response at fundamental and the 3nd harmonic.

Output amplitudes growth in proportion to the cube of the input amplitude.



4th- and 5th -order Responses

$$(\delta V_{in})^4 = (V_0/2)^4\left(z+z^{-1}\right)^4 = (V_0/2)^4\left(z^4z^{-0}+4z^3z^{-1}+6z^2z^{-2}+4z^1z^{-3}+z^0z^{-4}\right)$$

$$(\delta V_{in})^4 = (V_0/2)^4\left(z^4+z^{-4}\right)+(V_0/2)^4\left(4z^2+4z^{-2}\right)+(V_0/2)^4(6)$$

$$(\delta V_{in})^4 = (V_0/2)^4\cdot 2\cos(4\omega_0 t)+(V_0/2)^4\cdot 4\cdot 2\cos(2\omega_0 t)+(V_0/2)^4(6)$$

4^{th} - order repsonse generates DC and the 2nd and 4th harmonics

Output amplitudes growth in proportion to the 4th power of the input amplitude.

$$(\delta V_{in})^5 = (V_0/2)^5\left(z+z^{-1}\right)^5 = (V_0/2)^5\left(z^5z^{-0}+5z^4z^{-1}+10z^3z^{-2}+10z^2z^{-3}+5z^1z^{-4}+z^0z^{-5}\right)$$

$$(\delta V_{in})^5 = (V_0/2)^5\left(z^5+z^{-5}\right)+(V_0/2)^5\left(5z^3+5z^{-3}\right)+(V_0/2)^5\left(10z^1+10z^{-1}\right)$$

$$(\delta V_{in})^5 = (V_0/2)^5\cdot 2\cos(5\omega_0 t)+(V_0/2)^5\cdot 5\cdot 2\cos(3\omega_0 t)+(V_0/2)^5\cdot 10\cdot 2\cos(\omega_0 t)$$

5^{th} - order repsonse generates fundamental and the 3nd and 5th harmonics

Output amplitudes growth in proportion to the 5th power of the input amplitude.

General Picture....Nth Order responses

1st - order repsonses generate the 1st harmonic (i.e., the input)
Amplitudes grow in proportion to the 1st power of the input amplitude.

2nd - order repsonses generate the 0th, 2nd harmonics
Amplitudes grow in proportion to the 2nd power of the input amplitude.

3rd - order repsonses generate the 1st, 3rd harmonics
Amplitudes grow in proportion to the 3rd power of the input amplitude.

4th - order repsonses generate the 0th, 2nd, 4th harmonics
Amplitudes grow in proportion to the 4th power of the input amplitude.

5th - order repsonses generates 1st, 3rd, 5th harmonics
Amplitudes grow in proportion to the 5th power of the input amplitude..

Gain Compression

For a moment, let us look at the output *only* at the fundamental.

$$V_{out} = a_0 + a_1 \cdot (\delta V_{in}) + a_2 \cdot (\delta V_{in})^2 + a_3 \cdot (\delta V_{in})^3 + ...$$

$$V_{in}(t) = V_0 \cos(\omega_0 t)$$

Linear response $a_1 \cdot (\delta V_{in})^1 \rightarrow a_1 \cdot V_0 \cos(\omega_0 t)$.

Quadratic response $a_2 \cdot (\delta V_{in})^2 \rightarrow$ Responses at DC, $2\omega_0$, none at ω_0.

Cubic response $a_3 \cdot (\delta V_{in})^3 \rightarrow a_3 \cdot 3(V_0/2)^3 \cdot \cos(\omega_0 t)$, other response at $3\omega_0$.

4^{th}-order reponse $a_4 \cdot (\delta V_{in})^4 \rightarrow$ Responses at DC, $2\omega_0, 4\omega_0$, none at ω_0.

