## ECE 145B / 218B, notes set 8: Mixers

Mark Rodwell Doluca Family Chair University of California, Santa Barbara

rodwell@ece.ucsb.edu 805-893-3244, 805-893-3262 fax

## Mixers in Radio Receivers



The mixer generates sum and difference frequencies  $(f_{RF} - f_{LO})$  and  $(f_{RF} + f_{LO})$ .

One of these is, by design, the intermediate frequency (IF).

The other is rejected by the IF filter.

## Ideal Mixer as a Multiplication Element



 $V_{IF}(t) = V_{RF}(t) \cdot V_{LO}(t) / V_0 = (V_R V_L / V_0) (\cos(2(\omega_1 + \omega_2)t) + \cos(2(\omega_1 - \omega_2)t)).$ Sum and difference frequencies are generated

## Multiplication Through A Nonlinear Element (1)



Input to nonlinear element:  $V_{in}(t) = V_{RF}(t) + V_{LO}(t) = V_R \cos(\omega_{RF}t) + V_L \cos(\omega_{LO}t)$ 

Characteristics of nonlinear element:  $V_{out}(t) = a_1 V_{in}^1(t) + a_2 V_{in}^2(t) + a_3 V_{in}^3(t) + \dots$ (note that  $a_1$  has units of volts<sup>0</sup>,  $a_2$  units of volts<sup>-1</sup>, etc.)

+...

$$V_{out}(t) = a_1 \left( V_R \cos(\omega_{RF} t) + V_L \cos(\omega_{LO} t) \right) \longrightarrow \text{outputs at } \omega_{RF} \text{ and } \omega_{LO}$$
  
+  $a_2 \left( V_R \cos(\omega_{RF} t) + V_L \cos(\omega_{LO} t) \right)^2 \longrightarrow \text{outputs at } (\pm \omega_{RF} \pm \omega_{LO}), 2\omega_{RF}, 2\omega_{LO}, \text{DC}$   
+  $a_3 \left( V_R \cos(\omega_{RF} t) + V_L \cos(\omega_{LO} t) \right)^3 \longrightarrow \text{outputs at } \pm \begin{cases} \omega_{RF} \\ \text{or} \\ \omega_{LO} \end{cases} \pm \begin{cases} \omega_{RF} \\ \text{or} \\ \omega_{LO} \end{cases} \pm \begin{cases} \omega_{RF} \\ \text{or} \\ \omega_{LO} \end{cases} \pm \begin{cases} \omega_{RF} \\ \text{or} \\ \omega_{LO} \end{cases}$ 

## Multiplication Through A Nonlinear Element (2)



Consider just first 2 terms:  $V_{out}(t) = a_1 V_{in}^1(t) + a_2 V_{in}^2(t)$ 

$$V_{out}(t) = a_1 \left( V_R \cos(\omega_{RF} t) + V_L \cos(\omega_{LO} t) \right) \longrightarrow \text{outputs at } \omega_{RF} \text{ and } \omega_{LO} \\ + a_2 \left( V_R \cos(\omega_{RF} t) + V_L \cos(\omega_{LO} t) \right)^2 \longrightarrow \text{outputs at } (\pm \omega_{RF} \pm \omega_{LO}), 2\omega_{RF}, 2\omega_{LO}, \text{DC}$$

Output contains

desired \* mixing term.... $(\omega_{RF} - \omega_{LO})$ undesired \* mixing term.... $(\omega_{RF} + \omega_{LO})$ LO and RF signals .... $\omega_{RF}$  and  $\omega_{LO}$ .... "LO and RF leakage" DC and LO and RF harmonics.... $2\omega_{RF}$ ,  $2\omega_{LO}$ , DC

\* or vice - versa

#### Example of Nonlinear Element: PN or Schottky Diode

Forward bias : nonlinear conductance

$$I(V) = I_s(e^{qV/kt} - 1)$$



#### Reverse bias : nonlinear capacitance

$$Q(V) = Q_0 + Q_1 V + Q_2 V^2 + \dots$$



so if 
$$v(t) = v_1 e^{j\omega_1 t} + v_1 e^{j\omega_2 t}$$
, then,  $I(t) = \sum_{l,m} I_{l,m} e^{j(l\omega_1 + m\omega_2)t}$ 

