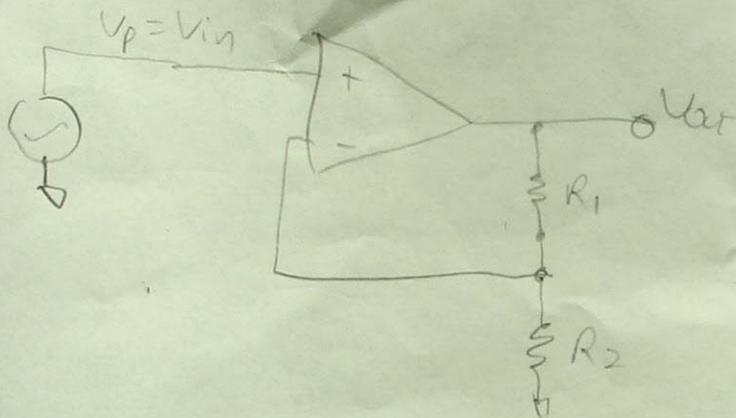


(a)



Nodal Analysis,

$$V_{out} = A_{diff} (V_p - V_m)$$

$$V_p = V_{in} \quad V_m = \frac{R_2}{R_1 + R_2} V_{out}$$

$$V_{out} = A_{diff} \left( V_{in} - \frac{R_2}{R_1 + R_2} V_{out} \right)$$

$$\left( \frac{1}{A_{diff}} + \frac{R_2}{R_1 + R_2} \right) V_{out} = V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\frac{1}{A_{diff}} + \frac{R_2}{R_1 + R_2}}$$

$$= \left( \frac{R_1 + R_2}{R_2} \right) \frac{1}{1 + \frac{1}{A_{diff}} \left( \frac{R_1 + R_2}{R_2} \right)}$$

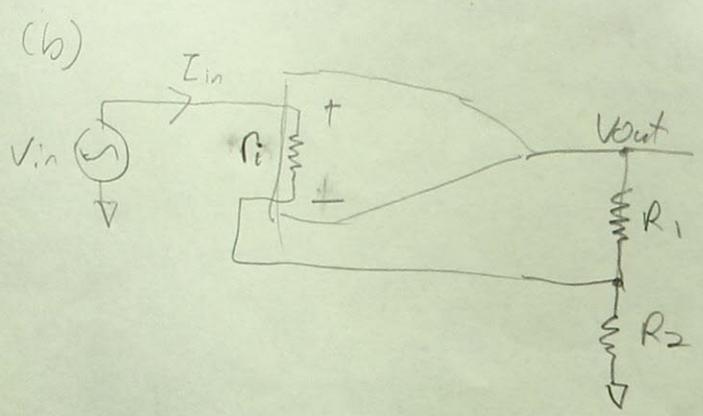
$$= \left( 1 + \frac{R_1}{R_2} \right) \frac{1}{1 + \frac{1}{A_{diff}} \left( 1 + \frac{R_1}{R_2} \right)}$$

Substituting values:

$$R_1 = 10 \text{ k}\Omega \quad R_2 = 0.1 \text{ k}\Omega \quad A_{diff} = 20,000$$

$$1 + \frac{R_1}{R_2} = 1 + \frac{10 \text{ k}\Omega}{0.1 \text{ k}\Omega} = 1 + 100 = 101$$

$$\frac{V_{out}}{V_{in}} = 101 \left( \frac{1}{1 + \frac{1}{20,000} (101)} \right) = 100.49$$



Solve for new  $\frac{V_{out}}{V_{in}}$

$$V_{out} = A_{diff} (V_p - V_m) = A_{diff} (V_{in} - V_m)$$

$$V_m = V_{in} - \frac{V_{out}}{A_{diff}}$$

$$\frac{V_p - V_m}{r_i} + \frac{V_{out} - V_m}{R_1} + \frac{-V_m}{R_2} = 0$$

$$\frac{1}{r_i} V_{in} + \frac{1}{R_1} V_{out} - \left( \frac{1}{r_i} + \frac{1}{R_1} + \frac{1}{R_2} \right) V_m = 0$$

$$\frac{1}{r_i} V_{in} + \frac{1}{R_1} V_{out} - \left( \frac{1}{r_i} + \frac{1}{R_1} + \frac{1}{R_2} \right) \left( V_{in} - \frac{V_{out}}{A_{diff}} \right) = 0$$

