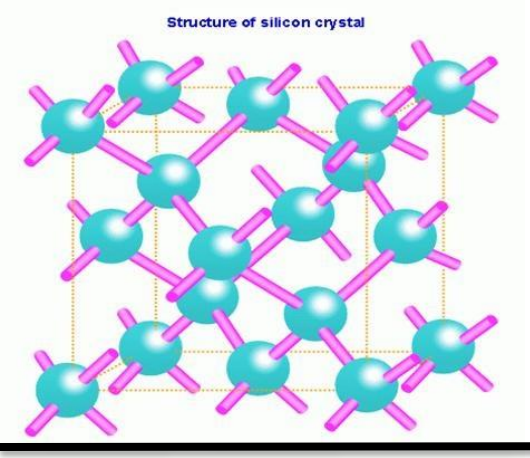

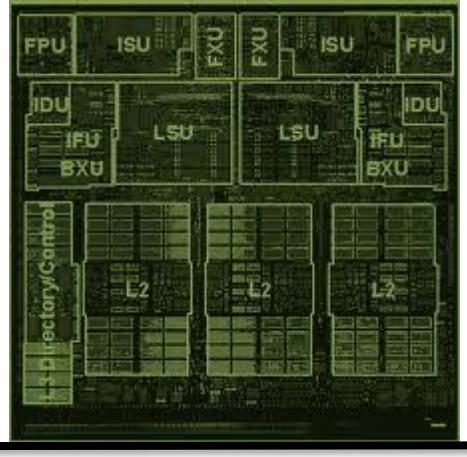

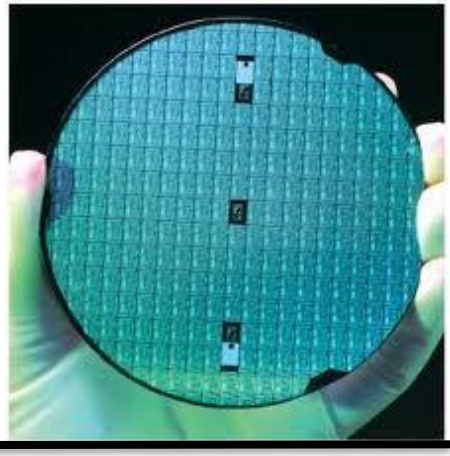

ECE 122A

VLSI Principles

Lecture 5

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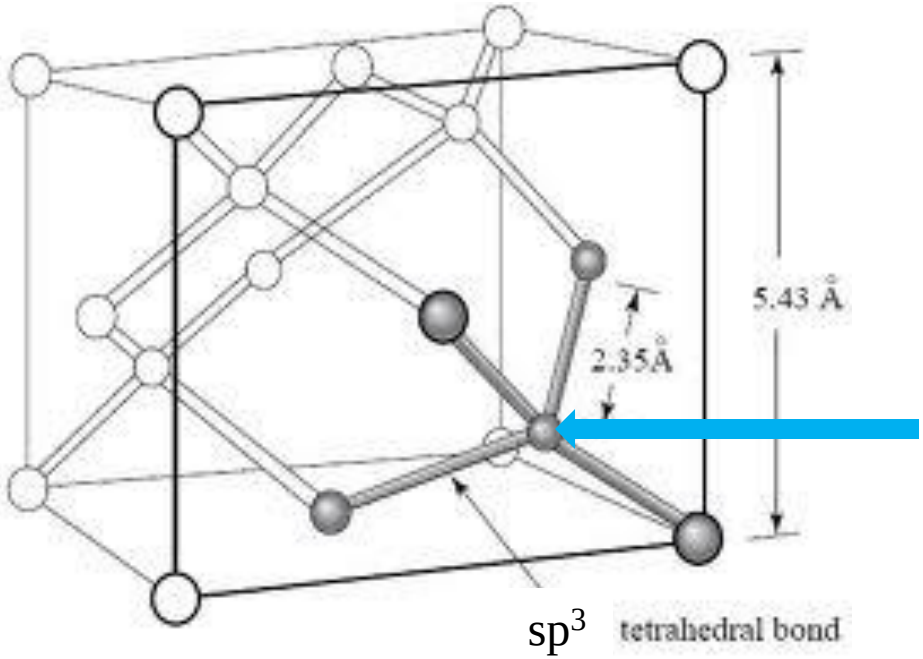
Semiconductors/Metals/Insulators

OUTLINE

- Energy band diagrams – metal/semi/insulator
- Fermi level E_F
- Intrinsic semiconductors
  - $n_0 = p_0$
  - Intrinsic Fermi level $E_i \sim \frac{1}{2} (E_V + E_C)$
- Extrinsic Semiconductors – Doping
  - n-doped semi: Donor Levels
  - p-doped semi: Acceptor Levels
- Current in Semiconductors

Bond Model

- Completed or broken electronic bonds…used to describe behavior of electrons and holes
- Consider the diamond-type crystal lattice (for Si and Ge): sp^3 bonding--one s-orbital and three p-orbitals undergo a sp^3 hybridization