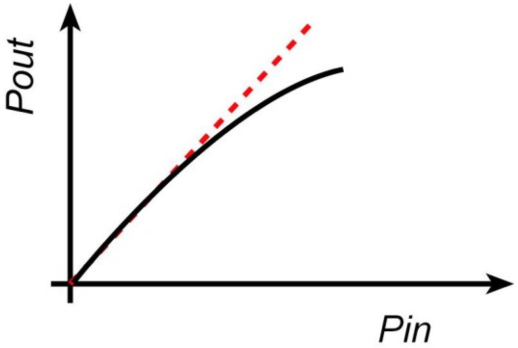
5^{th}-order reponse $a_5 \cdot (\delta V_{in})^5 \rightarrow a_5 \cdot (V_0/2)^5 \cdot 10 \cdot 2\cos(\omega_0 t)$, other responses at $3\omega_0$, $5\omega_0$.

$$V_{out} = \cos(\omega_0 t).\left(a_1 \cdot V_0 + a_3 \cdot 6(V_0/2)^3 + a_5 \cdot (V_0/2)^5 + ...\right),$$ responses at other frequencies.

This appears as saturation of the amplifier output power.

1 dB gain compression point: when gain has dropped by 1 dB....

Generally associated with hard clipping (cutoff, saturation, etc)



1dB Gain Compression Point: Cubic Model Only

$$V_{out} = a_1 \cdot V_{in}(t) + a_2 \cdot V_{in}^2(t) + a_3 \cdot V_{in}^3(t) + ... = a_1 \cdot \left(V_{in}(t) + b_2 \cdot V_{in}^2(t) + b_3 \cdot V_{in}^3(t) + ... \right)$$

$$V_{in}(t) = V_0 \cdot \cos(\omega_0 t)$$

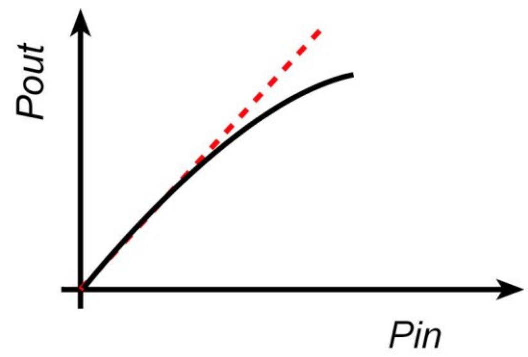
$$V_{out} = a_1 \cdot \cos(\omega_0 t) \cdot \left(V_0 + b_3 \cdot 6(V_0 / 2)^3 + b_5 \cdot (V_0 / 2)^5 + ... \right),\ \text{responses at other frequencies.}$$

1dB gain compression=0.891:1 reduction in gain

$$b_3 \cdot (3/4) V_0^2 = 0.1087$$

$$V_0^2 = 0.145 / b_3$$

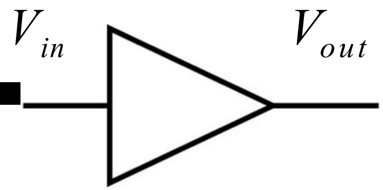
$$P_{1dB} = V_0^2 / 2Z_0 = 0.072 / b_3 Z_0$$



The above assumes a cubic-only model, i.e. that the V_{in}^5, V_{in}^7, etc. terms are negligible.

In general, this is *not* a good asssumption for an amplfier approaching hard clipping

Two-Tone Intermodulation (1)



Consider now an amplifier with $V_{in}(t) = V_0 \cos(\omega_1 t) + V_0 \cos(\omega_2 t)$

$$V_{in}(t) = (V_0/2)\left(e^{j\omega_1 t} + e^{-j\omega_1 t}\right) + (V_0/2)\left(e^{j\omega_2 t} + e^{-j\omega_2 t}\right) = (V_0/2)\left(z_1 + z_1^{-1} + z_2 + z_2^{-1}\right)$$

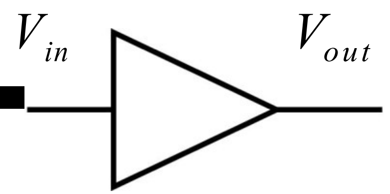
$$V_{out} = a_0 + a_1 \cdot (\delta V_{in}) + a_2 \cdot (\delta V_{in})^2 + a_3 \cdot (\delta V_{in})^3 + \ldots$$

$$(\delta V_{in}) = V_0 \cos(\omega_1 t) + V_0 \cos(\omega_2 t)$$

Again, linear response produces output frequencies equal to input frequencies

Output amplitude varies linearly with input amplitude.

