## Homework \#1 Digital Design Review

## Due Date: 10/09/2023, Monday 5:00 PM

## Problem 1 Number Systems (20')

(A) Convert (15) ${ }_{10}$ and (53) $)_{10}$ into binary, octal and hexadecimal representations. (5')
(B) Convert ( -15$)_{10}$ and $(-53)_{10}$ into two's complement binary numbers. Assume each number has 8 bits and the most significant bit (MSB) is the sign bit. (5')
(C) Calculate $15+(-53)$ (in 8 -bit representation) using two's complement binary number. Show your process. Explain briefly how to detect overflow in a two's complement sum. (10')

## Problem 2 Boolean Algebra (20')

(A) Use algebraic manipulation to convert the following equation to sum-of-product and product-of-sum forms: $(x+y w)\left(y^{\prime}+z^{\prime}\right)+x(y+w)^{\prime}+(y+x) z^{\prime} w$. ( $\left.10^{\prime}\right)$
(B) Make use of K-map to simplify $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum(3,4,6,8,11,13,14)+\mathrm{d}(0,9,10,2)$. Note that d corresponds to the don't cares. Choose the values of the don's cares to minimize the logic function. (10')

## Problem 3 Sequential Circuit (20’)

(A) What is this? What is the Input/Output properties? (5')
(B) If input is changing from $(0,0)$ to $(1,1)$, what will happen? $\left(5^{\prime}\right)$
(C) How to use it to implement a D-latch with Enable signal? (5')

(D) How to use two of the latches in (C) to implement an edge-triggered D-Flip-Flop? (5')

Problem 4 Boolean Algebra (30' $=5+(3+2)+(4+2+4)+(4+2+4)$ )
This question has four different parts:
a. Simplify the following expression:

$$
a+a^{\prime} b+a^{\prime} b^{\prime} c+a^{\prime} b^{\prime} c^{\prime} d+a^{\prime} b^{\prime} c^{\prime} d^{\prime} e+\cdots
$$

b. Represent the following sentences by a Boolean equation and then realize the equation using a minimum number of 2 -input AND and 2 -input OR gates. Assume complemented variables are available.
"The tape drive motor for a computer tape drive should be running iff:
(i) The tape is properly threaded
(ii) An end-of-tape signal is not present, and
(iii) The tape drive is in the manual mode and the motor start button has been pressed, or is in the automatic mode and the 'tape on' signal from the computer is present"
Definition of terms: $\quad$ R: Run motor
E: End-of-tape signal
S: Motor start button
T : Tape properly threaded
M: Manual (automatic = not manual)
O: Tape ON signal
c. Consider the four variable function $\mathrm{F}(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d})$. If $\mathrm{F}=1$ only if exactly two of the variables equal to 1 , list all the prime implicants. How many are essential? If F was an n variable function and F $=1$ iff exactly k of the variables were equal to 1 , how many prime implicants and essential prime implicants will this function have?
d. A three-input gate BOMB, whose characteristics are shown below, has been mass produced by an unfortunate company. Experimental evidence shows that the input combinations 101 and 010 causes the gate to "explode". Is this gate completely useless? If not, how can you externally modify it so that it may be used to implement any switching function without causing explosions? List all the possible switching functions. Can this gate be considered functionally complete?


## Problem 5 Combinational Circuit (20')

Sketch a transistor-level schematic for a single-stage CMOS logic gate for the following functions. You may assume you have both true and complementary versions of the inputs available. ( $2 \times 10^{\prime}$ )
(A) $\mathrm{Y}=\left(\mathrm{AB}+\mathrm{C}^{\prime} \mathrm{D}^{\prime}\right)$
(B) $\mathrm{Y}=\left(\mathrm{A}^{\prime} \mathrm{B}+\mathrm{AC}+\mathrm{B}^{\prime} \mathrm{C}\right)^{\prime}$

## Problem 6 Combinational Circuit (10')

Implement the Boolean expressions below in a complementary CMOS gate: (10')

1. $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum(0,1,2,8)+\mathrm{d}(3,4,12)$
