B

ECE 2C Final Exam

June 7 , 2011

Do not open exam until instructed to.

Closed book: Crib sheet and 2 pages personal notes permitted

There are 4 problems on this exam, and you have 3 hours.

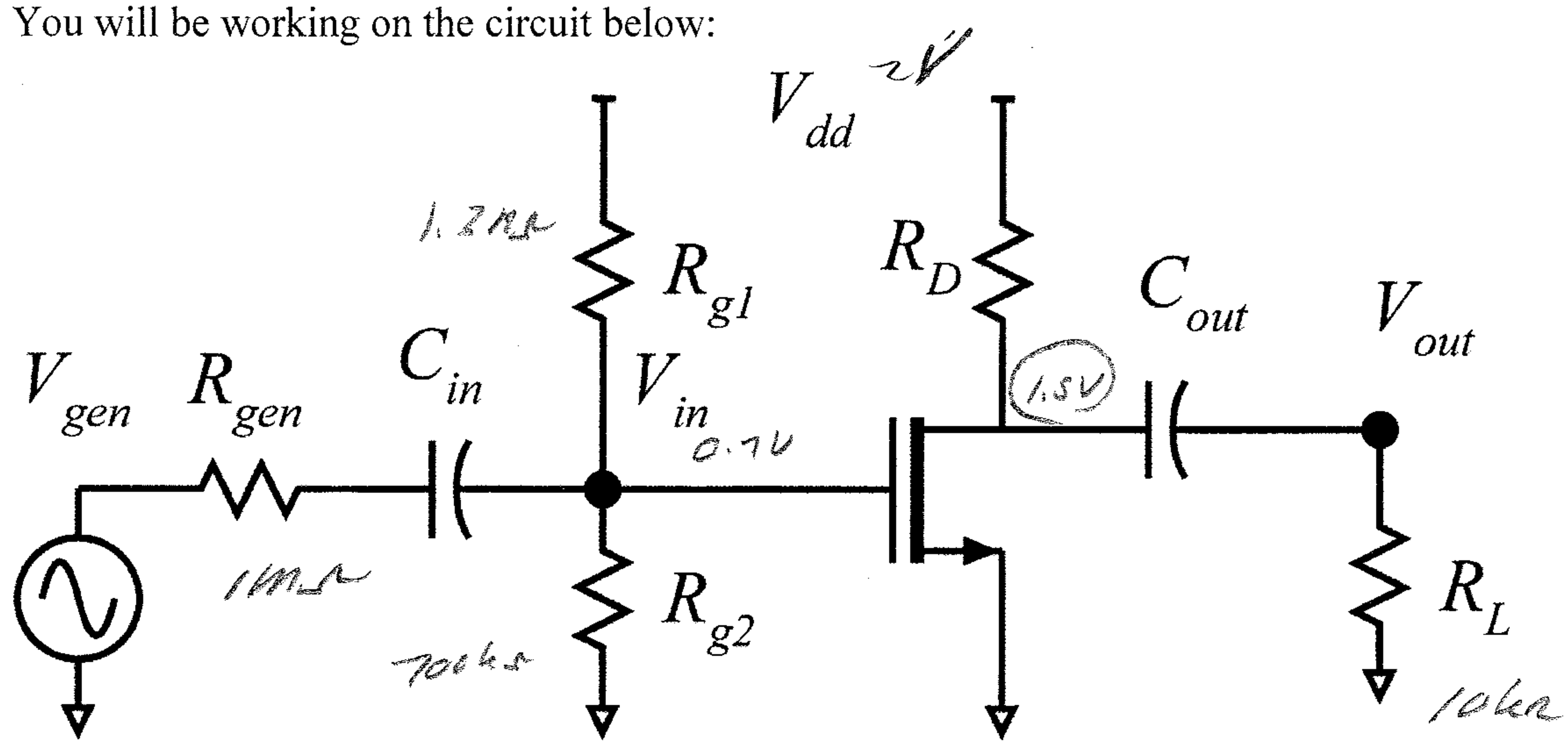
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING and approximately Justifying them.***

Name: Solution B

Problem	Points Received	Points Possible
1a		5
1b		2
1c		5
1d		5
1e		5
1f		3
1g		10
2a		10
2b		10
3a		10
3b		10
3c		5
3d		5
3e		5
4a		10
4b		5
4c		5
4d		5
4e		5
total		100

Problem 1, 25 points

You will be working on the circuit below:



Q1 is a mobility-limited FET, i.e. $I_d = (\mu c_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$ where $(\mu c_{ox} W_g / 2L_g) = 1\ \text{mA/V}^2$, $\lambda = 0.05\ \text{V}^{-1}$, and $V_{th} = 0.20$ V.

V_{dd} = +2.0 volts

Cin and Cout are very big and have negligible AC impedance.

RL=10 kOhm

Rgen=1 MOhm

part A soln..

$I_d = 1\text{mA/V}^2 \cdot (V_{gs} - V_{th})^2 = 1/4\ \text{mA}$

$V_{gs} - V_{th} = (1/4\ \text{V}^2)^{1/2} = 1/2\ \text{V}$ (1)

$V_{gs} = 0.7\text{V}$ (1)

current = 1µA (R_{g1}, R_{g2})

$R_{g2} = 0.7\text{V} \div 700\text{k}\Omega$ (1)

$R_{g1} = \frac{1.3\text{V}}{1\mu\text{A}} = 1.3\text{M}\Omega$ (1)

$R_d = \frac{2\text{V} - 1.5\text{V}}{1/4\ \text{mA}} \rightarrow \frac{0.5\text{V}}{0.25\text{mA}} = 2\text{k}\Omega$ (1)

Part a, 5 points

DC bias.

Q1 is to be biased with 1/4 mA drain current, and with 1.5 Volts drain voltage. Ignore λ while solving this part.