Two-Tone Intermodulation (2)



Consider now an amplifier with $V_{in}(t) = V_0\cos(\omega_1 t) + V_0\cos(\omega_2 t)$

$$V_{in}(t) = (V_0/2)\left(e^{j\omega_1 t} + e^{-j\omega_1 t}\right) + (V_0/2)\left(e^{j\omega_2 t} + e^{-j\omega_2 t}\right) = (V_0/2)\left(z_1 + z_1^{-1} + z_2 + z_2^{-1}\right)$$

$$V_{out} = a_0 + a_1\cdot(\delta V_{in}) + a_2\cdot(\delta V_{in})^2 + a_3\cdot(\delta V_{in})^3 + \ldots$$

$$(\delta V_{in})^2 = (V_0/2)^2\left(z_1 + z_1^{-1} + z_2 + z_2^{-1}\right)^2 = (V_0/2)^2\left(\left(z_1 + z_1^{-1}\right)^2 + 2\left(z_1 + z_1^{-1}\right)\left(z_2 + z_2^{-1}\right) + \left(z_2 + z_2^{-1}\right)^2\right)$$

$$(\delta V_{in})^2 = (V_0/2)^2\cdot\left[2\cos(2\omega_1 t) + 2 + 2\cos(2\omega_2 t)\right] \rightarrow \text{harmonic distortion terms...as before.}$$

$$+ (V_0/2)^2\, 2\left(z_1 + z_1^{-1}\right)\left(z_2 + z_2^{-1}\right)$$

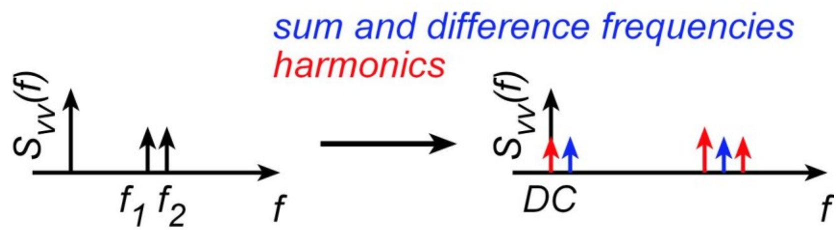
$$(\delta V_{in})^2 = \text{harm. dist. terms} + (V_0/2)^2\, 2\left(z_1 z_2 + z_1^{-1} z_2^{-1} + z_1^{-1} z_2 + z_1 z_2^{-1}\right) =$$

$$= \text{harm. dist. terms} + (V_0/2)^2\, 2\left(2\cos(2(\omega_1 + \omega_2)t) + 2\cos(2(\omega_1 - \omega_2)t)\right)$$

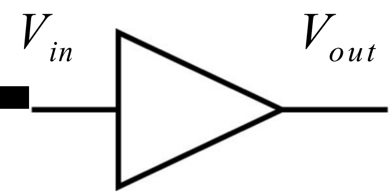
$$= (V_0/2)^2\cdot\left\{2 + 2\cos(2\omega_1 t) + 2\cos(2\omega_2 t) + 4\cos(2(\omega_1 + \omega_2)t) + 4\cos(2(\omega_1 - \omega_2)t)\right\}$$

2^{nd} - order response: outputs at DC, $2\omega_1$, $2\omega_2$, $(\omega_1 + \omega_2)$, and $(\omega_1 - \omega_2)$.

Output amplitude varies as the square of input amplitude.



Two-Tone Intermodulation (3)



$$(\delta V_{in})^3 = (V_0/2)^3\left(z_1 + z_1^{-1} + z_2 + z_2^{-1}\right)^3$$

$$= (V_0/2)^3\left(\left(z_1 + z_1^{-1}\right)^3 + 3\left(z_1 + z_1^{-1}\right)^2\left(z_2 + z_2^{-1}\right)^1 + 3\left(z_1 + z_1^{-1}\right)^1\left(z_2 + z_2^{-1}\right)^2 + \left(z_2 + z_2^{-1}\right)^3\right)$$

$$\left(\left(z_1 + z_1^{-1}\right)^3 + \left(z_2 + z_2^{-1}\right)^3\right) \rightarrow 2\cos(3\omega_1 t) + 3\cdot\cos(\omega_1 t) + 2\cos(3\omega_2 t) + 3\cdot\cos(\omega_2 t)\ldots\text{as before.}$$

$$3\left(z_1 + z_1^{-1}\right)^2\left(z_2 + z_2^{-1}\right)^1 + 3\left(z_1 + z_1^{-1}\right)^1\left(z_2 + z_2^{-1}\right)^2 = \ldots$$

$$\ldots = 3\left(z_1^2 + 2 + z_1^{-2}\right)\left(z_2 + z_2^{-1}\right) + \text{term with } z_1 \text{ and } z_2 \text{ interchanged.}$$