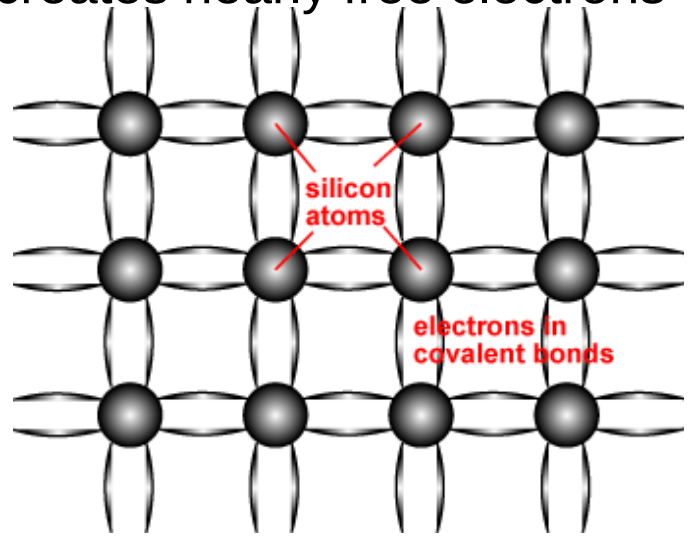


Unit cell of Si crystal:

- The sp^3 hybridized orbital has a tetrahedral symmetry (i.e., a central atom is located at the center with four substituents that are located at the corners of a tetrahedron. The bond angles are ≈ 109.5°

Bond Model…for Si

- Each atom has covalent bonds with its four nearest neighbors
- Two tightly bound electrons associated with each bond---one from each atom
  - At absolute zero temperature, all electrons are held in these bonds, none are free to move
  - At higher temperatures, thermal energy breaks some of the bonds and creates nearly free electrons



Band Model of Solids

- An electron acted on by the Coulomb potential of an atomic nucleus can have only certain allowed states below a reference energy taken as E = 0 (why?)
- Hint: think of the electron as a wave…

$$E_n = \frac{-Z^2 m_0 q^4}{8 \varepsilon_0^2 h^2 n^2}$$

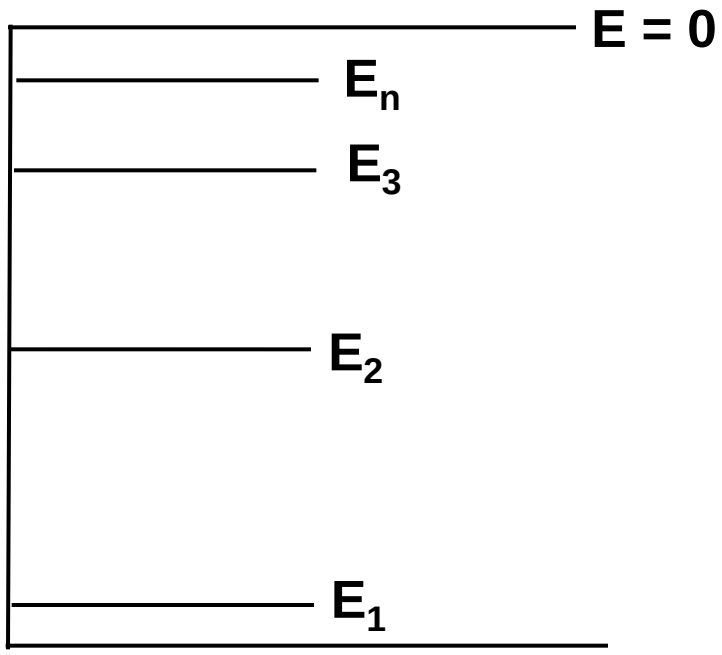
Z= # of protons in nucleus

q = charge of electron (= -1.6x10^{-19} C)

m_0 = free electron mass

For Hydrogen atom, Z=1, E_n =-13.6/n^2 eV

Pauli Exclusion Principle (PEP): at most 2 electrons (of opposite spins) can occupy an energy level



Band Model of Solids

- Now consider the electron in the highest occupied energy level (E_n)
  - when two atoms are separated by a large distance, electron associated with each atom has energy E_n
  - If the atoms approach one another, atomic core of first atom exerts a force on the second electron, changing the potential that determines the energy levels of the electron
  - All allowed energy levels for the electron are consequently modified….
  - Considering PEP for the two-atom system: an energy level E_n can contain at most 2 electrons----so when two atoms are brought close together--- need to find distinct energy levels for the 4 electrons!
  - Energy level E_n of the isolated atom must split into two with slightly differing energies to accommodate the 4 electrons
  - If N atoms are brought together, E_n will split into N different levels…forming an **energy BAND**….that can contain at most 2N electrons

Band Model of Solids (Cont'd)

- Number of atoms in a crystal is large: of the order of 10^{22} atoms/cm^{-3} and the total extent of the energy band is of the order of a few electron volts (eV) (Note: $1\ eV = 1.6 \times 10^{-19}\ J$)
- Hence, separation between the N different energy levels within a band is much smaller than the thermal energy (kT/q = ~1/40 eV) possessed by an electron at room temperature----therefore the electrons can move freely between these levels----thus we can speak of a continuous band of allowed energies
- This allowed BAND is bounded by maximum and minimum energies and may be separated from adjacent allowed bands by forbidden-energy gaps, or it may overlap other bands
- The size of the BAND gap essentially defines the electronic properties of a given material: conductors (zero band-gap), semiconductors (small band-gap), insulators (larger band-gap)