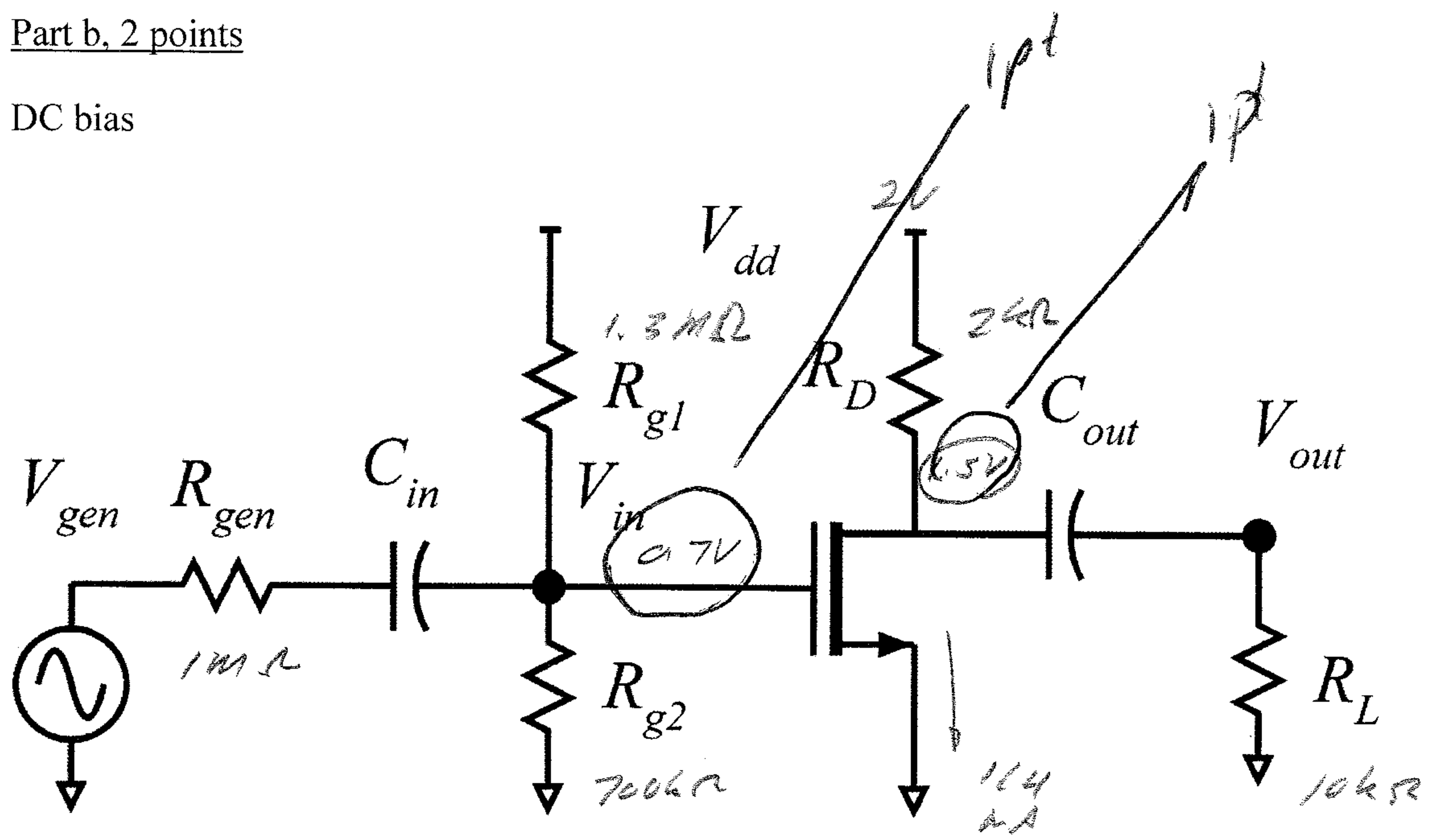
Find: Rg1= 700 kΩ , Rg2= 1.3 MΩ , Rd= 2 kΩ

The DC voltage at the gate of Q1. = 0.7V

Solution on page 2

Part b, 2 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

$I_d = 1\,mA/V^2\,(V_{gs}-V_{th})^2(1+\lambda V_{DS})$

Part c

(2.5)

$g_m = \frac{1mA}{V^2}\cdot(V_{gs}-V_{th})\cdot 2 = \frac{2mA}{V^2}\cdot 0.5V$

$= 1mA/V^2$ ok with or without $(1+\lambda V_{DS})$

(2.5)

$R_{ds} = \frac{1/\lambda + V_{DS}}{I_d} = \frac{20V + 1.5V}{I_d = 1/4\,mA} = 86k\Omega$ answer ok.

$\approx \frac{1}{\lambda I_D} = 80k\Omega$ answer ok

b

Part c, 5 points

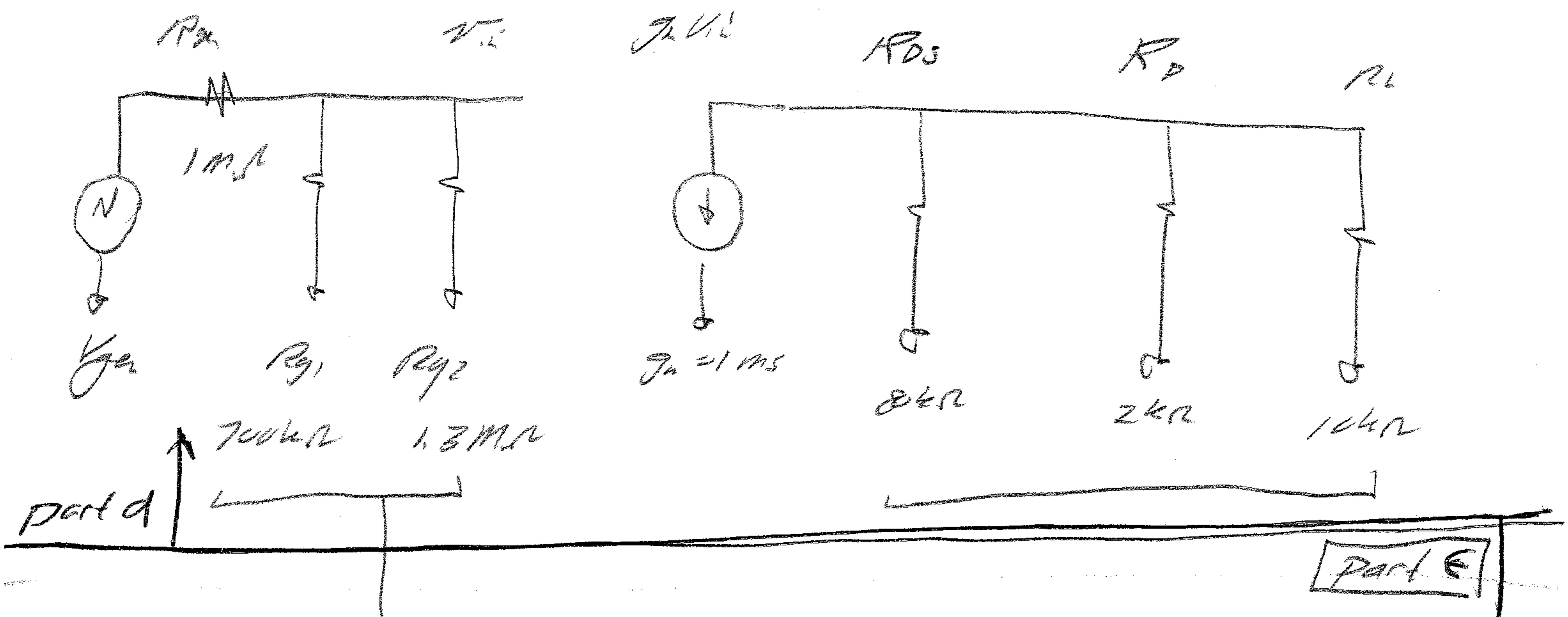
Find the small signal parameters of Q1. Use the mobility-limited model.

gm=___1 mS___
Rds=___80 kΩ___

SEE page 4

Part d, 5 points

Replacing the transistor with its small-signal model, draw a small-signal equivalent circuit diagram for the amplifier. Give values for all elements on the diagram.



$R_{in} = 700\,k\Omega \,||\, 1.3\,M\Omega$

$= 455\,k\Omega$

$R_{eq} = 1.63\,k\Omega$

$v_i / v_{gen} =$

Part e, 5 points.

Find the small signal voltage gain (Vout/Vin) of Q1.