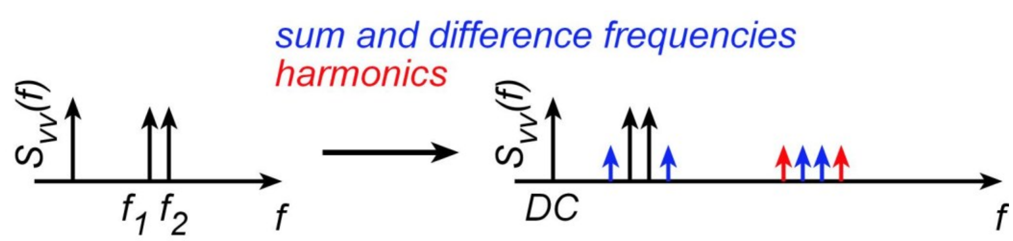
$$\ldots = 3\left(z_1^2 z_2 + z_1^{-2} z_2 + z_1^2 z_2^{-1} + z_1^{-2} z_2^{-1} + 2z_2 + 2z_2^{-1}\right) + \text{term with } z_1 \text{ and } z_2 \text{ interchanged.}$$

$$\ldots = 3\left(2\cos((2\omega_1 + \omega_2)t) + 2\cos((2\omega_1 - \omega_2)t) + 4\cos(\omega_2 t)\right) + 3\left(2\cos((2\omega_2 + \omega_1)t) + 2\cos((2\omega_2 - \omega_1)t) + 4\cos(\omega_1 t)\right)$$

$$(\delta V_{in})^3 = (V_0/2)^3\left\{\begin{matrix} 15\cdot\cos(\omega_1 t) + 15\cdot\cos(\omega_2 t) + 2\cos(3\omega_1 t) + 2\cos(3\omega_2 t) \\ + 6\cos((2\omega_1 + \omega_2)t) + 6\cos((2\omega_1 - \omega_2)t) + 6\cos((2\omega_2 + \omega_1)t) + 6\cos((2\omega_2 - \omega_1)t) \end{matrix}\right\}$$

3^{rd}-order response: outputs at $\omega_1,\ \omega_2, 3\omega_1,\ 3\omega_2,\ (2\omega_1 + \omega_2), (2\omega_1 - \omega_2), (2\omega_2 + \omega_1)$, and $(2\omega_2 - \omega_1)$.
Output amplitude varies as the cube of input amplitude.

This is the 2-tone intermodulation mentioned earlier.



General Picture....Nth Order responses

What frequencies are in $\left(V_0\cos(\omega_1 t)+V_0\cos(\omega_2 t)\right)^N$?

Binomial expansion: $(a+b)^N=\sum_{n=0}^{n=N}\binom{N}{n}a^{N-n}b^n=\sum_{n=0}^{n=N}\frac{N!}{n!(N-n)!}a^{N-n}b^n$

Pascal's triangle gives the binomial coefficients:

$$\begin{matrix} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1... \end{matrix}$$

$$\left(V_0\cos(\omega_1 t)+V_0\cos(\omega_2 t)\right)^N=(V_0/2)^N\left((z_1+z_1^{-1})+(z_1+z_2^{-1})\right)^N$$

$$=(V_0/2)^N\left(c_1(z_1+z_1^{-1})^N(z_1+z_2^{-1})^0+c_2(z_1+z_1^{-1})^{N-1}(z_1+z_2^{-1})^1+...+c_N(z_1+z_1^{-1})^0(z_1+z_2^{-1})^N\right)$$