Band Model of Solids (Cont'd)

- Although each energy level of the original isolated atom splits into a BAND composed of 2N levels, the range of allowed energies of each band can be different
- The higher energy bands generally span wider energy range than do those at lower energies….why?
  - Because the electrons in the higher energy levels are more loosely bound to the atomic nucleus and can move farther from it, thereby coming closer to the adjacent atoms and getting more strongly influenced by them

Metals, Semiconductors and Insulators

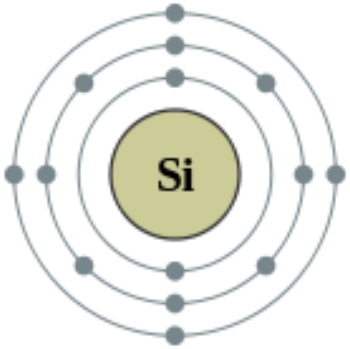
- Conduction happens if one band is neither empty nor full….*think of a half-full water bottle*
- Consider N sodium atoms
  - Total number of available states = 2N (at highest occupied energy level)
  - Factor of 2 because each energy level can hold 2 e^- with opposite spins
  - Na = $1s^2\ 2s^2\ 2p^6\ 3s^1$
  - Only $3s^1$ contributes to conduction: inner electrons are too tightly bound to their nuclei
  - So there are only N conduction electrons
- Thus Na has a half-filled band → **metallic**

Metals, Semiconductors and Insulators

- Alkaline earth metals (Be, Mg, Ca....)
- 2 conducting e^- per atom
- Lower band full and upper band empty, going by e^- count, one would expect them to be insulating
- However, conduction also happens if adjacent bands overlap in energy
- The upper band gets partially filled at the expense of the lower
- This makes alkaline earth metals metallic, though poorly so