Vout/Vin= -1.63

$R_{eq} = R_D \,//\, R_{DS} \,//\, R_L$

$= 1.63\ k\Omega$] 2.5

$v_o/v_{in} = -g_m R_{eq}$

$= -1mS \cdot 1.63k\Omega$] 2.5

$= -1.63$

Part f, 3 points

Find the *** amplifier *** input resistance, Vin/Vgen, and Vout/Vgen

Rin,amplifier = 455 kΩ

Vin/Vgen= 0.31

(Vout/Vgen) = −0.51

$R_{inA} = R_{g1} \parallel R_{g2} = 455k\Omega$ (1)

$\frac{v_i}{v_{gen}} = \frac{R_{inA}}{R_{inA} + R_{gen}} = \frac{455k\Omega}{455k\Omega + 1M\Omega} = 0.31$ (1)

$\frac{v_o}{v_{gen}} = \frac{v_o}{v_i} \cdot \frac{v_i}{v_{gen}} = 0.31 \cdot (-1.63)$

$= -0.51$ (1)

Part g, 10 points

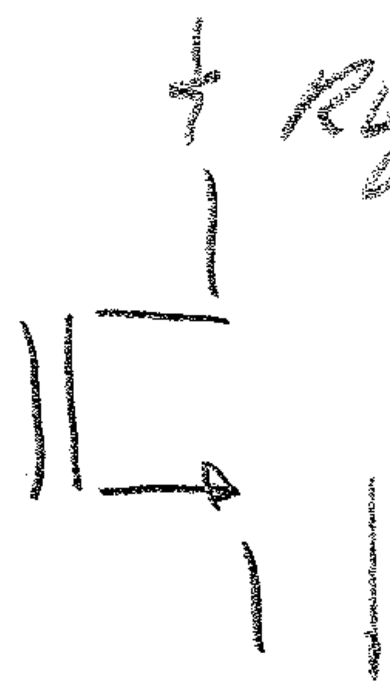
Now you must find the maximum signal swings. Find the output voltage due to the knee voltage and due to cutoff in Q1.

Cutoff of Q1; Maximum ΔVout resulting = + 0.41 V

Knee voltage of Q1; Maximum ΔVout resulting = − 1.0V.

(5)

Cutoff



$R_{sig} = 1.63\,k\Omega$

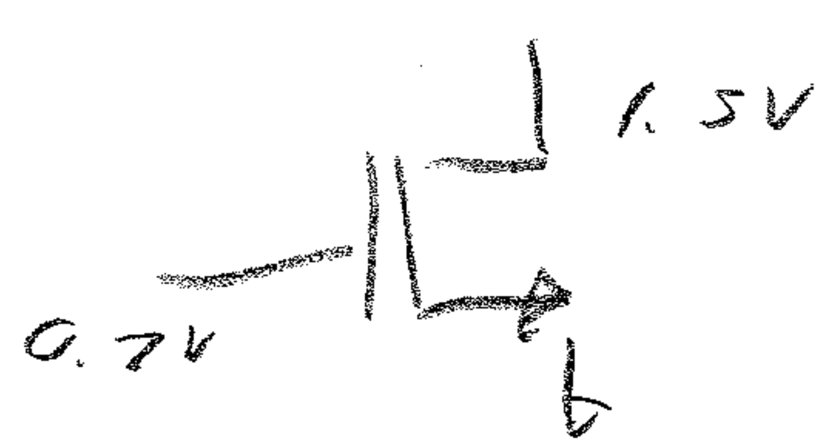
$I_{DQ} = 1/4\,mA$

$\Delta V = 1/4\,mA \cdot 1.63\,k\Omega = 0.4075V$ positive going

(5)

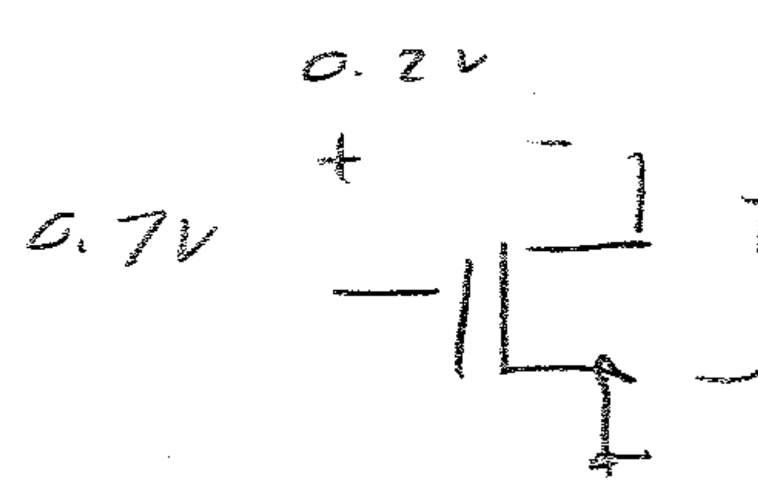
Knee voltage.

BIAS



$V_{KL} = 0.2V$

KNEE



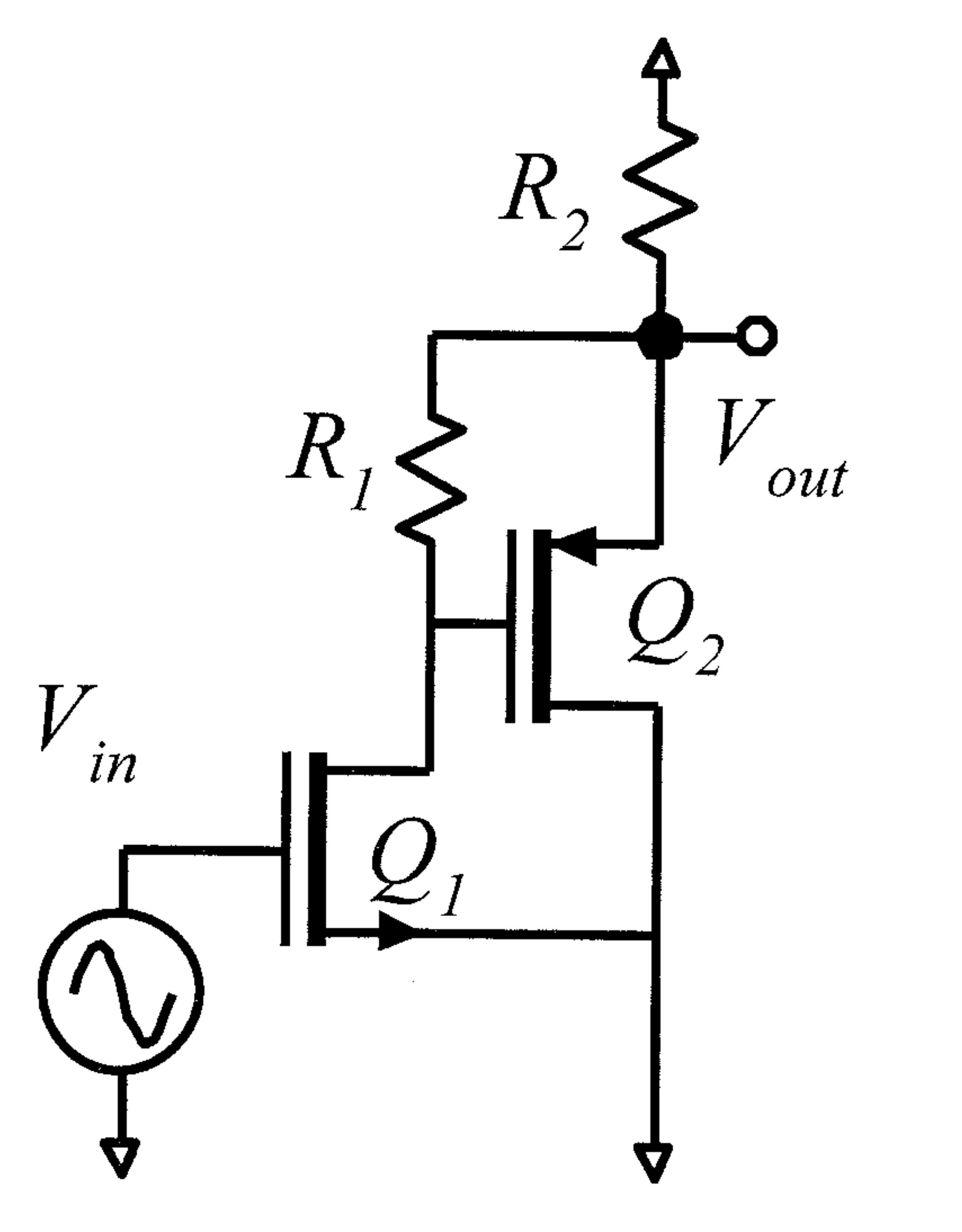
+ 0.5V = 0.7 − 0.2V (2.5)

