

B

### Mid-Term Exam, ECE-137B

Tuesday, May 3, 2016

#### Closed-Book Exam

There are 2 problems on this exam , and you have 75 minutes.

1) **show all work. Full credit will not be given for correct answers if supporting work is not shown.**

2) please write answers in provided blanks

3) Don't Panic !

4) 137a, 137b crib sheets, and 2 pages personal sheets permitted.

Use any, all reasonable approximations. After stating them. 5% accuracy is fine if the method is correct.

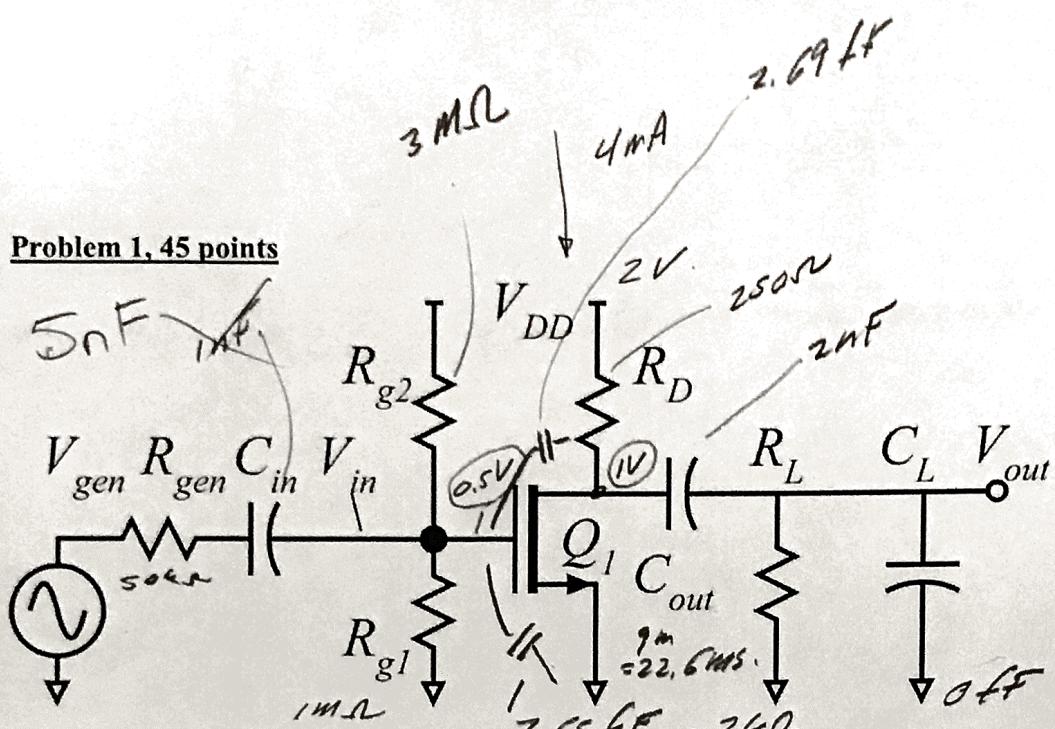
**Do not turn over cover page until requested to do so.**

Name: \_\_\_\_\_

Time function	LaPlace Transform
$\delta(t)$	1
$U(t)$	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s + \alpha}$
$e^{-\alpha t} \cos(\omega_d t)U(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_d^2}$
$e^{-\alpha t} \sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s + \alpha)^2 + \omega_d^2}$

Problem	Points Received	Points Possible
1a		2
1b		5
1c		4
1d		15
1e		7
1f		7
1g		5
2a		4
2b		6
2c		10
2d		5
3a		5
3b		13
3c		12
total		100

**Problem 1, 45 points**



Q1 has 0.8 nm oxide thickness,  $\varepsilon_r=3.8$ , 22 nm gate length, and a 0.25 V threshold.

Mobility is  $300 \text{ cm}^2/(\text{V}\cdot\text{s})$ , channel injection velocity is  $1\text{E}7 \text{ cm/s}$ ;  $\lambda = 0 \text{ Volts}^{-1}$ ,  $C_{gs} = \varepsilon_r \varepsilon_{ox} L_g W_g / T_{ox} + (0.5 \text{ fF}/\mu\text{m}) \cdot W_g$  and  $C_{gd} = (0.5 \text{ fF}/\mu\text{m}) \cdot W_g$ .

calculated for you:

$$\varepsilon_r \varepsilon_{ox} / T_{ox} = 4.21 \cdot 10^{-2} \text{ F/m}^2, (\mu c_{ox} W_g / 2L_g) = (2.87 \cdot 10^{-2} \text{ A/V}^2) \cdot (W_g / 1\mu\text{m})$$

$$(c_{ox} v_{ini} W_g) = (4.21 \cdot 10^{-3} \text{ A/V}^1) \cdot (W_g / 1\mu\text{m}), (v_{ini} L_g / \mu) = 73.3 \text{ mV}.$$

$$V_{DD} = +2\text{V}.$$

The FET is to be biased at 4mA drain current and 1 V drain bias voltage.