$$\left( \frac{1}{R_1} + \frac{1}{A_{diff}} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_i} \right) \right) V_{out} - \left( \frac{1}{R_1} + \frac{1}{R_2} \right) V_{in} = 0$$

$$\left( \frac{1}{R_1} + \frac{1}{A_{diff}} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_i} \right) \right) V_{out} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) V_{in}$$

$$A_{cc} = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{R_1} + \frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{A_{diff}} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_i} \right)}$$

$$= \frac{R_1 + R_2}{R_2 + R_1 + \frac{1}{A_{diff}} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_i} \right)}$$

$$= \left( 1 + \frac{R_1}{R_2} \right) \frac{1}{1 + \frac{1}{A_{diff}} \left( 1 + \frac{R_1}{R_2} + \frac{R_1}{r_i} \right)}$$

Now for  $R_{in,cl}$

$$V_p = V_{in} \quad V_m = V_{in} - \frac{V_{out}}{A_{diff}}$$

$$I_{in} = \frac{V_p - V_m}{r_{in}} = \frac{1}{r_{in}} \left( V_{in} - V_{in} + \frac{V_{out}}{A_{diff}} \right)$$

$$= \frac{1}{r_{in}} \frac{1}{A_{diff}} V_{out}$$

$$R_{in,cl} = \frac{V_{in}}{I_{in}} = r_{in} A_{diff} \left( \frac{V_{in}}{V_{out}} \right)$$

$$= r_{in} \frac{A_{diff}}{A_{cl}}$$

$$= r_{in} \left( \frac{R_2}{R_1 + R_2} \right) \left( 1 + \frac{R_1}{R_2} + \frac{R_1}{r_e} + A_{diff} \right)$$

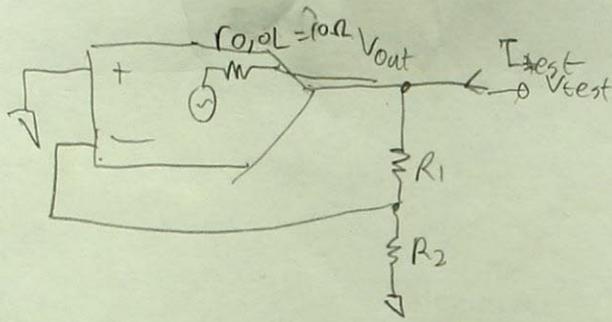
$$= r_{in} \left[ \left( \frac{R_2}{R_1 + R_2} \right) \left( A_{diff} + \frac{R_1}{r_e} \right) + \frac{R_2}{R_1 + R_2} \left( \frac{R_1 + R_2}{R_2} \right) \right]$$

$$= r_{in} \left[ 1 + A_{diff} \left( \frac{R_2}{R_1 + R_2} \right) + \left( \frac{R_1 || R_2}{r_e} \right) \right]$$

$$= R_1 || R_2 + r_{in} \left[ 1 + \frac{A_{diff}}{1 + \frac{R_1}{R_2}} \right]$$

$$= 0.09901k\Omega + 1k\Omega \left[ 1 + \frac{20,000}{101} \right]$$

$$R_{in,cl} = 299.1k\Omega$$



$$A_{diff} (V_{in+} - V_{in-}) = -A_{diff} (-V_{out}) = \frac{R_2}{R_1 + R_2} V_{out} = -\frac{A_{diff}}{1 + \frac{R_1}{R_2}} V_{out}$$

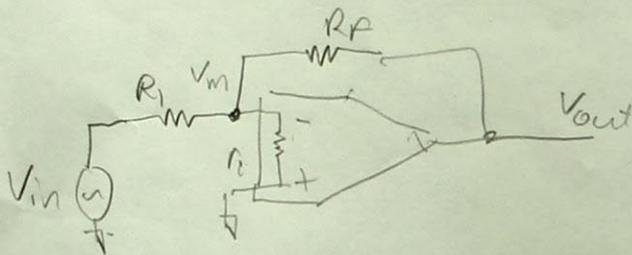
$$I_{test} = \frac{V_{test} - (-\frac{A_{diff}}{1 + \frac{R_1}{R_2}}) V_{test}}{R_{o,ol}}$$

$$= \frac{V_{test} \left(1 + \frac{A_{diff}}{1 + \frac{R_1}{R_2}}\right)}{R_{o,ol}}$$

$$R_{o,cl} = \frac{V_{test}}{I_{test}} = \frac{R_{o,ol}}{1 + \frac{A_{diff}}{1 + \frac{R_1}{R_2}}}$$