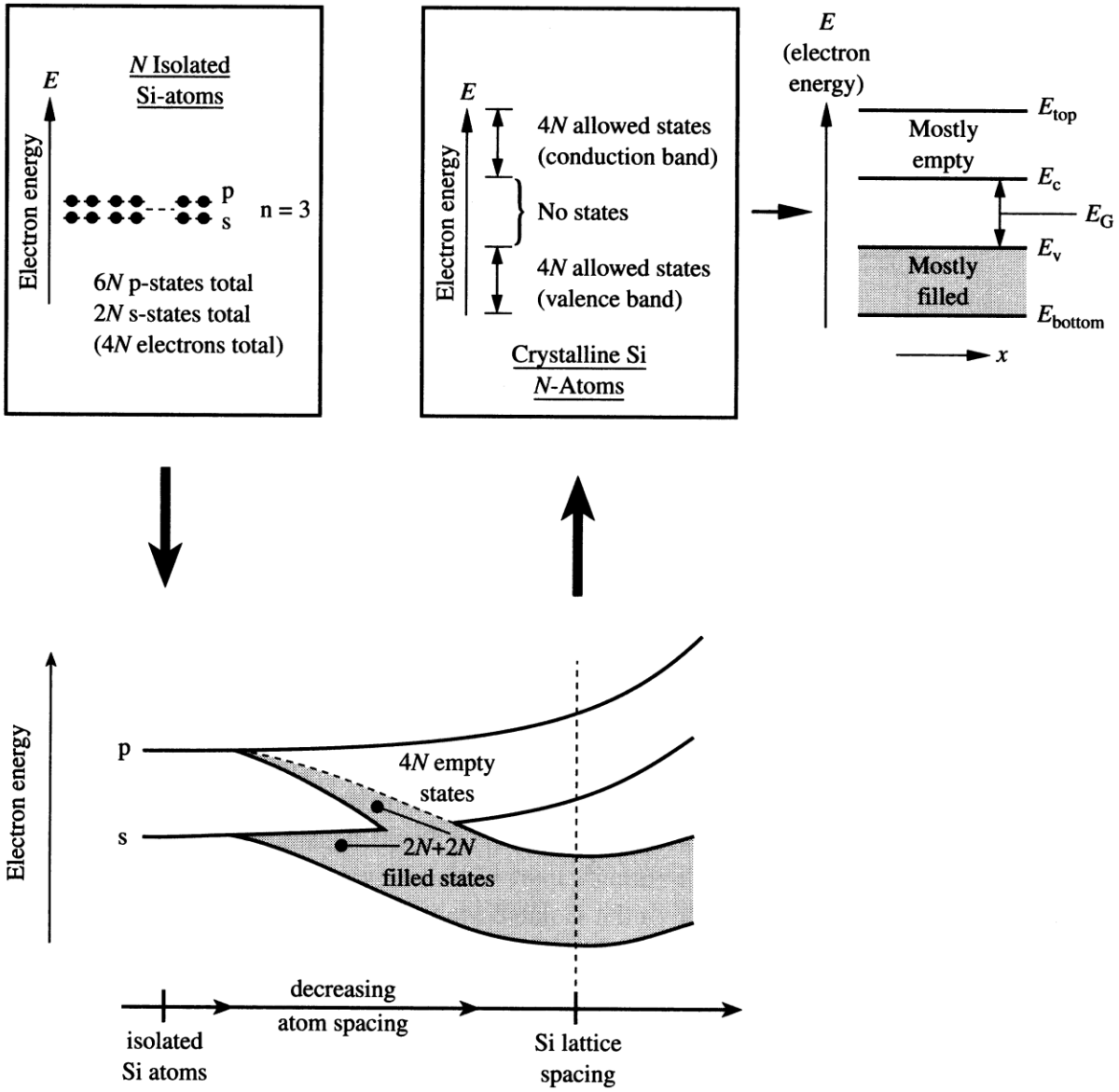
Band Structure of Silicon

- Atomic number of Si =14
- Hence, it has 14 protons and 14 electrons to keep it neutral
- The 14 electrons are placed into the following 3 energy levels and 5 orbitals: $1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^2$
- We have a total of 8 places (states) available in the last two s and p orbitals….but there are only 4 electrons
- These are the 4 valence electrons



4 valence electrons

Energy-Band Model of Silicon



Metals, Semiconductors and Insulators

- Conditions for insulator
  - Non-overlapping bands
  - Fully filled valence band
  - Empty conduction band

- Examples: pure diamond, silicon, germanium at absolute zero

- Why absolute zero? To ensure no thermal excitation of e^- from VB to CB

Metals, Semiconductors and Insulators

- So what distinguishes insulators from semiconductors?
  - *Ans*: The magnitude of the band-gap
- Semiconductors have smaller band-gap than that of insulators
- Impurities in semiconductors are almost fully ionized at room temperatures
- Negligibly so in insulators….
- Si with E_G=1.12 eV is a semiconductor
- SiO_2 with E_G ~ 5 eV is an insulator
  - ***Ref.: Charles Kittel, 'Introduction to Solid State Physics', 7th edition***

Electron Concentration (n) in a Semiconductor

- Calculation of **n** needs two concepts
  - Density of States (DOS)
  - Fermi-Dirac Occupation Function

- DOS:
  - Definition: D(E) dE = # of "allowed" energy states between E and E+dE per unit volume
  - Expressed in cm^{-3} (for 3D or bulk semiconductors like Si)

Electron Concentration in a Semiconductor

- Fermi-Dirac Occupation Function

$$f_D(E) = \frac{1}{1 + e^{\frac{-(E_F - E)}{k_B T}}}$$

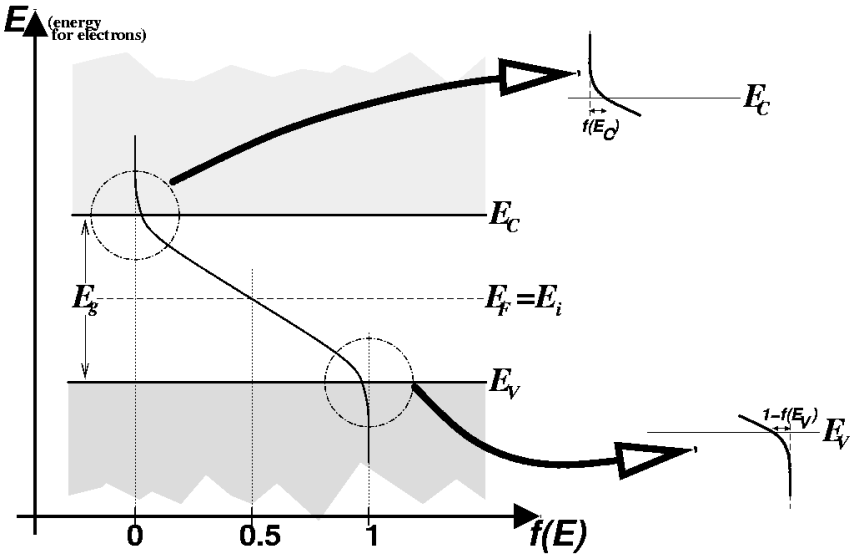
- $f_D(E)$ = 0.5 at E = E_F (E_F is the Fermi level…)

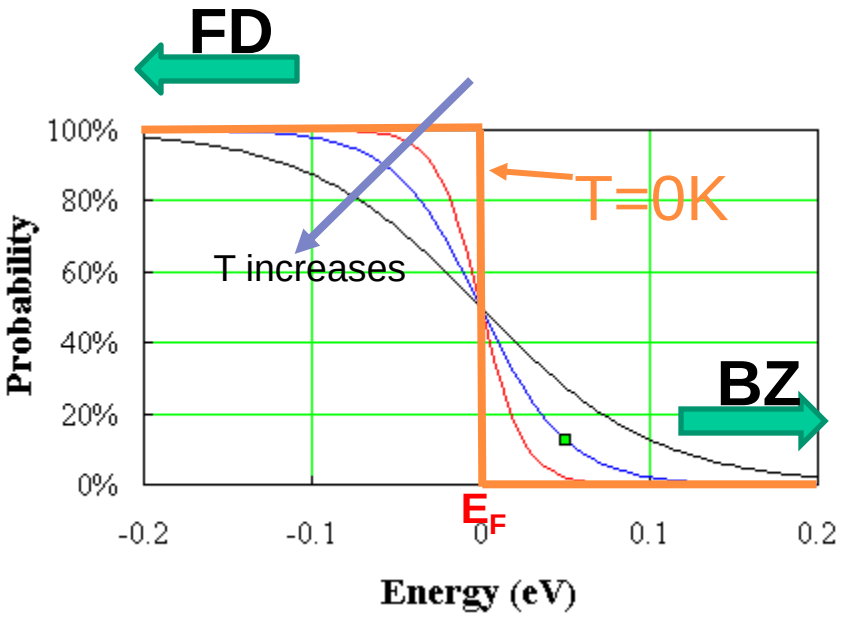
- $f_D(E)$ = probability of occupation of a state of energy E, given that such a state exists.
  - Probability of a hole in a state of energy E is $(1 - f_D(E))$

(Why?)