## Idealized Diode Mixer



#### Idealized Diode Mixer



Diode voltage

 $\delta V_{diode}(t) = V_{RF}(t) + V_{LO}(t) + V_{IF}(t) = V_R \cos(\omega_{RF}t) + V_L \cos(\omega_{LO}t) + V_I \cos(\omega_{IF}t)$ Diode current

$$I_{d} = I_{bias} \exp((V_{bias} + \delta V_{diode}) / V_{t}) \text{ where } V_{t} = kT / q$$
  

$$I_{d} = I_{bias} \left(1 + \left(\delta V_{diode} / V_{t}\right) + \left(\delta V_{diode} / V_{t}\right)^{2} / 2 + \left(\delta V_{diode} / V_{t}\right)^{3} / 6 + \dots\right)$$

(Over) approximate by limiting series to  $2^{nd}$  order, and assume that  $(\omega_{RF} - \omega_{LO}) = \omega_{IF}$ :

$$I_{d} = (I_{bias} / V_{t}) \cdot (V_{R} \cos(\omega_{RF}t) + V_{L} \cos(\omega_{LO}t) + V_{I} \cos(\omega_{IF}t)) \leftarrow \text{ linear resistance of diode junction.} \\ + (I_{bias} / 4V_{t}^{2}) \cdot \begin{pmatrix} V_{R}V_{L} \cos(\omega_{IF}t) \\ V_{R}V_{L} \cos(\omega_{LO}t) \\ V_{L}V_{I} \cos(\omega_{LO}t) \\ V_{L}V_{I} \cos(\omega_{RF}t) \end{pmatrix} \leftarrow \text{ mixing of RF signal to IF port} \\ \leftarrow ??? \\ \downarrow_{L}V_{L} \cos(\omega_{RF}t) \end{pmatrix} \leftarrow \text{ backwards mixing of IF signal to RF port}$$

+ terms at other frequencies.....eliminated by filters

## Idealized Diode Mixer

These equations are represented clearly by an equivalent circuit :



We see that the nonlinearity introduces a frequency shift. This causes forward from the RF to the IF port. We also see \* reverse \* coupling from the IF to the LO ports.

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Passive mixers are strongly * bilateral*.
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## Mixer as time-dependent conductance

 $I_d = I_{bias} \exp((V_{bias} + \delta V_{diode}) / V_t)$  where  $V_t = kT / q$ LO voltage modulates the diode current. If the LO is small,  $I_d(t) = I_{bias} + I_{IO} \cos(\omega_{IO} t)$  $V_{RF}(t) + V_{IF}(t)$ The diode conductivity is then  $g(t) = I_d(t) / V_t = g_0 + g_{10} \cos(\omega_{10}t)$ The small RF and IF signals are applied to g(t):  $V_{d}(t) = V_{RE} \cos(\omega_{RE}t) + V_{IE} \cos(\omega_{IE}t)$  $I_d(t) = V_d(t)g(t)$  $= g_0 V_{RE} \cos(\omega_{RE} t) + g_0 V_{IE} \cos(\omega_{IE} t)$ +  $(g_{LO}/2)V_{RE}(\cos((\omega_{RE}+\omega_{LO})t)\cos((\omega_{RE}-\omega_{LO})t))$ +  $(g_{LO}/2)V_{IF}(\cos((\omega_{IF}+\omega_{LO})t)\cos((\omega_{IF}-\omega_{LO})t))$ 

#### The time - varying conductivity produces the sum and difference frequencies



#### The LO drive should be big !



$$I_{d}(t) = \frac{g_{0}}{V_{RF}} \cos(\omega_{RF}t) + g_{0}V_{IF} \cos(\omega_{IF}t) \leftarrow \text{Resistive currents (loss!)} + (\frac{g_{LO}}{2}/2)V_{RF} (\cos((\omega_{RF} + \omega_{LO})t)\cos((\omega_{RF} - \omega_{LO})t)) \leftarrow \text{mixing terms} + (\frac{g_{LO}}{2}/2)V_{IF} (\cos((\omega_{IF} + \omega_{LO})t)\cos((\omega_{IF} - \omega_{LO})t)) \leftarrow \text{mixing terms}$$