\*\*You will pick the FET width  $W_g$  such that  $V_{gs} = 0.5 \text{ Volts}$ \*\*\*

Rgen=50kOhm, Rg1=1MOhm, RL=2000 Ohms, CL=0fF.

$C_{in}=5\text{nF}$ ,  $C_{out}=2\text{nF}$ .

$$V_{th} = 0.25V \quad \Delta V = v_{inj} \cdot \lg \mu = 73mV$$

in which is  $\geq V_{th} + \Delta V \rightarrow 105V$

$V_{th} = 0.25V$ ,  $\Delta V = v_{inj}$   $\rightarrow$   $v_{sys} = 0.5V$  which is  $> V_{th} + \Delta V \rightarrow$  velocity limited

$$\rightarrow I_d = 4.21 \cdot 10^{-3} \frac{A}{V} \text{ Wykres: } \frac{V_{ds}}{0.5V} = \frac{(V_{gs} - V_{th} - 4V)}{0.25V} = \frac{36.5mV}{36.5mV}$$

$$\Rightarrow w_g = 4.45 \mu m \Rightarrow g_m = 18.7 mS$$

$$R_2 = 16 / 4 \text{ mA} = 250 \Omega$$

$$R_{Q2} = \frac{1.5V}{0.5mA} = 3M\Omega$$

Part a, 2 points

Find the following:

$$W_g = \underline{4.45 \mu m}$$

*Calculations on previous page.*

Part b, 5 points

*small-signal parameters*  
Find the following

$$C_{gs} = \frac{6.35 \text{ fF}}{18.7 \text{ mS}}, \quad C_{gd} = \frac{2.23 \text{ fF}}{350 \text{ GHz}}$$

$$g_m = \frac{22 \mu\text{m}}{4.45 \mu\text{m}}$$

$$\boxed{C_{gs} = 4.21 \cdot 10^{-2} \text{ F/m}^2 \cdot \frac{L_g \cdot W_g}{22 \mu\text{m}} + 0.5 \text{ fF}/\mu\text{m} \cdot 5.37 \mu\text{m}} \\ = 4.12 \text{ fF} + 223 \text{ fF} = 6.35 \text{ fF}$$

$$\boxed{C_{gd} = 0.5 \text{ fF}/\mu\text{m} \cdot 5.37 \mu\text{m} = 2.23 \text{ fF}}$$

$$\boxed{g_m = 4.21 \frac{\text{mA}}{\text{V}} \cdot \frac{5.37 \mu\text{m}}{1 \mu\text{m}} = 18.7 \text{ mS}}$$

$$\boxed{f_T = \frac{0.159 \cdot g_m}{C_{gs} + C_{gd}} = 348 \text{ GHz}}$$

Part c: 4 points

*Mid Band Analysis:*

Find the following:

$$R_{in, \text{Amplifier}} = \frac{750 \text{ k}\Omega}{-4.15} \quad R_{L,eq} = \frac{222 \text{ }\Omega}{0.9375}$$

$$\left[ R_{in, \text{Amplifier}} = 1 \text{ M}\Omega // 3 \text{ M}\Omega = \frac{1}{\frac{1}{1 \text{ M}\Omega} + \frac{1}{3 \text{ M}\Omega}} = 750 \text{ k}\Omega \right]$$

$$\left[ V_{in}/V_{gen} = \frac{750 \text{ k}\Omega}{800 \text{ k}\Omega} = \frac{15}{16} = 0.9375 \right]$$

$$\left[ R_{L,eq} = 24 \text{ }\Omega // 280 \text{ }\Omega = 222.2 \text{ }\Omega \right]$$

$$\left[ v_o/v_i = -g_m \cdot R_{L,eq} = -18.7 \text{ m}^{-5} \cdot 222 \text{ }\Omega = -4.15 \right]$$