Occupation Function….

❑ Fermi-Dirac Function f(E):





- At higher temperatures, **$(E-E_F) >> kT$**
- **Fermi-Dirac (FD) distribution function reduces to the Boltzmann (BZ) distribution function: $f(E) = \exp(-(E-E_F)/kT)$**

Electron and Hole Concentrations

$$n = \int_{E_C}^{\infty} D(E) f(E) dE$$

- Integrating:

$$n_0 = N_C e^{\frac{(E_F - E_C)}{k_B T}}$$

- Analogous expression for hole concentration:
  - (A hole is a vacant state in a band)

$$p_0 = N_V e^{\frac{(E_V - E_F)}{k_B T}}$$

N_C and N_V are called the "effective density of states"….why?

Equilibrium Concentrations (1)

- Subscript 0 in n_0 and p_0 indicates concentration in thermal equilibrium
  - (no applied bias, no light shining on the semiconductor....only thermal generation and recombination of electrons and holes)

- **Nowhere in the derivation of n_0 or p_0 have we explicitly assumed intrinsic (undoped) semiconductor**

- Doping moves E_F closer to either CB (n-type) or VB (p-type)

- Exercise: calculate n_0 for intrinsic (undoped, pure) Si at 300K
  - $N_c = 3.2 \times 10^{19}$ cm^{-3}
  - $E_i - E_V = 0.5506$ eV
  - $E_g = 1.11$ eV

- Result: $n_0 = 1.45 \times 10^{10}$ cm^{-3} = p_o

Equilibrium Concentrations (2)

- Intrinsic semiconductor: $n_0 = p_0 = n_i$
- n_i is called the **intrinsic concentration**
- For Si, $n_i = 1.45 \times 10^{10}$ cm^{-3}
- Assume $N_C \sim N_V$: Then for intrinsic semiconductor:

$$E_F = (E_C + E_V)/2$$

- Using previous equations for n_0 and p_0, it can be shown that for *any* doping:

$$n_0 p_0 = n_i^2$$

⇒ Valid for all semiconductors, intrinsic or extrinsic

Equilibrium Concentrations (3)

- One more useful relationship between n_0, p_0, n_i:

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

- Given position of Fermi level, find n_0 and p_0

OR

- Given n_0 and p_0, find location of Fermi level
- Sanity check: for intrinsic, $E_F = E_i$, and $n_0 = p_0 = n_i$

Extrinsic Semiconductors

- To increase conductivity, *doping* adds impurities to silicon crystal

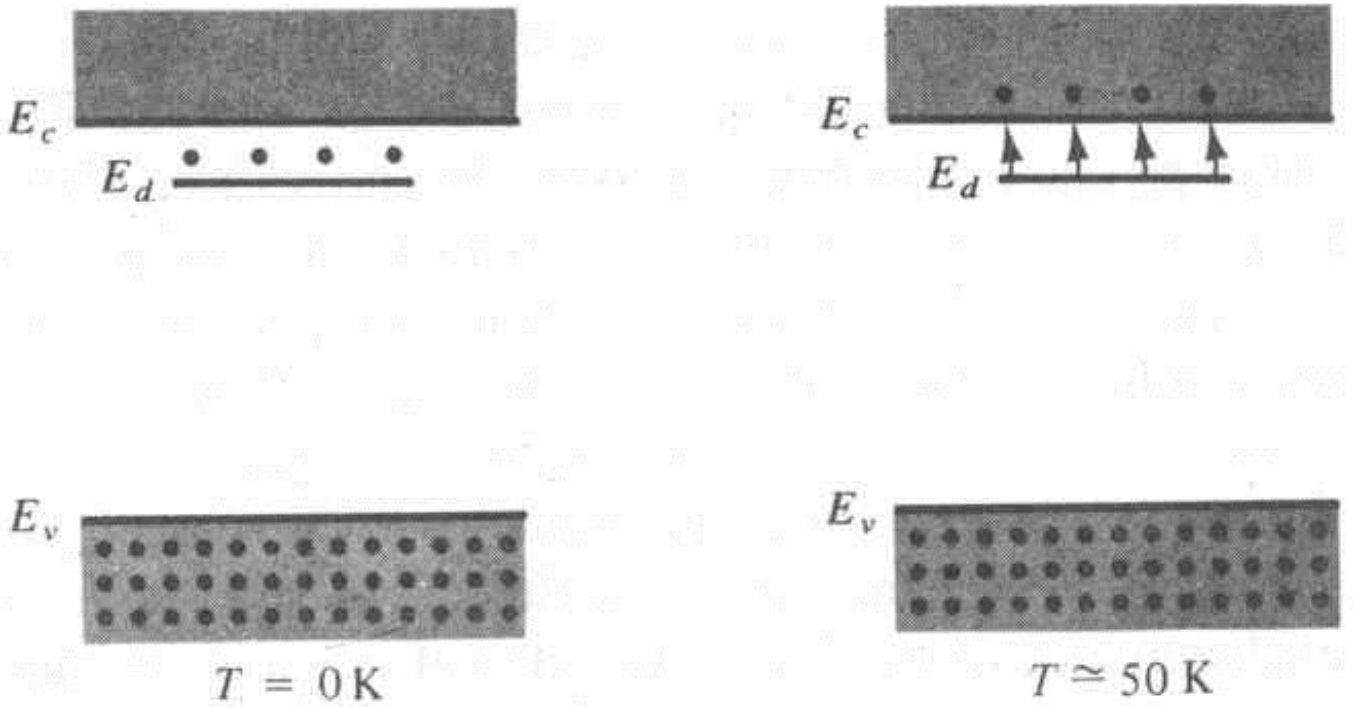
- Two types of impurities:
  - **donors**: Loosely-bound electron, donated to crystal lattice
  - **acceptors**: Free electron orbit, accept electron from lattice
- Impurities are ionized when they donate or accept an electron
  - Note: Semiconductor is still electrically neutral!