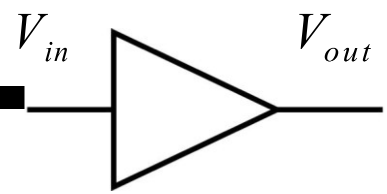
$$=(V_0/2)^N\cdot\text{sum of terms, each of form }(z_1^a\cdot(z_1^{-1})^b z_2^a\cdot(z_2^{-1})^d)\text{, each having }a+b+c+d=N$$

$$\text{i.e. each of form }(\exp(j((a-b)\omega_1+(c-d)\omega_2)t)$$

$$\text{output frequencies are }\pm\begin{Bmatrix}\omega_1\\ \text{or}\\ \omega_2\end{Bmatrix}\pm\begin{Bmatrix}\omega_1\\ \text{or}\\ \omega_2\end{Bmatrix}\pm....\pm\begin{Bmatrix}\omega_1\\ \text{or}\\ \omega_2\end{Bmatrix}\text{...and amplitude varies as }V_0^N$$

$$----N\text{ terms}----$$

Two-Tone Intermodulation: summary



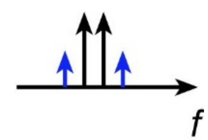
2^{nd} - order response: outputs at $\pm n\omega_1 \pm m\,\omega_2$, with $n+m=2$

$\rightarrow$ no responses near ω_1, ω_2

Output amplitude varies as the 2^{nd} power of input amplitude.

3^{rd} - order response: outputs at $\pm n\omega_1 \pm m\,\omega_2$, with $n+m=1,3$

$\rightarrow$ responses at $(2\omega_1 - \omega_2)$ and $(2\omega_1 - \omega_2)$, both near ω_1, ω_2



Output amplitude varies as the 3^{rd} power of input amplitude.

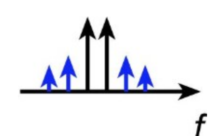
4^{th} - order response: outputs at $\pm n\omega_1 \pm m\,\omega_2$, with $n+m=2,4$

$\rightarrow$ no responses near ω_1, ω_2

Output amplitude varies as the 4^{th} power of input amplitude.

5^{th} - order response: outputs at $\pm n\omega_1 \pm m\,\omega_2$, with $n+m=1,3,5$

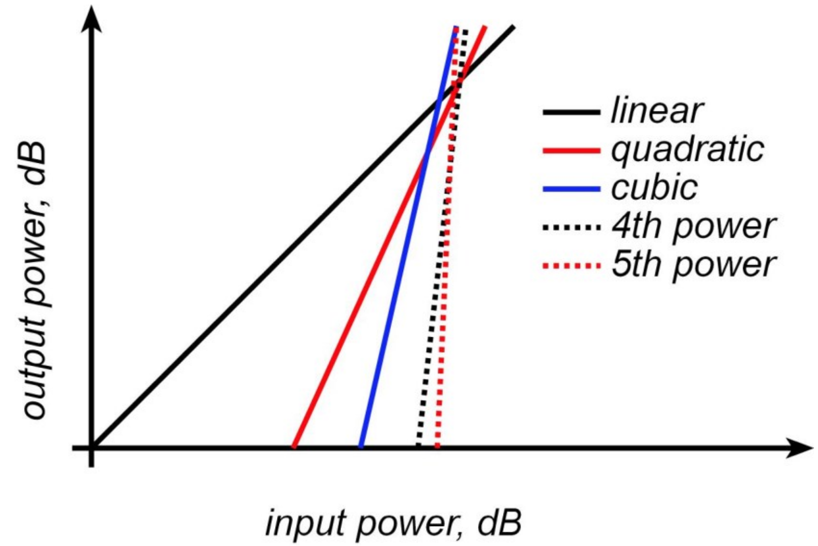
$\rightarrow$ responses at $(2\omega_1 - \omega_2)$, $(2\omega_1 - \omega_2)$, $(3\omega_1 - 2\omega_2)$, $(3\omega_2 - 2\omega_1)$, all near ω_1, ω_2



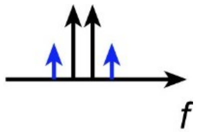
Output amplitude varies as the 5^{th} power of input amplitude.