Part d: 15 points

*High-Frequency Analysis: Poles*

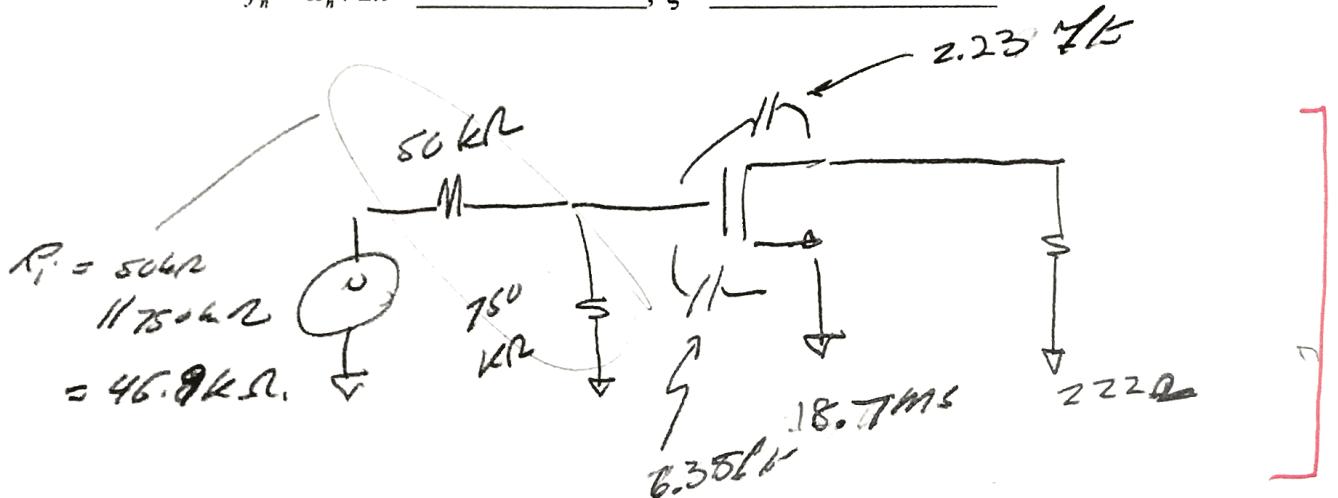
Find the frequencies, in Hz, of the two poles limiting the high-frequency response of the amplifier. You can either use MOTC, or use the results derived in class (and written down on the class amplifier crib sheet). No need to do nodal analysis. Hint: assume  $C_{in}$  and  $C_{out}$  are short-circuits for this calculation

If the poles are real, give the 1 or 2 pole frequencies in Hz:

$$f_{p1,HF} = \underline{190 \text{ MHz}} \quad f_{p2,HF} = \underline{905 \text{ GHz}}$$

If there are 2 poles, and they are complex, give  $f_n = \omega_n / 2\pi$  and the damping factor  $\zeta$ :

$$f_n = \omega_n / 2\pi = \underline{\quad}, \quad \zeta = \underline{\quad}$$



$$\begin{aligned} a_1 &= 46.9k\Omega \cdot 6.35\text{nf} \\ &\quad + 2.23M\Omega [46.9k\Omega(1+4.15) + 222\Omega] \\ &= 0.30\text{ns} + 0.54\text{ns} = .84\text{ns.} \end{aligned}$$

by motc.

$$\begin{aligned} a_2 &= \frac{R_1 C_1 C_2}{1 + \frac{R_2}{R_1 C_1 C_2}} \left[ \frac{R_2}{L} \right] \\ &= \frac{46.9k\Omega}{1 + \frac{750k\Omega}{50k\Omega \cdot 100pF \cdot 100pF}} \left[ \frac{750k\Omega}{222\Omega} \right] \\ &= 1.047 \cdot 10^{-22} \text{ sec}^2. \end{aligned}$$



$$q_2/q_1 = \frac{1.4\pi \cdot 10^{-22} \text{ sec}^2}{0.84 \text{ m}} = 0.175 \text{ ps.}$$

$\ll q_1$   
so SPS works.

$$f_{p1} \approx \frac{1}{2\pi q_1} = 190 \text{ MHz.}$$

$$f_{p2} \approx \frac{1}{2\pi(q_2/q_1)} = 905 \text{ GHz (').}$$

Part e: 7 points

*High-Frequency Analysis: Zeros*

Find the frequencies of any zeros (there may be zero, one or two present) in the transfer function. You can either use nodal analysis, or use the results derived in class (and written down on the class amplifier crib sheet).

$$f_{z1} = 1.33 \text{ THz}, f_{z2} = \underline{\hspace{2cm}}, \dots$$

$$\boxed{f_z = \frac{0.159 \cdot 9m}{C_{gd}}} = 1.33 \text{ THz} \quad \left\{ \begin{array}{l} \text{THz} \\ (10^{12} \text{ Hz}) \end{array} \right\}$$

Right half plane zero.