The local oscillator should \* strongly \* modulate the conductivity

#### With big LO drive, mixer becomes a switch



$$G(t) = \left[\frac{G_{on}}{2}\right] + G_{on} \cdot \frac{2}{\pi} \left[\cos(\omega_{LO}t) + \frac{\cos(3\omega_{LO}t)}{3} + \frac{\cos(5\omega_{LO}t)}{5} + \dots\right]$$
$$\left[\frac{G_{on}}{2}\right] \rightarrow \text{this will give us direct RF} \rightarrow \text{IF coupling ...not good.}$$
$$G_{on} \cdot \frac{2}{\pi} \left[\cos(\omega_{LO}t)\right] \rightarrow \text{generates desired mixing terms } \dots (\omega_{RF} \pm \omega_{LO})$$
$$G_{on} \cdot \frac{2}{\pi} \left[\frac{\cos(3\omega_{LO}t)}{3}\right] \rightarrow \text{generates * harmonic * mixing terms } \dots (\omega_{RF} \pm 3\omega_{LO})$$

#### **Diode Double Balanced Mixer**



The 4 switches are implemented with (Schottky) diodes. Positive LO  $\rightarrow$  Positive RF - IF connection Negative LO  $\rightarrow$  crossed diodes on, negative RF - IF connection

#### Idealized (switch) double-balanced mixer



Input is multiplied by +1,-1,+1,-1,...., i.e. by a squarewave  $V_{IF}(t) = M(t)V_{RF}(t)$  where M(t) is a squarewave The squarewave has a Fourier series :

$$M(t) = \frac{4}{\pi} \left[ \cos(\omega_{LO}t) + \frac{\cos(3\omega_{LO}t)}{3} + \frac{\cos(5\omega_{LO}t)}{5} + \dots \right]$$
  
So we are multiplying  $V_{RF}(t)$  with  $\cos(\omega_{LO}t)$   
First hint of trouble : there's also  $3\omega_{LO}$ ,  $5\omega_{LO}$ ,...

#### Typical mixer presentation then moves to circuits:



But, before we concentrate on circuits, how well do we understand mixing? Insertion loss? Noise figure? Image responses? Harmonic responses? Input - output isolation?

#### Ideal switch-based mixer



Insertion loss? Noise figure? Image responses? Harmonic responses? Input - output isolation?

We can learn much by studying an ideal mixer

#### What is our goal ?



Question : Why do we use LNAs in receivers ? Answer : to reduce the mixer's noise figure contribution. Question : Why do mixers have 3 - 6 dB noise figures ?

If low mixer noise figure  $\rightarrow$  eliminate LNA.

Lower cost, higher receiver IP3.

Note : IF amp operates at lower frequency, can have low  $F_{\min}$ 

### Even ideal mixers have:

Image response  $\rightarrow$  out - of - band response, added noise.

LO harmonic response  $\rightarrow$  out - of - band response, added noise.

Attenuation : becuase input signal is converted to several frequencies

Bilateral response : output couples back to input. At several frequencies.



#### Image response $\rightarrow$ interference, loss of SNR

$$V_{IF}(t) = M(t)V_{RF}(t)$$
$$M(t) = \frac{4}{\pi} \left[ \cos(\omega_{LO}t) + \frac{\cos(3\omega_{LO}t)}{3} + \frac{\cos(5\omega_{LO}t)}{5} + \dots \right]$$

Image response :

If  $f_{IF} = f_{RF} - f_{LO}$ , then  $f_{image} = f_{LO} - f_{IF} = f_{RF} - 2f_{LO}$ Signals and noise at  $f_{image}$  also mix to  $f_{IF}$ .