N-Type Semiconductors

- N-Type: Large concentration of electrons in conduction band
- Created by donor impurities
  - One extra electron than Silicon
  - Silicon: periodic table column IV
  - Donors: periodic table column V
    - Phosporus (P)
    - Arsenic (As)
    - Antimony (Sb)
- Doping concentration: N_D (atoms/cm^3)

N-Type Semiconductors (2)

- Donor impurities create "donor level" in energy band diagram

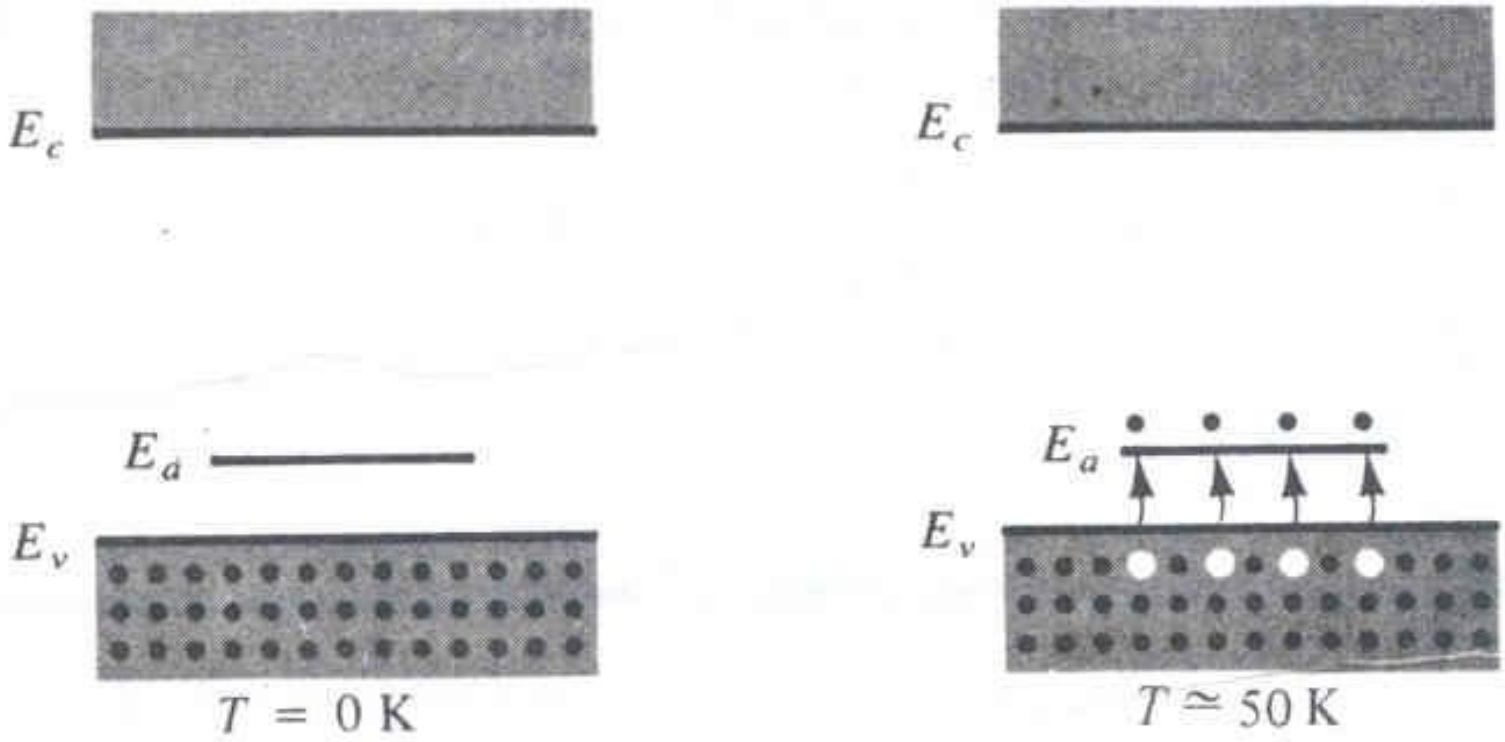


P-Type Semiconductors

- P-Type: Large concentration of holes in valence band
- Created by acceptor impurities
  - Acceptors: one fewer electron than Silicon
  - Silicon: periodic table column IV
  - Donors: periodic table column III
    - Boron (B)
    - Aluminum (Al)
    - Gallium (Ga)
- Doping concentration: N_A