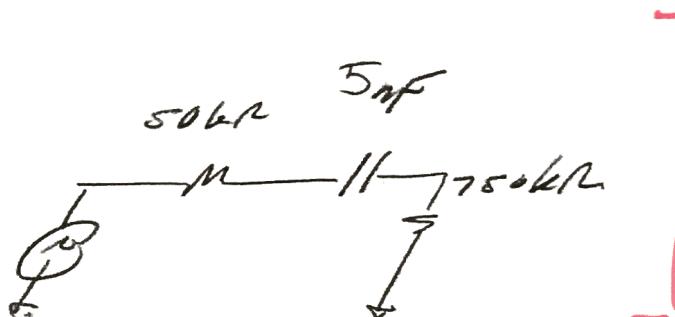
Part f: 7 points

Low-Frequency Analysis:

Find the frequency in Hz, of the poles, due to  $C_{out}$  and  $C_{in}$ , limiting the low-frequency response of the amplifier. Use any method of analysis you choose.

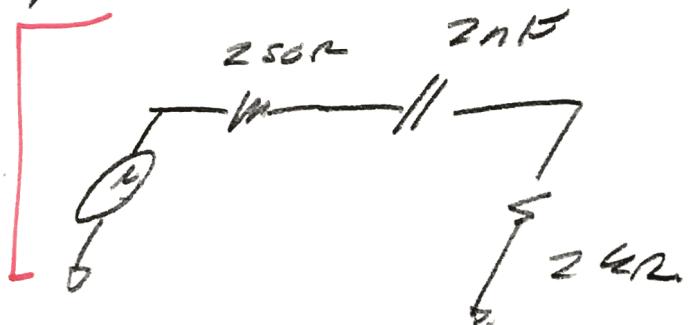
$$f_{p1,LF} = \frac{39.6 \text{ Hz}}{25.3 \text{ kHz}}$$

Input:



$$f_{low} = \frac{0.159}{5\text{nF} \cdot 800\text{k}\Omega} = 39.6 \text{ Hz}$$

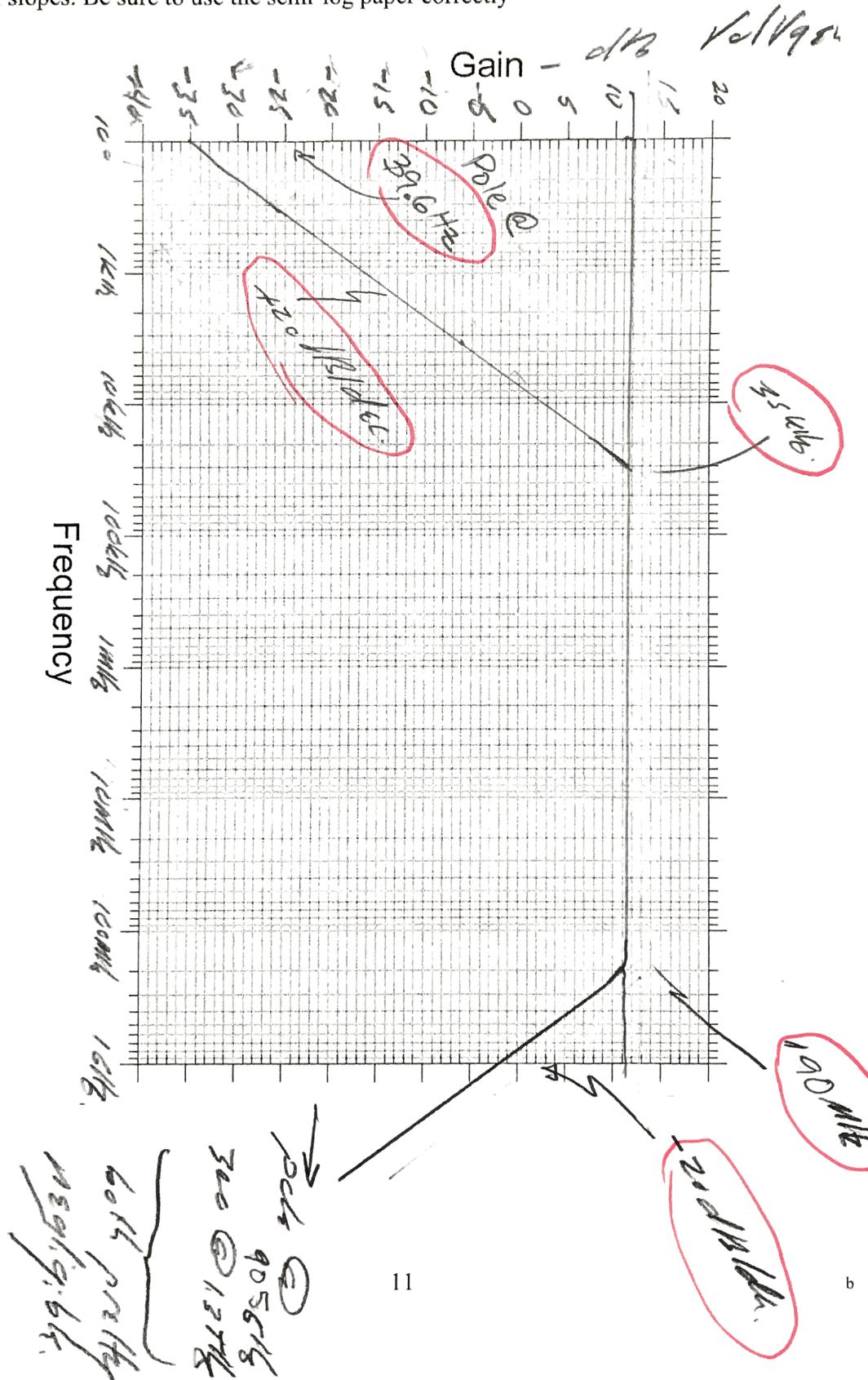
Output:



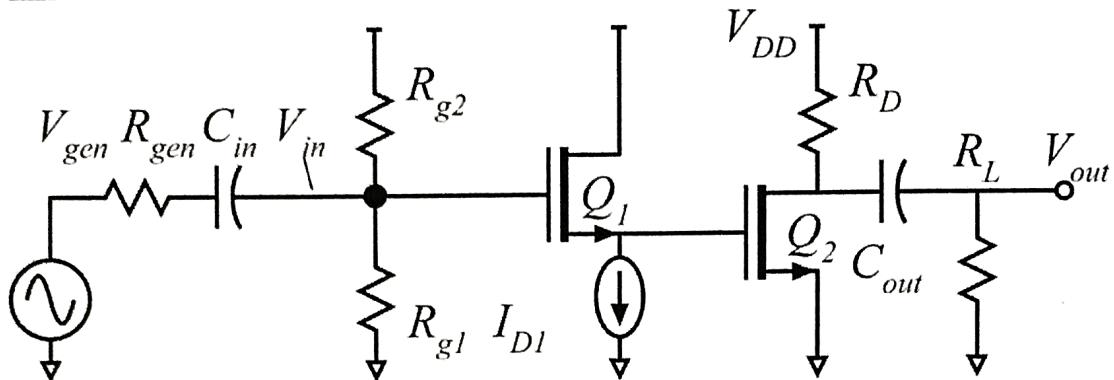
$$f_{low} = \frac{0.159}{2\text{nF} \cdot 2.25\text{k}\Omega} = 35.3 \text{ kHz}$$

Part g: 5 points

Draw a clean asymptotic Bode Magnitude plot of  $V_{out}/V_{gen}$  as a function of frequency in Hz. Be sure to label and dimension the axes clearly, label pole and zero frequencies and gain slopes. Be sure to use the semi-log paper correctly



**Problem 2, 25 points**



In the amplifier above,

$$R_{gen} = 10 \text{ kOhm}, R_{g1} = R_{g2} = 10 \text{ MOhm}, R_{out} = 10 \text{ kOhm} \quad R_D = R_L = 4 \text{ kOhm}$$

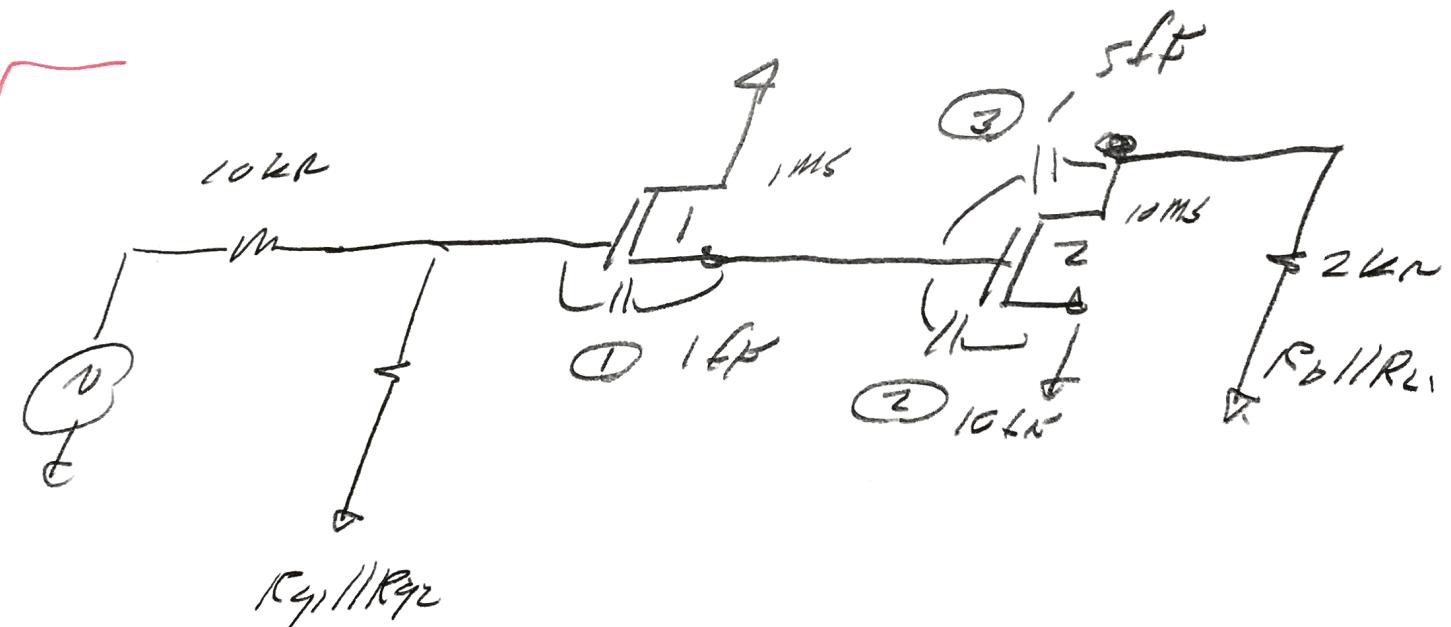
$C_{in}$  and  $C_{out}$  are very very large: treat as infinite