Problem #1 is interference : RF front - end needs filter to reject  $f_{image}$ 

Problem #2 is loss in SNR due to  $f_{image}$ Mixer input noise power spectral density @  $f_{RF} = kTFG_{LNA}(f_{RF})$ Mixer input noise power spectral density @  $f_{image} = kTFG_{LNA}(f_{image})$ Poor front - end filtering ?  $\rightarrow$  Image response adds significant noise





## LO harmonics also produce images:

$$V_{IF}(t) = M(t)V_{RF}(t)$$
$$M(t) = \frac{4}{\pi} \left[ \cos(\omega_{LO}t) + \frac{\cos(3\omega_{LO}t)}{3} + \frac{\cos(5\omega_{LO}t)}{5} + \dots \right]$$

At the RF port, other frequencies also mix to  $f_{IF}$   $3f_{LO} \pm f_{IF}, 5f_{LO} \pm f_{IF}, 7f_{LO} \pm f_{IF}...$  And, given imperfect LO symmetry:  $2f_{LO} \pm f_{IF}, 4f_{LO} \pm f_{IF}, ...$  Problem : out - of - band interference  $\rightarrow$  need good RF filter Problem : out - of - band noise contribution  $\rightarrow$  need good RF filter

Further, an input at  $f_{RF}$  mixes to many output frequencies  $f_{RF} + f_{LO_1} f_{RF} \pm 2f_{LO_1} f_{RF} \pm 3f_{LO_1} \dots$ 

signal power at these  $\rightarrow$  less power at  $f_{IF} \rightarrow$  more attenuation







## Ideally: eliminate spurious responses with filters

$$V_{IF}(t) = M(t)V_{RF}(t)$$
$$M(t) = \frac{4}{\pi} \left[ \cos(\omega_{LO}t) + \frac{\cos(3\omega_{LO}t)}{3} + \frac{\cos(5\omega_{LO}t)}{5} + \dots \right]$$

#### But:

off - wafer filters cost money, increase product size on - wafer filters occupy die area, & are low - Q If  $f_{IF} / f_{RF}$  is small, then the filter must be very high Q

\*And\*:

In a real mixer, the switches are diodes or transistors these have RC parasitics, shot noise generators.

- 1) the resistors & transistor junctions will introduce kTF, shot noise directly at  $f_{RF}$ ,  $f_{IF}$
- 2) the resistors also generate noise at all the image frequencies.
- These will also mix into the receiver passband;
  - and external filters cannot prevent this.





#### Real mixers are bilateral, not unilateral



#### RF port :

response @  $f_{LO} + f_{IF} = f_{RF}$ response @  $f_{LO} - f_{IF}$ response @  $2f_{LO} + f_{IF}$ response @  $2f_{LO} - f_{IF}$ , etc.

Just like other circuits, mixers are bilateral  $(S_{21}S_{12} \neq 0)$ Passive mixers  $S_{21} = S_{12}$ Active mixers  $S_{12} \ll S_{21}$  but  $S_{12}$  nevertheless  $\neq 0$ .

Note:  $S_{ii}$  here defined at different frequencies for input & output ports.

#### Diode Double Balanced Mixer: Two-Port Representation



If we apply filters, as shown, to restrict the signal frequencies at the two ports, we can again represent the mixer with a 2-port network, where the two ports have signals at frequencies  $\omega_{RF}$  and  $\omega_{IF}$ .

 $\rightarrow$  Mixers have MAG, optimum impedances, etc.

Derivation is not hard. But, we will not pursue here. For ideal switches, Y, Z matrices will have infinities. in that case, use S matrix.

## Bilateral mixing→ spurious RF responses

- Example: filter at IF port, not at RF port
- Apply signal at  $f_{RF}$  to the IF port
- $\rightarrow$  produces signal at  $f_{IF}$  at the IF port
- $\rightarrow$  then produces signal at the RF port
  - (a)  $f_{LO} + f_{IF} = f_{RF}$ (a)  $f_{LO} - f_{IF}$ (a)  $2f_{LO} + f_{IF}$ (a)  $2f_{LO} - f_{IF}$ , etc.