(2.5)

1.5V − 0.5V = 1.0V negative going

Problem 2: 20 points

Nodal analysis, transistor circuit models

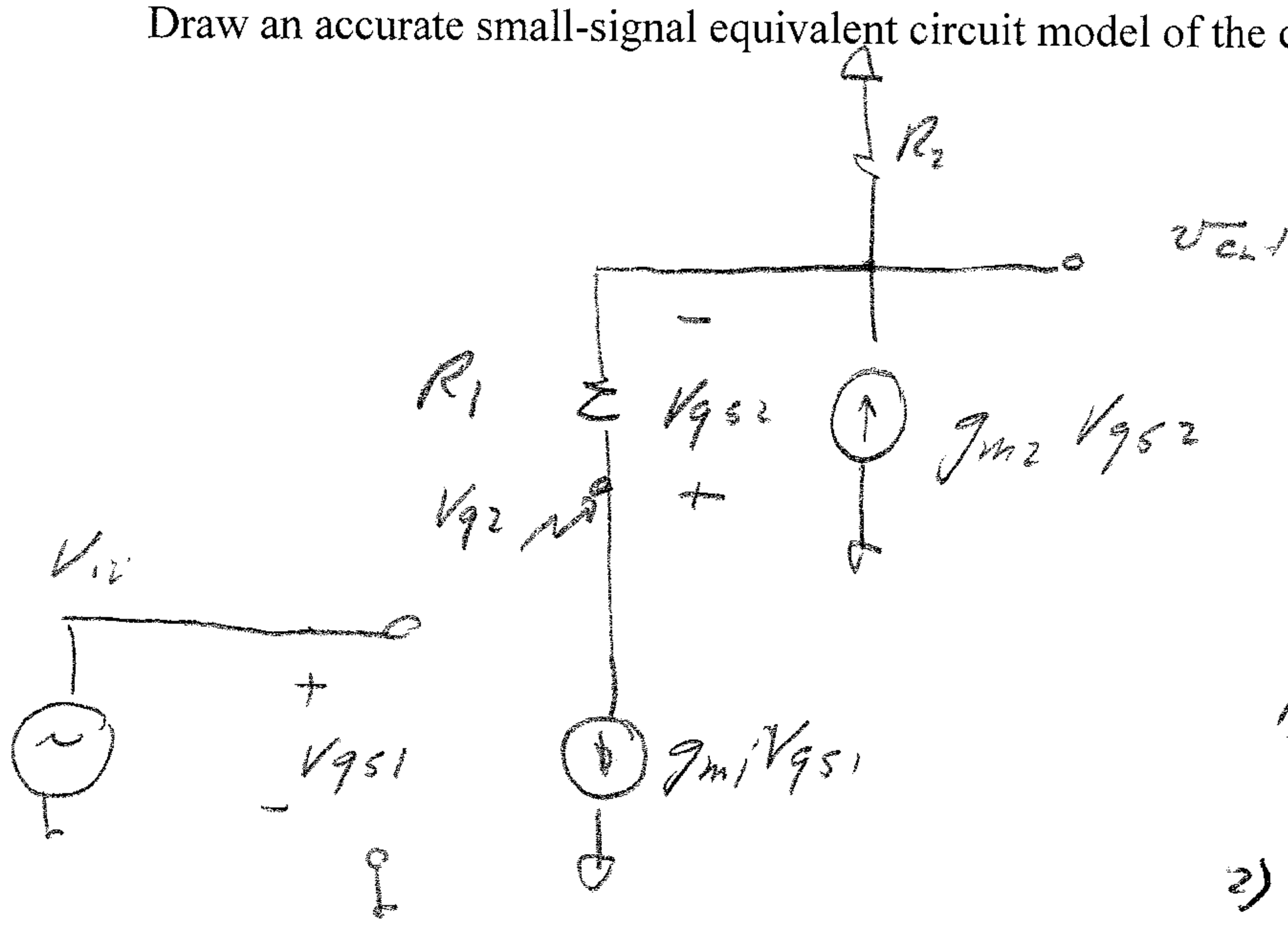


Ignore DC bias; you don't need it.

The transistors have zero C_{gs} and C_{gd}, and have transconductance g_{m1} and g_{m2}. Both have zero G_{ds}

Part a, 10 points

Draw an accurate small-signal equivalent circuit model of the circuit above.



-3 pts penalty each for:

1) topological / connection errors

2) Missing control voltages on gm elements.

3) unlabelled control voltages

Part b, 10 points

Using NODAL ANALYSIS, find V_{out}/V_{in}. Give both an algebraic expression, then find the numerical value with with gm1=10 mS, gm2=20 mS, R1=1000 Ohms, R2=10,000 Ohms.

$\frac{V_o}{V_{in}} = -g_{m1}(R_1 R_2)(g_{m2}+G_1)$ (algebraic expression)

$\frac{V_o}{V_{in}} = -2,100$ (1)

(value with gm1=10 mS, gm2=20 mS, R1=1000 Ohms, R2=10,000 Ohms)

$\Sigma I = 0$ @ V_{g2}:

$$g_{m1} V_{gs1} + (V_{g2} - V_{out}) G_1 = 0$$

$$g_{m1} V_{in} + G_1 V_{g2} + V_{out}(-G_1) = 0$$

$$V_{g2}(-G_1) + V_{out}(+G_1) = +g_{m1} V_{in}$$ ← (3)

$\Sigma I = 0$ @ V_{out}

$$V_{out}(G_2) - g_{m2} V_{gs2} + (v_{out} - V_{g2}) G_1 = 0$$

$$v_{out}(G_1 + G_2) + g_{m2}(v_{out} - v_{g2}) - V_{g2} G_1 = 0$$

$$v_{out}(G_1 + G_2 + g_{m2}) + v_{g2}[-g_{m2} - G_1] = 0$$ (3)

$$V_{g2}(+G_1) + V_{out}(-G_1) = -g_{m1}V_{in}$$

$$V_{g2}(-G_1 - g_{m2}) + V_{out}(G_1 + G_2 + g_{m2}) = 0$$

$$\frac{V_{out}}{V_{in}} = \frac{N}{D}$$

$$N = \begin{vmatrix} G_1 & -g_{m1} \\ -G_1 - g_{m2} & 0 \end{vmatrix} = -g_{m1}(g_{m2} + G_1)$$

$$D = \begin{vmatrix} G_1 & -G_1 \\ -G_1 - g_{m2} & G_1 + G_2 + g_{m2} \end{vmatrix} \begin{array}{l} G_1G_1 + G_1G_2 + G_1g_{m2} \\ = -G_1G_1 - G_1g_{m2} \\ = G_1G_2 \end{array}$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m1}(g_{m2} + G_1)}{G_1G_2} = \boxed{-g_{m1}R_1R_2(g_{m2} + G_1)}$$