Q1:  $gm = 1 \text{ mS}$ ,  $C_{gs} = 1 \text{ fF}$ ,  $C_{gd} = 0 \text{ fF}$ ,  $R_{ds} = \text{infinity Ohms}$

Q2:  $gm = 10 \text{ mS}$ ,  $C_{gs} = 10 \text{ fF}$ ,  $C_{gd} = 5 \text{ fF}$ ,  $R_{ds} = \text{infinity Ohms}$

**Part a: 4 points**

draw below a small-signal representation of the circuit, but with the transistors represented by transistor symbols, not small-signal hybrid-pi models



$$R_i = 5 \text{ M}\Omega // 10 \text{ k}\Omega$$

$$\approx 10 \text{ k}\Omega$$

Part b, 6 points

Find the small-signal **mid-band** voltage gain of the two stages:

$$V_{out1}/V_{in1} = V_s1/V_{g1} = \underline{\hspace{2cm}}$$

$$V_{out}/V_{in2} = V_d2/V_{g2} = \underline{\hspace{2cm}}$$

gain of Q2:

$$\begin{aligned} A_v &= -g_2 \cdot R_{eg} = -10mS \cdot 2k\Omega \\ &= -20 \end{aligned}$$

gain of Q1

$$A_v = 1 \quad (\text{no load resistance})$$

Part c, 10 points

using the method of time constants, find a1 and a2 of the circuit transfer function:

$$a_1 = \frac{126\text{ps}}{1.26 \cdot 10^{-21} \text{sec}^2}$$

$R_{11}' C_1$        $R_{11}' = R_{\text{gen}} [1 - A_{V1}] + 1/g_{m1} = 1/g_{m1} = 1\text{k}\Omega$ .

$R_{11}' C_1 = 1\text{k}\Omega \cdot 1\text{fF} = 1\text{ps}$ .

$R_{22}' C_2$ :       $R_{22}' = 1/g_{m1} = 1\text{k}\Omega$ .

$$R_{22}' C_2 = 1\text{k}\Omega \cdot 10\text{fF} = 10\text{ps}$$

$R_{33}' C_3$ :       $R_{33}' = (1/g_{m1}) [1 - A_{V2}] + 2\text{k}\Omega = 1\text{k}\Omega [2] + 2\text{k}\Omega$   
 $= 23\text{k}\Omega$

$R_{33}' C_3 = 23\text{k}\Omega \cdot 5\text{fF} = 115\text{ps}$

$a_1 = 1 + 10 + 115\text{ps} = 126\text{ps}$

$R_{11}' C_1 C_2 R_{22}'$   $\Rightarrow R_{22}' = R_{\text{gen}} = 10\text{k}\Omega$ .

$L$   $= 1\text{k}\Omega \cdot 1\text{fF} \cdot 10\text{fF} \cdot 10\text{k}\Omega = 10^{-22} \text{sec}^2$ .