Out - of - band signal responses. Antenna will re - radiate. Suppressed by LNA  $S_{12}$ , if present. Suppressed by filter, if present, and if filter is sufficiently narrow.

(one response is (a)  $f_{RF} - 2f_{IF}$ )



This is in addition to LO leakage: also radiates from antenna; much stronger signal

## Eliminating Noise from Image Response

Image - reject mixer suppresses both image signal and image noise response

Trap provides zero available noise power at image frequency

Filtering :  $\sim kT$  noise at image frequency but  $\sim kTFG$  noise at signal frequency







## System-Level Mixer Noise analysis

Citation: TUTORIAL 5594: System Noise-Figure Analysis for Modern Radio Receivers By: Charles Razzell, Maxim Integrated Products, Inc.

Model of Mixer's internal noise. Adds  $N_s$  at  $f_{signal}$ ; spectral density  $S_s$ Adds  $N_i$  at  $f_{image}$ ; spectral density  $S_I$ Adds  $N_{IF}$  at  $f_{IF}$ ; spectral density  $S_{IF}$   $G_s$  = mixer gain for signal frequency  $G_I$  = mixer gain for image frequency https://pdfserv.maximintegrated.com/en/an/TUT5594.pdf



Total noise at IF port

$$S_{mixer} = S_S G_S + S_I G_I + S_{IF}$$

Again, this is just the mixer's internal noise.

## System-Level Mixer Noise analysis

Receiver model:

LNA adds noise  $N_{LNA,S}$  at  $f_{signal}$ LNA has gain  $G_{LNA,S}$  at  $f_{signal}$ LNA adds noise  $N_{LNA,I}$  at  $f_{image}$ LNA has gain  $G_{LNA,I}$  at  $f_{image}$ Include filter 2 frequency reponse in  $G_{LNA}$ 

IF signal power

 $P_{\rm IF} = P_{\rm signal} G_{\rm LNA,S} G_{\rm S}$ 

IF noise power spectral density

$$S_{IF} = (kT + N_{LNA,S})G_{LNA,S}G_S + (kT + N_{LNA,I})G_{LNA,I}G_{IM} + S_{mixel}$$
$$= kTF_{LNA,S}G_{LNA,S}G_S + kTF_{LNA,I}G_{LNA,I}G_{IM} + S_{mixer}$$

Component of IF noise power spectral density from RF source  $@f_s$ 

 $S_{IF,\text{from antenna}@f_s} = kTG_{LNA,S}G_S$ 

System Noise figure

$$F_{\text{system}} = S_{IF} / S_{IF,\text{from antenna}@f_{s}} = \frac{F_{LNA,S}G_{LNA,S}G_{S} + F_{LNA,I}G_{LNA,I}G_{IM} + S_{mixer} / kT}{G_{LNA,S}G_{S}}$$

$$F_{\text{system}} = F_{LNA,S} + F_{LNA,I} \frac{G_{LNA,I}G_{IM}}{G_{LNA,S}G_S} + \frac{S_{mixer}}{kTG_{LNA,S}G_S}$$



## System-Level Mixer Noise analysis

#### System Noise figure

$$F_{\text{system}} = F_{LNA,S} + F_{LNA,I} \frac{G_{LNA,I}G_{IM}}{G_{LNA,S}G_S} + \frac{S_{mixer}}{kTG_{LNA,S}G_S}$$

Suppose : no RF filtering of image,

 $G_{LNA,I} = G_{LNA,S}, G_{IM} = G_S$  $F_{\text{system}} = F_{LNA,S} + F_{LNA,I} + \frac{S_{mixer}}{kTG_{LNA,S}G_S}$ 

we have doubled the LNA noise contribution

Suppose : perfect RF filtering of image,  $G_{LNA,I} = 0$ 

$$F_{\text{system}} = F_{LNA,S} + \frac{S_{mixer}}{kTG_{LNA,S}G_S}$$

we have elimnated the image noise contribution except for that internal to the mixer





## System-Level Mixer Noise analysis: another case

Assume:

filter 2 provides perfect RF filtering of image, but we then have another gain stage (LNA 2).