ok

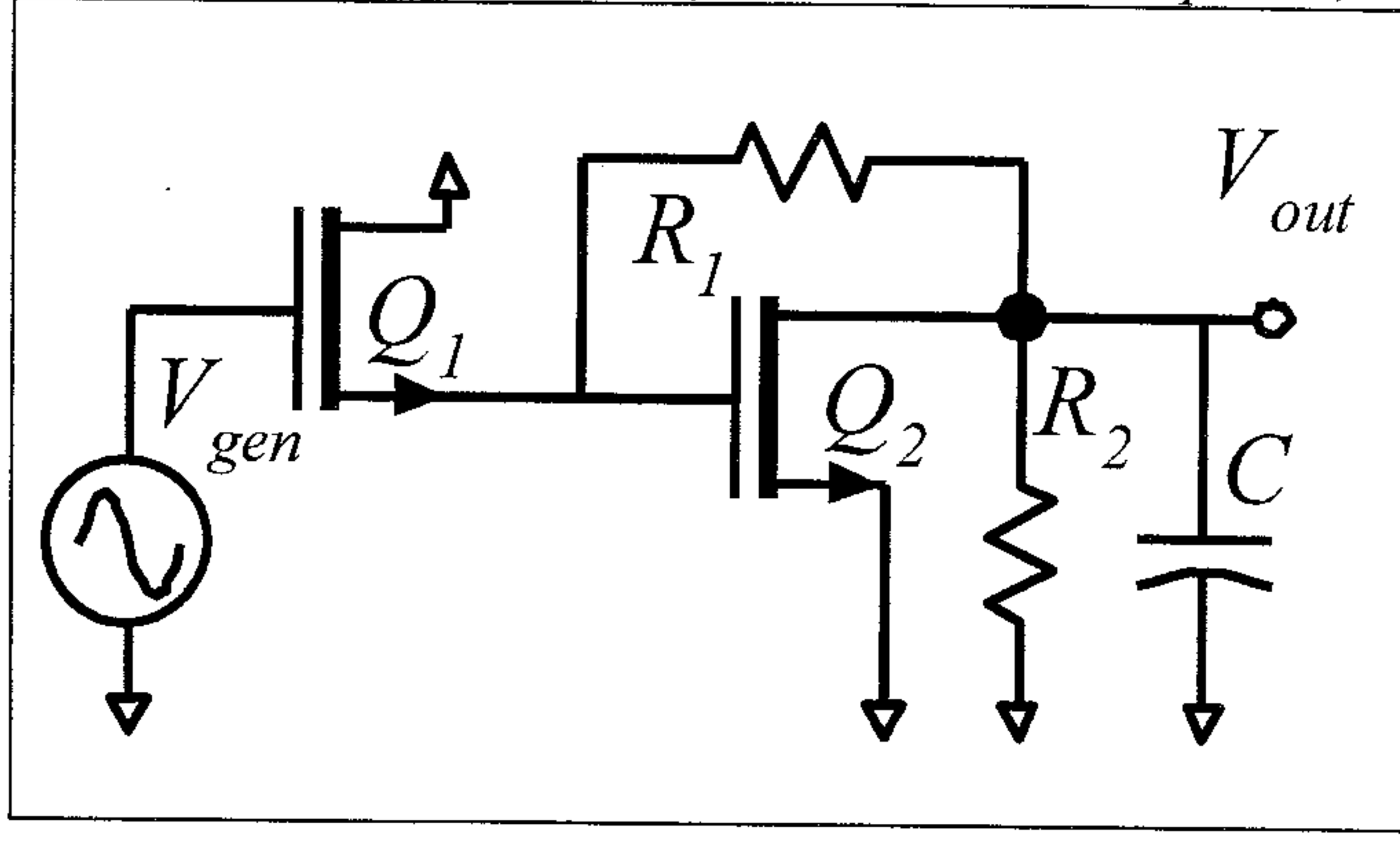
(2)

or ok

$$= -g_{m1}g_{m2}R_1R_2 - g_{m1}R_2 \quad \boxed{= -g_{m1}(1 + g_{m2}R_1)R_2}$$

Problem 3: 35 points

Nodal analysis, frequency and transient response, transistor circuit models



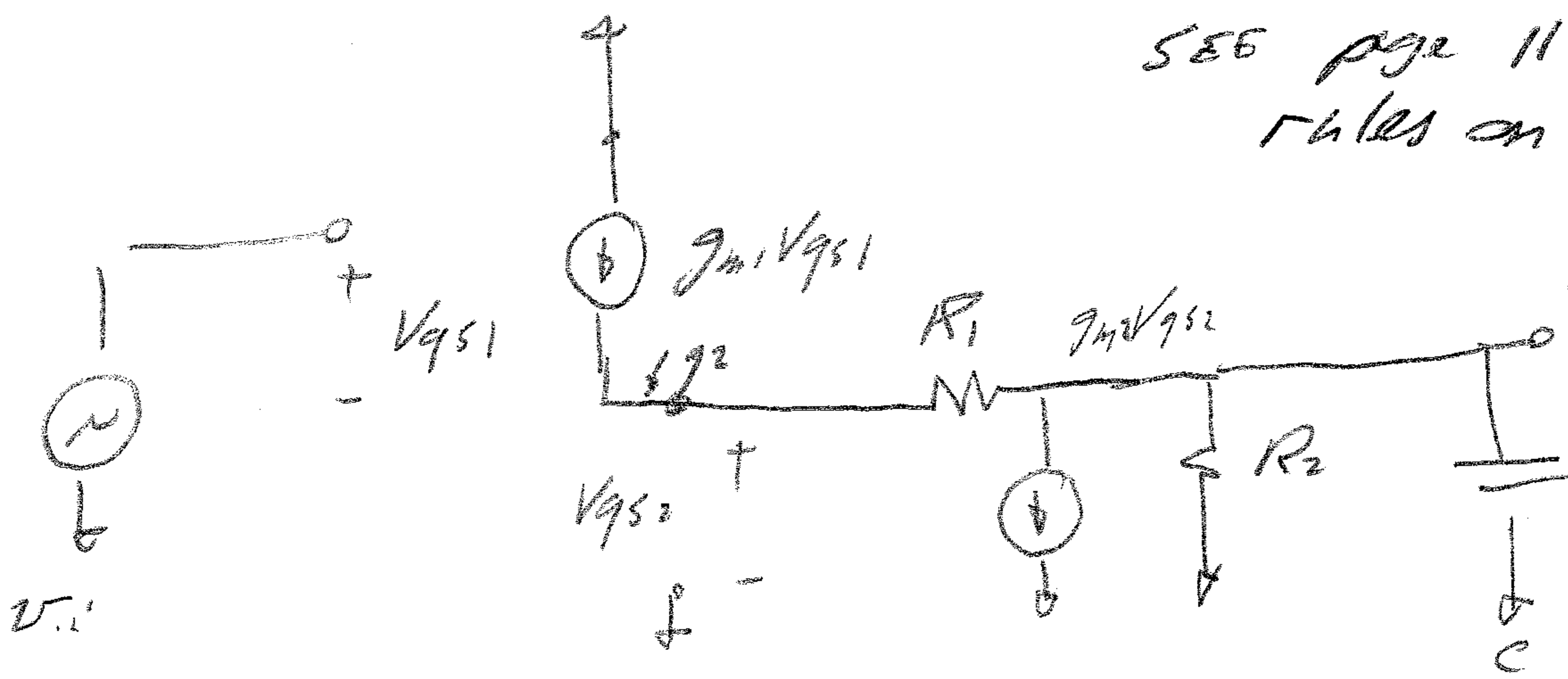
Ignore DC bias; you don't need it.

The transistors have zero C_{gs} and C_{gd}, and have transconductance g_{m1} and g_{m2}. Both have zero G_{ds}

Part a, 10 points

Draw an accurate small-signal equivalent circuit model of the circuit above.