$R_{11}' C_1 C_3 R_{33}'$   $\Rightarrow R_{33}' = 10\text{k}\Omega [1 + 20] + 2\text{k}\Omega = 212\text{k}\Omega$

$L$   $= 1\text{k}\Omega \cdot 1\text{fF} \cdot 5\text{fF} \cdot 212\text{k}\Omega = 1.06 \cdot 10^{-21} \text{sec}^2$

$R_{22}' C_2 C_3 R_{33}'$   $\Rightarrow R_{33}'^2 = R_b \parallel R_L = 2\text{k}\Omega$

$L$   $= 1\text{k}\Omega \cdot 10\text{fF} \cdot 5\text{fF} \cdot 2\text{k}\Omega = 10^{-22} \text{sec}^2$

$a_2 = \text{sum of these} = 1.26 \cdot 10^{-21} \text{sec}^2$

Part d, 5 points

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:

$$f_{p1} = \underline{\hspace{2cm}}, f_{p2} = \underline{\hspace{2cm}}$$

If there are 2 poles, and they are complex, give  $f_n = \omega_n / 2\pi$  and the damping factor  $\zeta$ :

$$f_n = \omega_n / 2\pi = \underline{\hspace{2cm}}, \zeta = \underline{\hspace{2cm}}$$

try SPA:  $\frac{a_2}{a_1} = 10 \quad \frac{1.20 \cdot 10^{-21} \text{ sec}^2}{126 \text{ ps}} = 10 \text{ ps}.$

$\frac{a_2}{a_1}$  is about  $\frac{1}{13}$  of  $a_1$ , so SPA is ok.

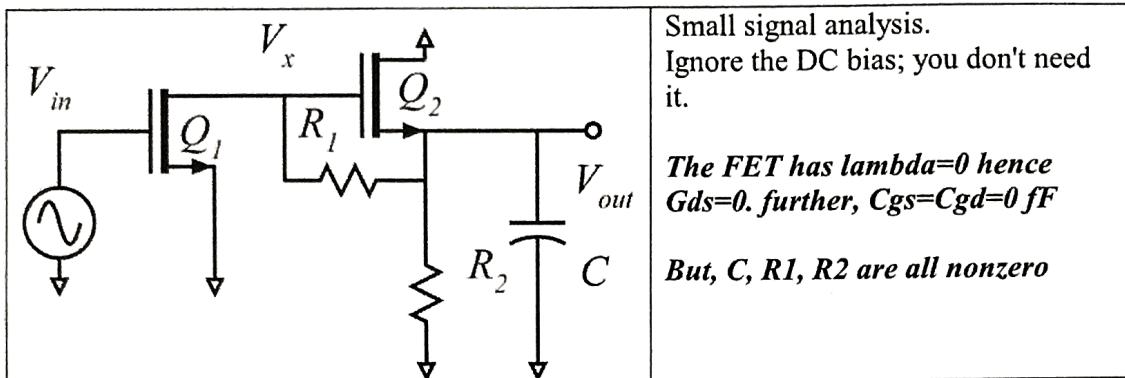
$$f_{p1} = \frac{0.1154}{a_1} = 1.26 \text{ GHz}$$

$$f_{p2} = \frac{0.1154}{a_2/a_1} = 15.96 \text{ GHz.}$$

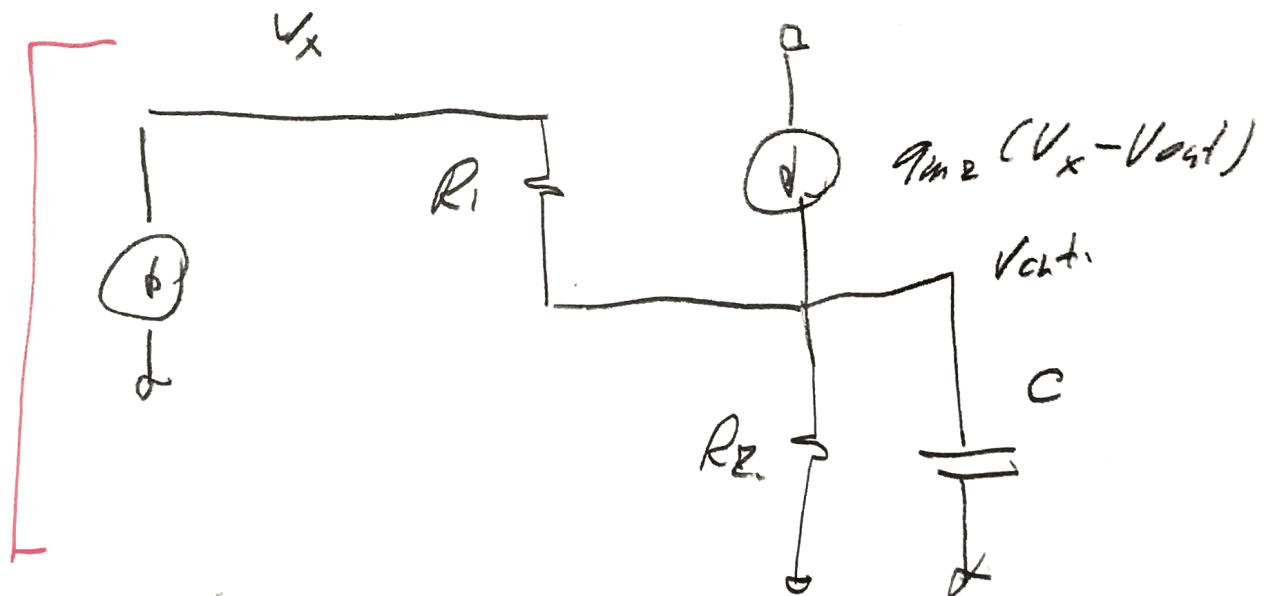
B

Problem 3, 30 points

Part a 5 points



Replacing the transistor with its high frequency small-signal model, draw a small-signal equivalent circuit diagram.