P-Type Semiconductors (2)

- Acceptor impurities create "acceptor level" in energy band diagram



Strongly-Doped Semiconductors

- Strong doping: concentration of ionized dopants is much higher than equilibrium concentration n_i
- In that case, assume all free electrons (N-type) or free holes (P-type) are due to impurity.
- Example:
  - Intrinsic Si:

    $n_0 = 1.45 \times 10^{10}$ cm^{-3}
  - n-type Si doped with 10^{17} P atoms/cm^3:

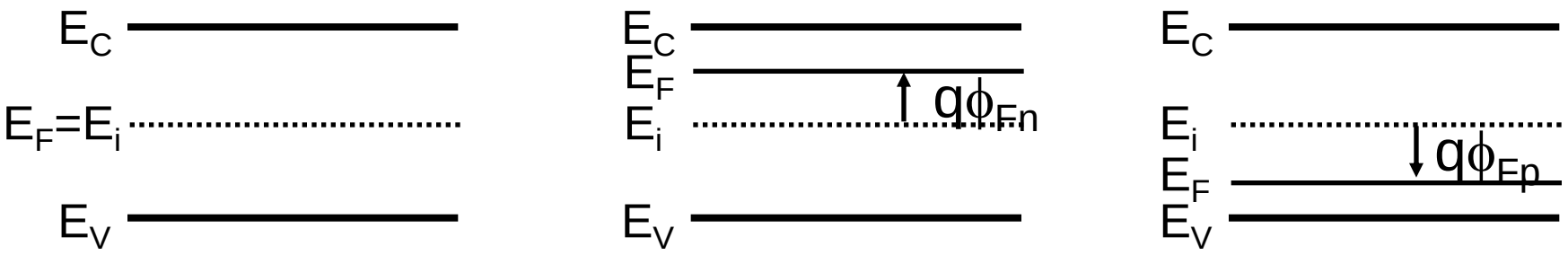
    $n_0 = 10^{17}$ cm^{-3}

Example: Extrinsic Doping

- Silicon is doped with 10^{16} As atoms/cm^3
  - Assume all of the As is ionized
- What type of semiconductor is produced?
- What are equilibrium electron and hole concentrations?

Extrinsic Energy Band Diagram

- Effect of doping on Fermi level E_F
  - N-Type: Fermi level moves up
  - P-Type: Fermi level moves down



ϕ_F is called the Fermi potential

$$q\phi_F = E_F - E_i$$

ϕ_F: V
q ϕ_F: eV

Extrinsic Energy Band Diagram (2)

- Recalling these equations:

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

- New position of E_F can be found for any doping level

- Example: find position of E_F for last example with 10^{16} As atoms/cm^3

Extrinsic Energy Band Diagram (3)

For n-type semi

$$n_0 = N_D = n_i e^{(E_F - E_i)/kT}$$

$$E_F - E_i = kT \ln \frac{N_d}{n_i}$$

$$q\phi_{Fn} = kT \ln \frac{N_D}{n_i}$$

(since $q\phi_F = E_F - E_i$)