SEE page 11 for rules on scoring



Part B

$\Sigma I = 0$ @ V_{g2}

$$V_{g2}[g_{m1} + G_1] + V_{out}[-G_1] = v_{in}[g_{m1}]$$ (3)

$\Sigma I = 0$ @ V_{out}

$$V_{g2}[g_{m2} - G_1] + V_{out}[G_1 + G_2 + sC] = 0$$ (3)

Part b, 10 points

Using NODAL ANALYSIS, find the transfer function Vo(s)/Vgen(s)

The answer must be in standard form $\frac{V_o(s)}{V_{gen}(s)} = \frac{V_o}{V_{gen}}\Big|_{low-frequency-value} \times \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots}$,

$\frac{V_o(s)}{V_{gen}(s)} =$ ________________________

$$\begin{bmatrix} g_{m1} + G_1 & -G_1 \\ g_{m2} - G_1 & G_1 + G_2 + sC \end{bmatrix} \begin{bmatrix} V_{g2} \\ V_{out} \end{bmatrix} = \begin{bmatrix} g_{m1} \\ 0 \end{bmatrix} \cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{N}{D}$$

$$N = \begin{vmatrix} g_{m1} + G_1 & g_{m1} \\ g_{m2} - G_1 & 0 \end{vmatrix} = -g_{m1}(g_{m2} - G_1)$$

$$D = \begin{vmatrix} g_{m1} + G_1 & -G_1 \\ g_{m2} - G_1 & G_1 + G_2 + sC \end{vmatrix} = g_{m1}G_1 + g_{m1}G_2 + G_1G_1 + G_1G_2 + g_{m2}G_1 - G_1G_1 + sC(g_{m1} + G_1)$$

$$= g_{m1}(G_1 + G_2) + (g_{m2} + G_2)G_1 + sC(g_{m1} + G_1)$$

$$\frac{V_o}{V_{gs}}(s) = \frac{-g_{m1}(g_{m2} - G_1)}{g_{m1}(G_1+G_2) + (g_{m2}+G_2)G_1 + sC(g_{m1}+G_1)}$$

low frequency gain

$$= \frac{-g_{m1}(g_{m2} - G_1)}{g_{m1}(G_1+G_2) + (g_{m2}+G_2)G_1} \cdot \frac{1}{1+s\tau}$$

where $\tau = C \dfrac{g_{m1} + G_1}{g_{m1}(G_1+G_2) + (g_{m2}+G_2)G_1}$

credit 2 of 4 if done numerically only

(4)

part c, D

$g_{m1} = 10mS$, $g_{m2} = 5mS$, $R_1 = 1k\Omega$, $R_2 = 10k\Omega$

$C = 1pF$ $G_1 = 1mS$ $G_2 = 0.1mS$

$$\text{low frequency gain} = \frac{-g_{m1}(g_{m2}-G_1)}{g_{m1}(G_1+G_2)+(g_{m2}+G_2)G_1} = \frac{-10mS\cdot(4mS)}{10mS(1.1mS)+(5.1mS)1mS}$$

$= -2.48 \rightarrow 20\log_{10}(2.48) = -7.90\,dB$

$$\frac{g_{m1}+G_1}{g_{m1}(G_1+G_2)+(g_{m2}+G_2)G_1} = \frac{10mS + 1mS}{16.1mS} = 683\Omega$$

$\tau = 683\Omega \cdot 1pF = 683ps$

Part c, 5 points

g_{m1}=10 mS, g_{m2}=5 mS. R_1=1,000 Ohms. R_2=10,000 Ohms. C= 1 pF.
How many poles are there in the transfer function ?
Give its frequency / their frequencies:
f_{p1}= 233 MHz , f_{p2}= X , f_{p3}= X ....

See previous page.

(2) $\tau = \dfrac{g_{m1} + G_1}{g_{m1}(G_1+G_2)+(g_{m2}+G_2)G_1} \cdot C = 683\,\Omega \cdot C$

(1) $\tau = 683$ ps

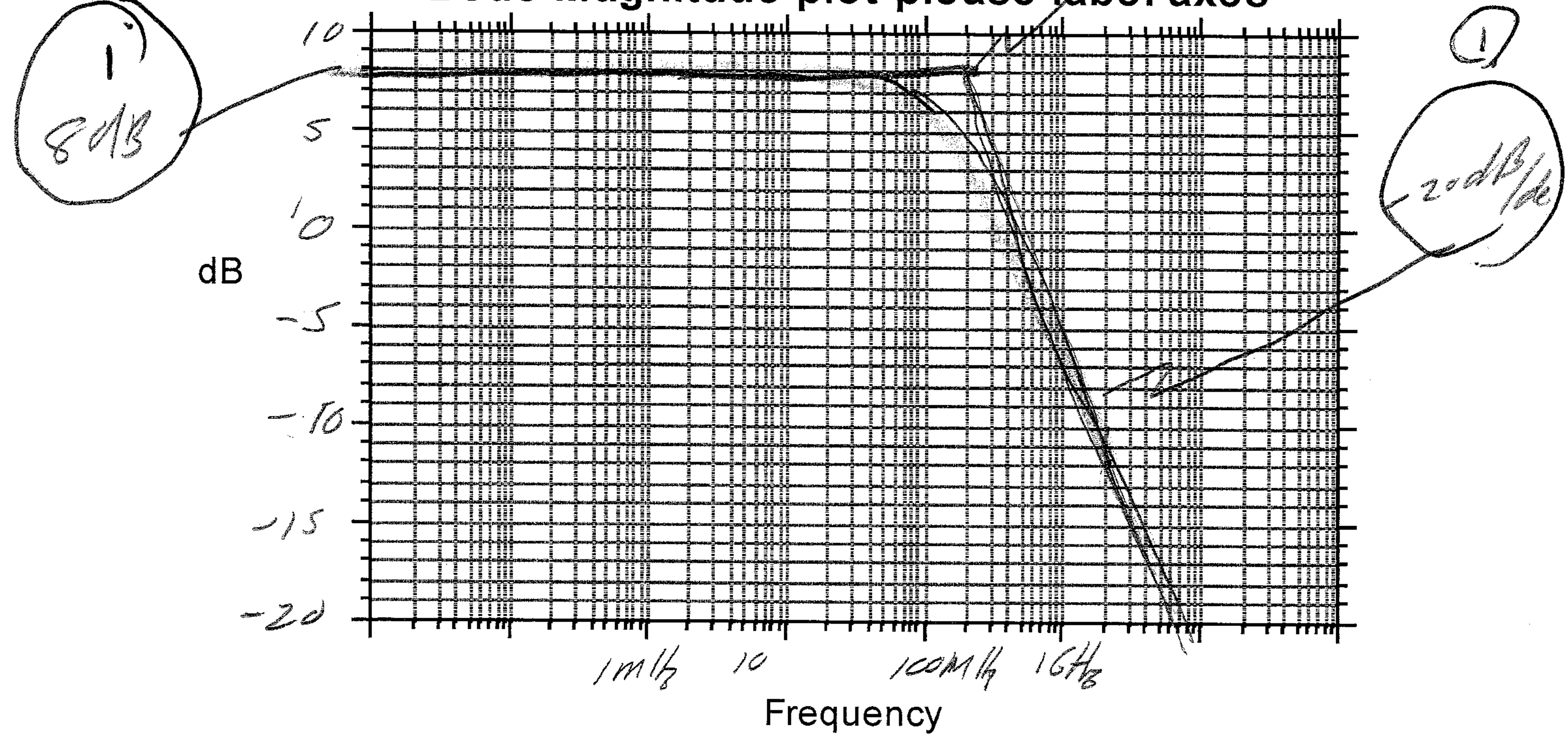
(1) $f_p = \dfrac{1}{2\pi\tau} = 233$ MHz

Part d, 5 points

Make an accurate Bode plot of Vout/Vgen, labeling all slopes, and all key gain and frequency values. Make sure you draw the straight-line asymptotes, and then sketch the true curve.