Part b, 13 points

**USING NODAL ANALYSIS**, compute  $V_{out}(s)/V_{in}(s)$  in ratio-of-polynomials form:

$$V_{out}(s)/V_{in}(s) = A_{v, \text{mid-band}} \times (s\tau)^m \times \frac{1+b_1s+b_2s^2+\dots}{1+a_1s+a_2s^2+\dots} = \frac{1+b_1s+b_2s^2+\dots}{1+a_1s+a_2s^2+\dots}$$

here  $m$ , an integer, can be positive or negative or zero

$$G_1 = 1/R_1$$

$$G_2 = 1/R_2$$

$$\sum I = 0 \quad @ V_x$$

$$\underbrace{q_m V_{in} + (V_x - V_{out}) G_1 = 0}_{\boxed{V_x [G_1] + V_{out} [-G_1] = -q_m V_{in}}}$$

$$\sum I = 0 \quad @ V_{out}$$

$$(V_{out} - V_x)(G_1 + q_{m2}) + (G_2 + SC)V_{out} = 0.$$

$$\boxed{V_x [- (G_1 + q_{m2})] + V_{out} [(G_1 + q_{m2}) + SC + G_2] = 0}$$

$$\begin{bmatrix} G_1 & -G_1 \\ -(G_1 + q_{m2}) & (G_1 + q_{m2}) + G_2 + SC \end{bmatrix} \begin{bmatrix} V_x \\ V_{out} \end{bmatrix} = \begin{bmatrix} -q_m V_{in} \\ 0 \end{bmatrix}$$

$$D = \begin{vmatrix} G_1 & -G_1 \\ -G_1 & G_1 + g_{m2} + G_2 + SC \end{vmatrix}$$

$$\begin{aligned}
 &= G_1^2 + G_1(G_1 + g_{m2} + G_2 + SC) - G_1^2 - G_1 g_{m2} \\
 &= G_1 g_{m2} + G_1 G_2 + G_1 SC - G_1 g_{m2} \\
 &= G_1 (G_2 + SC)
 \end{aligned}$$

$$N = \begin{vmatrix} G_1 & -g_{m1} v_{in} \\ -G_1 - g_{m2} & 0 \end{vmatrix} = -g_{m1} (G_1 + g_{m2}) v_{in}$$

$$V_{out} = N/D \Rightarrow$$

$$\frac{V_o}{V_{in}} = \frac{-g_{m1} (G_1 + g_{m2})}{G_1 (G_2 + SC)} = \frac{-g_{m1} (G_1 + g_{m2})}{G_1 G_2} \frac{1}{1 + SCR_2}$$

$$= -g_{m1} R_1 R_2 \left( \frac{1}{R_1} + g_{m2} \right) \frac{1}{1 + SCR_2}$$

$$\boxed{\frac{V_o}{V_{in}} = -g_{m1} R_2 \left( 1 + g_{m2} R_1 \right) \frac{1}{1 + SCR_2}}$$

Part c, 12 points

$g_{m1} = 1 \text{ mS}$ ,  $g_{m2} = 2 \text{ mS}$ ,  $R_1 = 10 \text{ kOhm}$ ,  $R_2 = 20 \text{ kOhm}$ ,  $C = 1 \text{ pF}$

Find the frequencies of any zeros (there may be zero, one or two present) in  $V_{out}(s)/V_{in}(s)$ :

$$f_{z1} = \cancel{\quad}, f_{z2} = \cancel{\quad}, \dots$$

There may be either 1 or 2 poles in  $Z(s)$ .

If the poles are real, give the 1 or 2 pole frequencies in Hz:

$$f_{p1} = \cancel{7.95 \text{ MHz}}, f_{p2} = \cancel{\quad}$$

If there are 2 poles, and they are complex, give  $f_n = \omega_n / 2\pi$  and the damping factor  $\zeta$ :

$$f_n = \omega_n / 2\pi = \cancel{\quad}, \zeta = \cancel{\quad}$$

no zeros, one pole

$$f_p = \frac{0.159}{e^{R_2}} = 7.95 \text{ MHz}$$

↓      ↑  
 1PF     $20 \text{ k}\Omega$