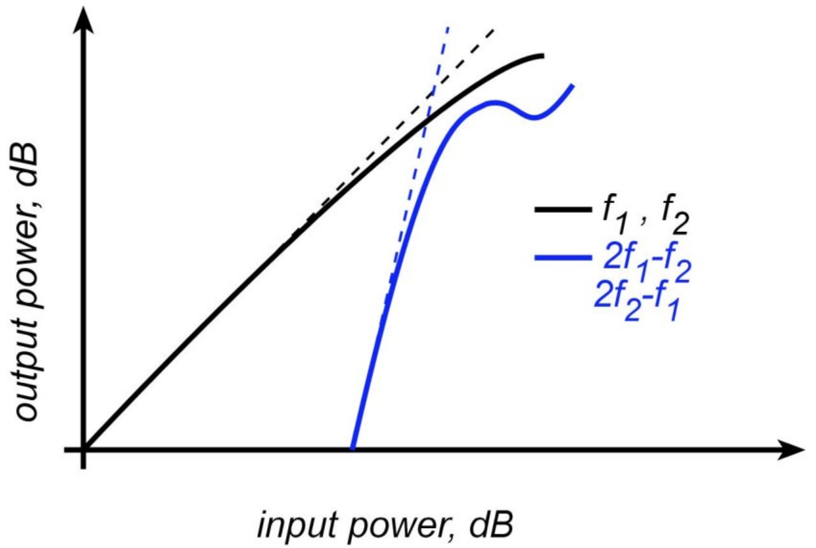
Intermodulation: Variation with Power

The linear, quadratic, cubic, 4th and 5th power terms of course vary with 1:1, 2:1, 3:1, 4:1 and 5:1 slopes on a plot of dB(Pout) vs. dB(Pin)

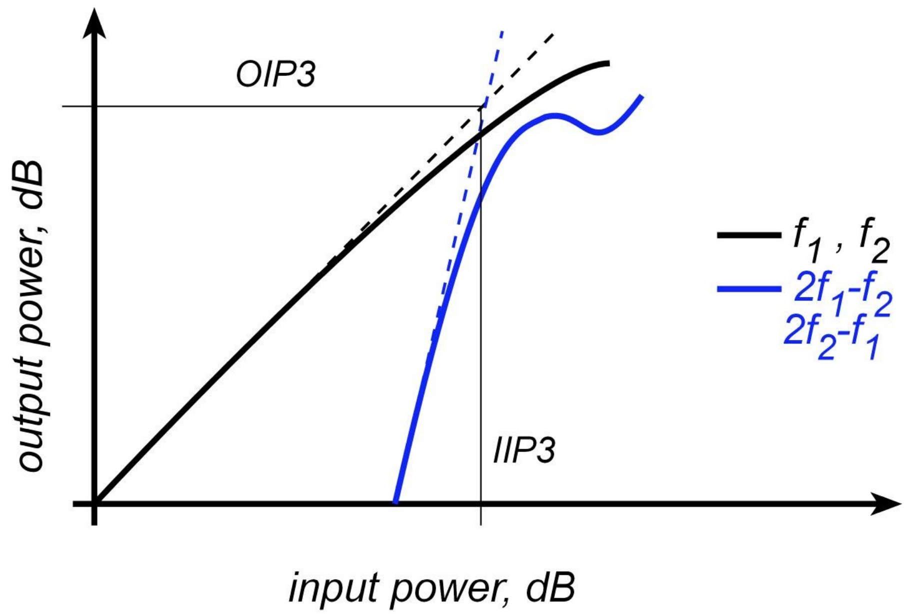


But the power at $(2\omega_1-\omega_2),(2\omega_2-\omega_1)$ involves $3^{rd},5^{th},7^{th}$ order terms, and varies with 3:1 slope only at low inputs where the cubic term dominates.





Third-Order Intercept



If we extrapolate the linear and two-tone response, the point of interception is the third-order intercept

Third order intercept

$$V_{out} = a_1 \cdot V_{in}(t) + a_2 \cdot V_{in}^2(t) + a_3 \cdot V_{in}^3(t) + ... = a_1 \cdot \left(V_{in} + b_2 \cdot V_{in}^2(t) + b_3 \cdot V_{in}^3(t) + ... \right)$$