 $G_{LNA1,I}=0$ 



$$F_{\text{system}} = F_{LNA1,S} + \frac{F_{LNA2,S} - 1}{G_{LNA1,S}} + \frac{F_{LNA2,I} - 1}{G_{LNA1,S}} \frac{G_{LNA2,I} G_{IM}}{G_{LNA2,S} G_{S}} + \frac{S_{mixer}}{kTG_{LNA1,S} G_{LNA2,S} G_{S}}$$

We have doubled the noise contribution of LNA 2.

## System-Level Mixer Noise analysis: Big Picture

If unfiltered, image responses will add LNA (etc) noise at image frequency to that at signal frequency

 $\rightarrow$  increased receiver noise

Receiver noise model



Mixer internal noise at image frequency adds to that at signal frequency → increased mixer noise This can't be filtered.

Image responses also arise from LO harmonics

Clearly: try to minimize mixer harmonic and image responses

#### FET passive switch double balanced mixer



Key point: FETs operate in their resistive regions, as switched resistors. This is unlike the FET Gilbert cell mixer, where FETs operate in the constant-current regions

## BJT (HBT) Unbalanced Mixer: g<sub>m</sub>(t) modulation

LO voltages modulates  $I_E(t)$ This modulates  $g_m(t)$ . Larger LO drive: components of  $g_m(j2\pi f)$ at  $f_{LO}$ ,  $2f_{LO}$ ,...



Network  $(I_0 || C_E)$  forces constant time-average emitter current, independent of LO drive.



# BJT (HBT) Unbalanced Mixer: mixing of RF and LO



If we now apply a small  $V_{RF}(t)$ , then  $I_E(t)$  containts a component  $I_{mix}(t) = g_m(t)V_{RF}(t)$  $\rightarrow$  Mixing

Multiplication in time domain = convolution in frequency domain

 $\rightarrow$  Sum and difference frequencies.

#### BJT (HBT) Unbalanced Mixer: Combining Signals



In unbalanced mixers, LO, RF, and IF filters must be seperated (isolated) by filters. The associated filter design can be very difficult.

The filter design will be extremely difficult if  $f_{LO} \approx f_{RF}$ .

#### Quadrature or 90° Hybrids

power at the 2 inputs splits equally at the 2 outputs. sums with  $90^{\circ}$  phase difference

Branch line coupler: one for of  $90^{\circ}$  hybrid.



Lange couplers are also  $90^{\circ}$  hybrids.

https://www.keysight.com/us/en/assets/7018-08094/technical-overviews-archived/5989-8911.pdf

#### Balanced mixer using Quadrature Hybrid



If the LO and RF frequencies are similar, a 90 degree hybrid can provide the LO to RF isolation

## FET Unbalanced Mixer: g<sub>m</sub>(t) modulation





Similar to the BJT/HBT design. Only major difference:

without the  $I_C = I_S e^{qV_{be}/kT}$ , we don't need the current source to regulate the bias current.

#### FET Unbalanced Mixer using Cascode Pair



....signal path is on.

When LO is low,  $V_{ds}$  of Q1 is reduced to zero, reducing  $g_{m1}$  to zero. ....signal path is off.

IF current is RF wave form mulitiplied by square wave.

IF port also has strong LO and RF currents.

#### FET Single-balanced Mixer



LO drive voltage is sufficient for to fully switch upper FET pair Upper FETs operate as common-gate stages, not as resistive switches Lower FETs operate as common-source stages, with  $V_{DS} > V_{knee}$ 

RF signal no longer appears at IF output. LO, unfortunately - - - does

#### FET double-balanced Mixer (Gilbert Cell)



LO drive voltage is sufficient for to fully switch upper FET quad Upper FETs operate as common-gate stages, not as resistive switches Lower FETs operate as common-source stages, with  $V_{DS} > V_{knee}$ 

Neither RF nor LO signals appear at IF output. ....to the extent that the circuit is perfectly balanced...

#### BJT/HBT double-balanced Mixer (Gilbert Cell)



Neither RF nor LO signals appear at IF output. ....to the extent that the circuit is perfectly balanced...