$f_c = 233\,MHz$

Bode Magnitude plot-please label axes



(2) See page 17

$= \frac{-g_{m1}(g_{m2}-g_1)}{g_{m1}(G_1+G_2)+(g_{m2}+G_2)G_1} = -2.49$

Low frequency gain = 7.9 dB

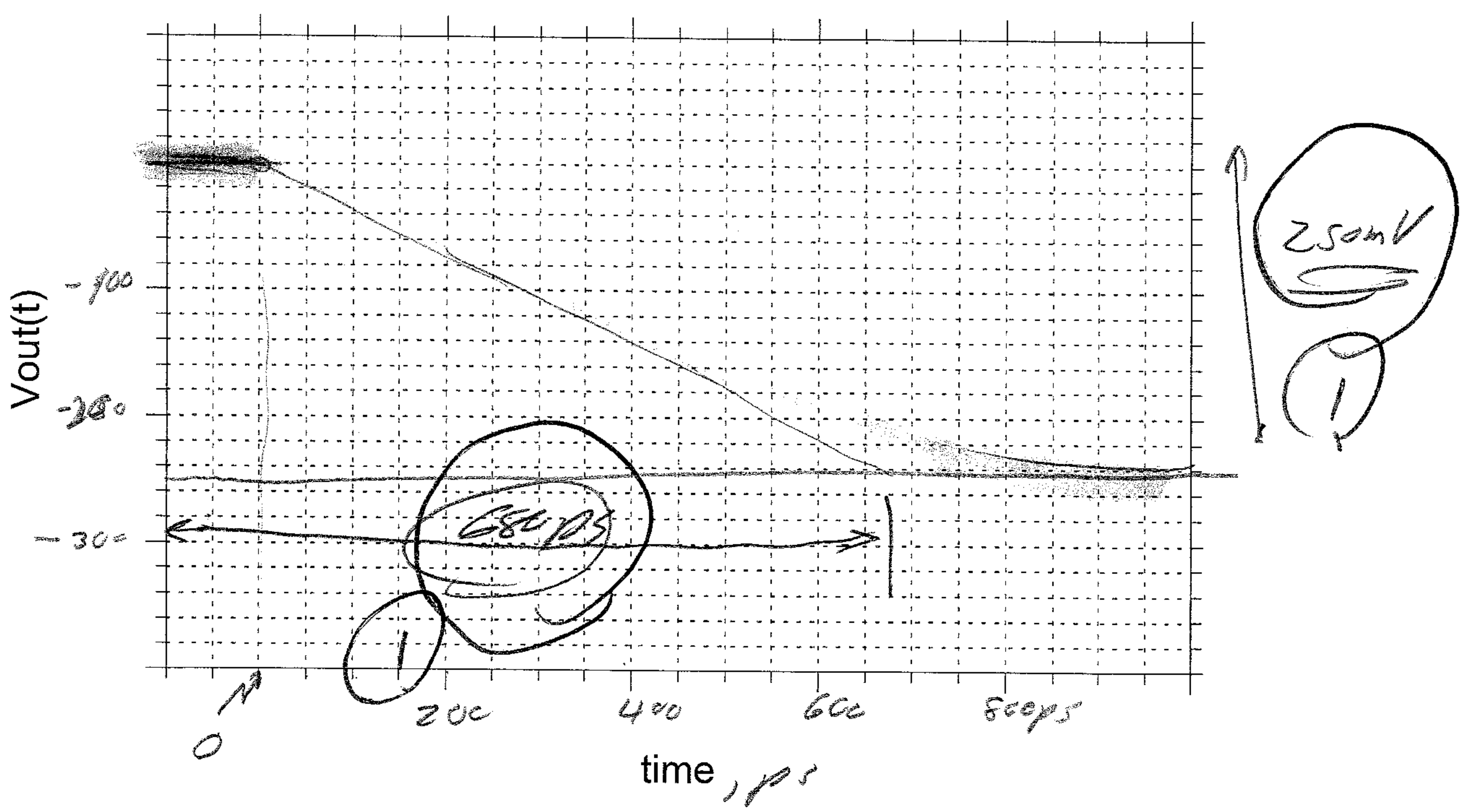
$\tau_{pole} = 683\,ps$

$f_{pole} = 233\,MHz$

Part e, 5 points

If Vgen(t) is a 10 mV step-function, find and ***accurately*** plot Vout(t). ***Be sure to label both axes and give units.***

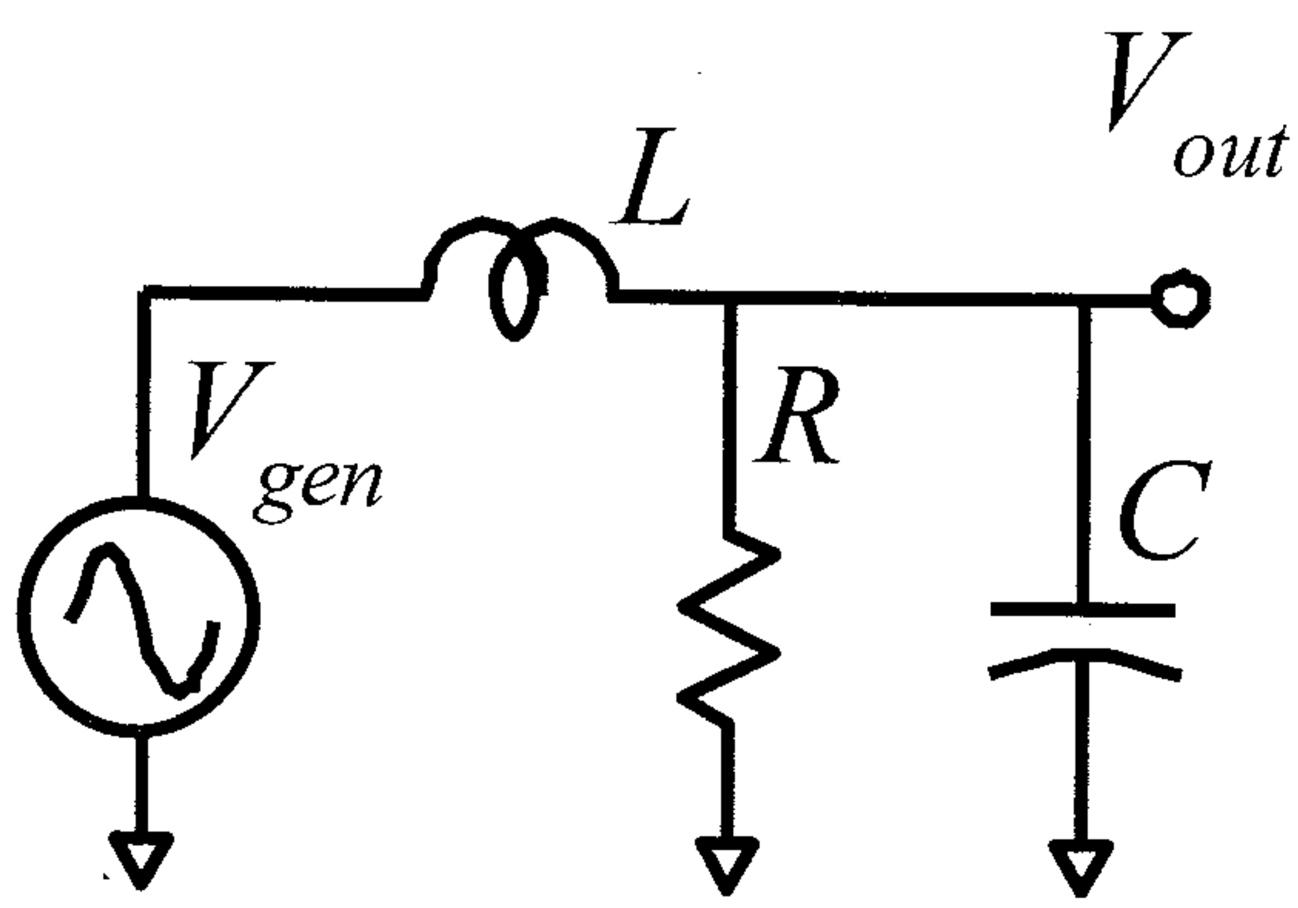
Vout(t)=________________________________