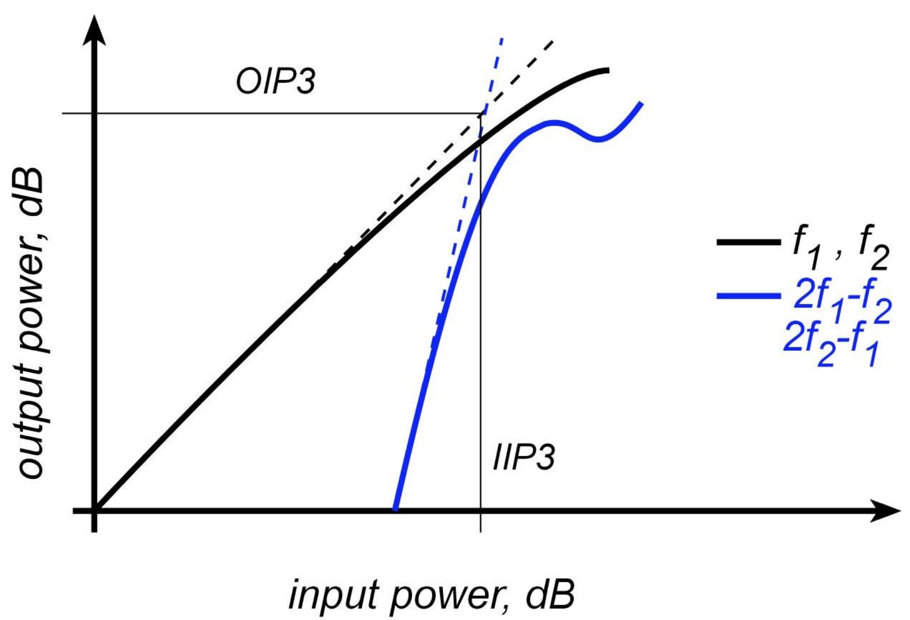
$$V_{in}(t) = V_0 \cdot \cos(\omega_1 t) + V_0 \cdot \cos(\omega_2 t)$$

$$b_3 \cdot V_{in}{}^3 = b_3 \cdot (V_0 / 2)^3 \left\{ 6\cos((2\omega_1 - \omega_2)t) + 6\cos((2\omega_2 - \omega_1)t) + \text{terms at other frequencies} \right\}$$

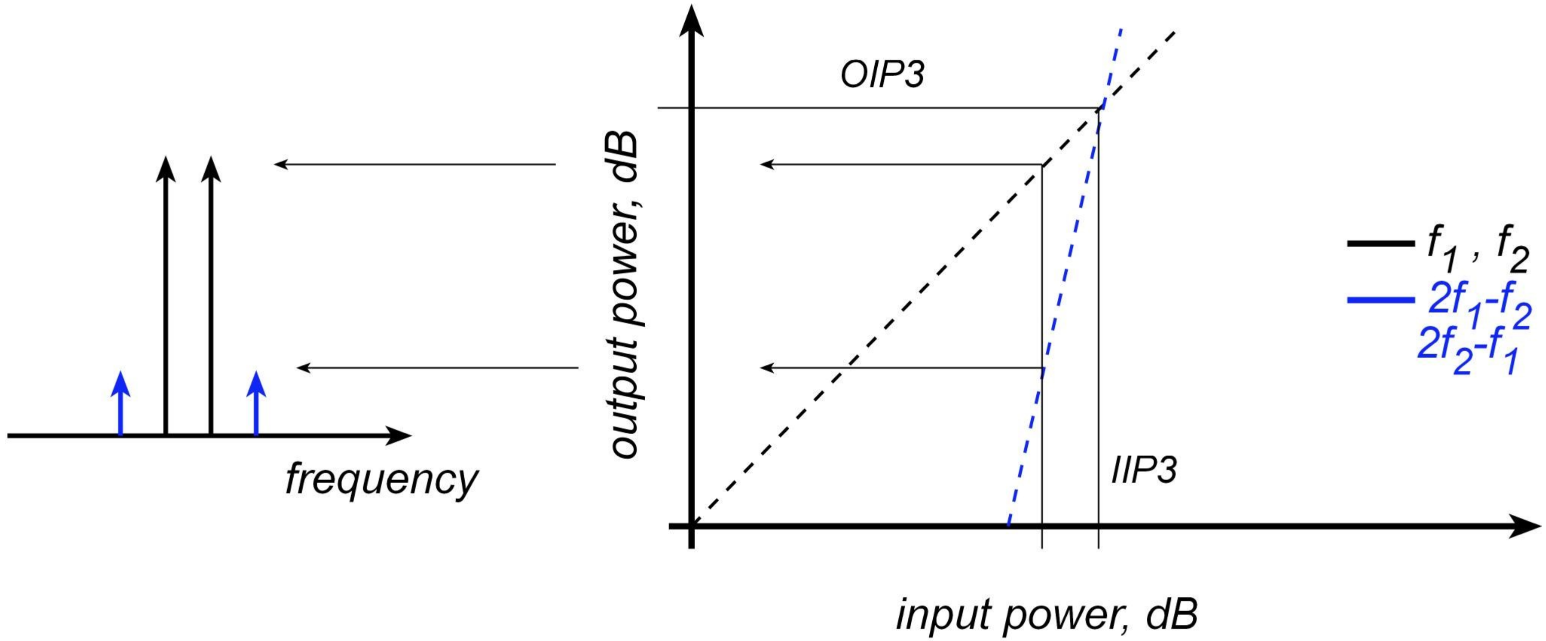
To find the third-order intercept, set V_0 such that the linear and cubic terms are equal