For p-type semi

$$p_0 = N_A = n_i e^{(E_i - E_F)/kT}$$

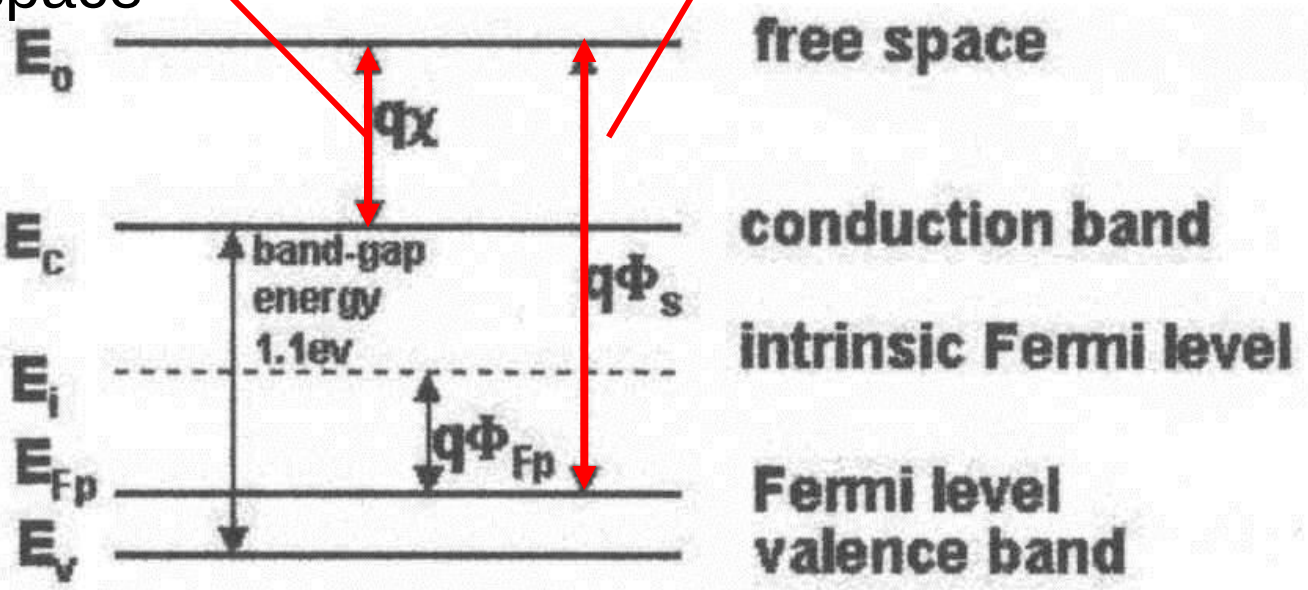
$$E_i - E_F = kT \ln \frac{n_i}{N_A}$$

$$q\phi_{Fp} = kT \ln \frac{n_i}{N_A}$$

Extrinsic Semiconductor

Electron affinity: energy to move electron from conduction band to free space

Work function: energy to move electron from Fermi level to free space



Energy band diagram of a p-type silicon

Electron Currents

- Two types of current in semiconductors
  - Drift current: Electron motion due to electric field
  - Diffusion current: Electron motion due to differences in carrier concentration
- Semiconductors rely on interaction between these two currents!
- Current density (J) is always used rather than actual current value: A/cm^2
- Actual current $I = J \bullet A$

Drift Current

- With no electric field, there is no net motion of electrons, but each electron moves randomly….why?
- With electric field, there is net force on each electron, causing acceleration
- Acceleration causes collisions, which balance electric field
- Force on electron: $F_x = -q\mathrm{E}_x$
- Average net velocity: $\langle v_x \rangle = -\frac{q\bar{t}}{m_n^*} E_x$

Drift Current (2)

- Given average velocity, find current density:

$$J_x = \frac{nq^2\bar{t}}{m_n^*}\mathrm{E}_x$$

Hint: estimate the **charge** crossing an area **A** in time **dt….. = nq<v>Adt**

- Define μ_n = electron mobility: ease with which electrons drift in a material

$$\mu_n = \frac{q\bar{t}}{m_n^*}$$

$$J_x = qn\mu_n\mathrm{E}_x$$

J_x = A/cm^2
n = electrons/cm^3
μ = cm^2/V-s

Drift Current (3)

- Therefore current density is proportional to electric field (Ohm's Law):

$$J_x = \underbrace{qn\mu_n}\mathrm{E}_x$$

$$\sigma = qn\mu_n$$

$$\mathrm{J}_x = \sigma E_x$$

or V = I.R
Since, J = I/A = E/ρ
And E = V/L

- σ is the conductivity of the material (σ=1/ρ, where ρ=resistivity)

Drift Current Example

- Find the approximate electron current for intrinsic silicon
  - Size = 1 cm^3
  - Voltage applied: 1V
  - μ_n = 600 $cm^2/V \bullet s$
    q = 1.6 x 10^{19} C

Diffusion Current

- Diffusion is due to electron or hole concentration gradient
  - Concentration gradient = d_n/d_x or d_p/d_x
- Flux density ϕ_n, ϕ_p is rate of electron or hole flow, per unit area

$$\phi_n = -D_n \frac{dn}{dx} \qquad \phi_p = -D_p \frac{dp}{dx}$$

- D_n, D_p is diffusion coefficient

$$J_n = qD_n \frac{dn}{dx} \qquad J_p = -qD_p \frac{dp}{dx}$$

Total Current: Electrons

- Drift:
  - Electrons drift opposite to the electric field
  - Drift current is in the same direction as the electric field
- Diffusion:
  - Electrons diffuse in the direction of decreasing concentration
  - Diffusion current is in opposite direction to decreasing concentration

$$J_n(x) = \underbrace{q\mu_n n(x)E(x)}_{\text{drift}} + \underbrace{qD_n \frac{dn(x)}{dx}}_{\text{diffusion}}$$

Total Current: Holes

- Drift:
  - Holes drift with the electric field
  - Drift current is in the same direction as the electric field
- Diffusion:
  - Holes diffuse in the direction of decreasing concentration
  - Diffusion current is in same direction as decreasing concentration

$$J_p(x) = \underbrace{q\mu_p p(x)E(x)}_{\text{drift}} \underbrace{- qD_p \frac{dp(x)}{dx}}_{\text{diffusion}}$$

Equilibrium

- At equilibrium, no current flows

- Any diffusion current must be balanced by an equal drift current, and vice-versa.
- For example (for electrons):

$$q\mu_n n(x)E(x) = -qD_n \frac{dn(x)}{dx}$$

- Equilibrium Fermi level must be flat:

$$\frac{dE_F}{dx} = 0$$