$A_{v_1} = -2.48, \quad \tau = 680\,ps.$

③ $V_{out}(t) = -248\,mV \cdot (1 - e^{-t/680ps}) \cdot u(t)$

Problem 4: 25 points
frequency and transient response



You are solving frequency and transient response of the network to the left.

Part a , 10 points

Using NODAL ANALYSIS, find the transfer function Vo(s)/Vgen(s)

The answer must be in standard form $\frac{V_o(s)}{V_{gen}(s)} = \frac{V_o}{V_{gen}}\bigg|_{low-frequency-value} \times \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots}$,

$$\frac{V_o(s)}{V_{gen}(s)} = \frac{1}{1 + sL/R + s^2 LC}$$

(5) $\Sigma I = 0$ @ V_{out}:

$$V_{out}\left(sC + G + 1/sL\right) + V_i\left(-1/sL\right) = 0$$

(5)

$$\frac{V_{out}}{V_i} = \frac{1/sL}{sC + G + 1/sL} = \frac{1}{1 + sLG + s^2 LC}$$

Part b, 5 points

Now evaluate with L=31.8 nH, C=0.796 pF, R=1000 Ohm

How many poles are there in the transfer function ?

If there are one or two poles, and if they are real, give f_{p1} and possibly f_{p2}:

f_{p1}=______________ , f_{p2}=______________

If the two dominant poles are complex, give $f_n = \omega_n / 2\pi$ and ζ :

$f_n = \omega_n / 2\pi =$ 10^9 Hz , $\zeta =$ 0.10

$$H(s) = \frac{1}{1 + sL/R + s^2 LC} = \frac{1}{1 + s\,2\zeta/\omega_n + s^2/\omega_n^2}$$

$$\Rightarrow \omega_n = 1/\sqrt{LC} \rightarrow f_n = \frac{1/2\pi}{\sqrt{LC}} = 10^9 \text{ Hz}$$

(3)

$$\Rightarrow \zeta = \frac{1}{2R}\sqrt{\frac{L}{C}} = 0.10$$

(2)

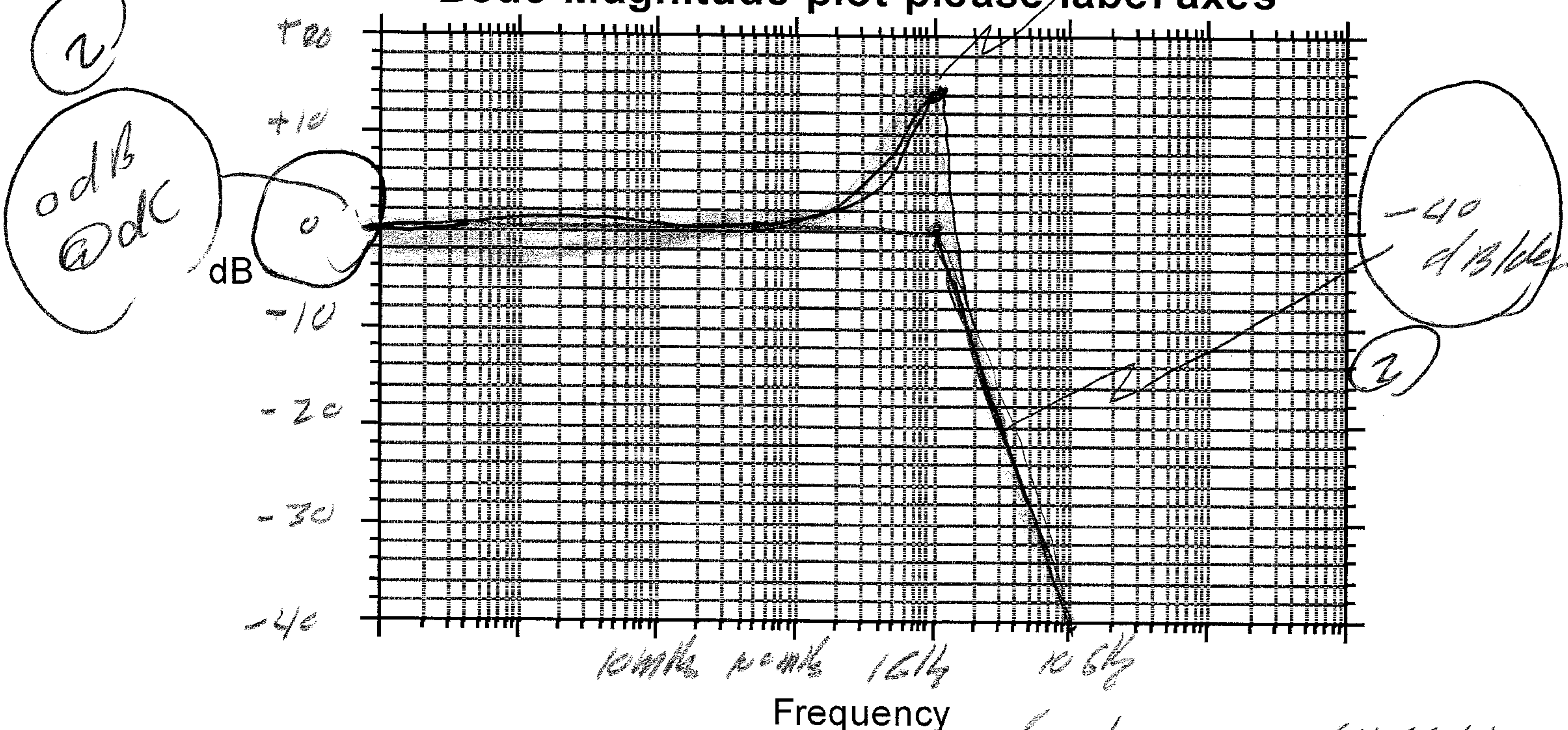
(1)

Part d, 5 points

Make an accurate Bode plot of Vout/Vgen, labeling all slopes, and all key gain and frequency values. Make sure you draw the straight-line asymptotes, and then sketch the true curve.

14dB peak @ 1GHz

Bode Magnitude plot-please label axes



$$H(\omega) = \frac{1}{1 + j\omega 2\zeta/\omega_n - \omega^2/\omega_n^2} = \begin{cases} 1 & \omega \ll \omega_n \\ \frac{-j}{2\zeta} & \omega = \omega_n \\ \frac{\omega_n^2}{\omega^2} & \omega \gg \omega_n \end{cases}$$

$$\frac{1}{2\zeta} = \frac{1}{0.2} = 14\,dB$$

$f_n = 1\,GHz$