$$V_0 = 6b_3 \cdot (V_0 / 2)^3 = (3/4) b_3 V_0^3 \rightarrow V_0^2 = 4 / 3b_3$$

Single-tone power =IIP3=$V_0^2 / 2Z_0 = 2 / (3b_3 Z_0)$



Distortion Powers From Third-Order Intercept



3rd-Order distortion has 3:1 slope on log-log (dB) plot:

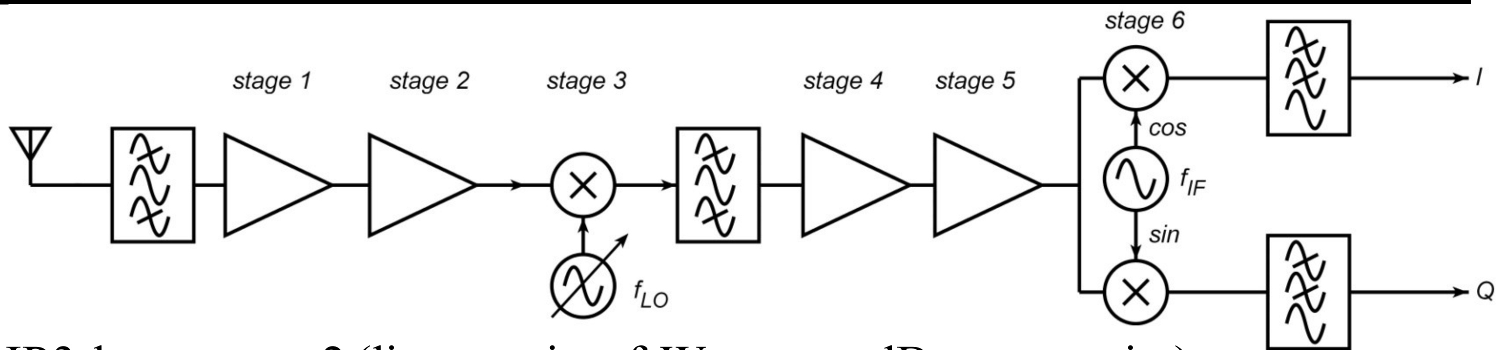
$$\text{OIP3(dBm)} - P_{out,(2f_1-f_2)}\text{(dBm)} = \text{OIP3(dBm)} - P_{out,(2f_2-f_1)}\text{(dBm)} = 3\cdot\left[\text{OIP3(dBm)} - P_{out,linear}\text{(dBm)}\right]$$

Therefore:

$$P_{out,linear}\text{(dBm)} - P_{out,(2f_1-f_2)}\text{(dBm)} = P_{out,linear}\text{(dBm)} - P_{out,(2f_2-f_1)}\text{(dBm)} = 2\cdot\left[\text{OIP3(dBm)} - P_{out,linear}\text{(dBm)}\right]$$

Each 1 dB drop in Pout below the third order intercept point improves the signal/distortion ratio by 2 dB.

Input-Referred Third-Order Intercept



Input-referred IP3 due to stage 2 (linear units of Watts, not dB power units)

$$\text{Receiver IIP3}_{\text{due to stage2}} = \frac{\text{IIP3}_{\text{Stage2}}}{G_1}$$

$$\text{Receiver IIP3}_{\text{due to stage3}} = \frac{\text{IIP3}_{\text{stage3}}}{G_1 G_2}.$$

...etc

Note that it is hard to develop an expression of an *aggregate* IP3 involving all stages, as the *phase angles* of the IM3 products may vary between successive stages. Distortion products may add in-phase, out-of-phase, or at some intermediate angle.

To compute the phase of the IM3 products requires a Volterra series analysis.