For given values

$$R_{o,cl} = \frac{10 \Omega}{1 + \frac{20000}{101}} = 0.05025 \Omega = 50.25 \text{ m}\Omega$$



$$V_{out} = A_d (V_p - V_m) = -A_d V_m$$

$$\frac{V_{in} - V_m}{R_1} + \frac{V_{out} - V_m}{R_F} + \frac{-V_m}{r_i} = 0$$

$$\frac{1}{R_1} V_{in} - \left( \frac{1}{R_1} + \frac{1 + A_d}{R_F} + \frac{1}{r_i} \right) V_m = 0$$

$$\frac{V_m}{V_{in}} = \frac{V_{in}}{\left( \frac{1}{R_1} + \frac{1 + A_d}{R_F} + \frac{1}{r_i} \right) V_{in}}$$

$$= \frac{1}{1 + \frac{R_1}{R_F} (1 + A_d) + \frac{R_1}{r_i}}$$

$$= \frac{R_F}{R_1} \frac{1}{1 + A_d + \frac{R_F}{R_1} + \frac{R_F}{r_i}}$$

$$= \frac{R_F}{R_1} \frac{1}{A_d} \frac{1}{1 + \frac{1}{A_d} \left( 1 + \frac{R_F}{R_1} + \frac{R_F}{r_i} \right)}$$

$$\frac{V_{out}}{V_{in}} = -A_d \left( \frac{V_m}{V_{in}} \right) = -\frac{R_F}{R_1} \frac{1}{1 + \frac{1}{A_d} \left( 1 + \frac{R_F}{R_1} + \frac{R_F}{r_i} \right)}$$

Substituting

$$\frac{V_{out}}{V_{in}} =$$

Substituting

$$R_F = 50 \text{ k}\Omega \quad R_i = 10 \text{ k}\Omega \quad R_{i'} = 0.1 \text{ k}\Omega$$

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= -\frac{50 \text{ k}\Omega}{10 \text{ k}\Omega} \left( \frac{1}{1 + \frac{1}{10^4} \left( 1 + \frac{50 \text{ k}\Omega}{10 \text{ k}\Omega} + \frac{50 \text{ k}\Omega}{0.1 \text{ k}\Omega} \right)} \right) \\ &= -5 \left( \frac{1}{1 + 10^{-4} (1 + 5 + 500)} \right) \\ &= \frac{-5}{1.0506} = -4.759 \end{aligned}$$

$$A_d(s) = 10^6 \frac{1}{\left(1 + \frac{s}{2\pi \times 50\text{Hz}}\right) \left(1 + \frac{s}{2\pi \times 50 \times 10^6\text{Hz}}\right) \left(1 + \frac{s}{2\pi \times 200 \times 10^6\text{Hz}}\right)}$$

$$A_d(f) = 10^6 \frac{1}{\left(1 + \frac{jf}{50\text{Hz}}\right) \left(1 + \frac{jf}{50 \times 10^6\text{Hz}}\right) \left(1 + \frac{jf}{200 \times 10^6\text{Hz}}\right)}$$

(a)

$$\beta(f) = 1$$

$$\rightarrow \frac{A(f)}{\beta(f)} = 1$$

General shape of  $A_d(s)$ . Asymptotes

DC - 50 Hz flat at +120 dB

50 Hz - 50 MHz falls at 20 dB/dec to +120 dB to 0 dB

50 MHz - 200 MHz falls at 40 dB/dec

from 0 dB = 1 V/V

to  $\frac{1}{16}$  V/V = -24.08 dB

200 MHz on falls at 60 dB/dec from  $\frac{1}{16}$  V/V

$$\text{@ } 16\text{MHz} \left(\frac{1}{16}\right) \left(\frac{1}{125}\right) = \frac{1}{2000} = -66.02\text{dB}$$

These are from Asymptotes

To get actual curve, let's calculate some values near the 50 MHz, 200 MHz poles

$$|Ad(10\text{MHz})| = \frac{10^6}{\left( \sqrt{1 + \frac{(10\text{MHz})^2}{50\text{Hz}^2}} \right) \times \left( \sqrt{1 + \frac{(10\text{MHz})^2}{50\text{MHz}^2}} \right) \times \left( \sqrt{1 + \frac{(10\text{MHz})^2}{200\text{MHz}^2}} \right)}$$

$$= \frac{10^6}{|1 + j200,000| \times |1 + j\frac{1}{5}| \times |1 + j\frac{1}{20}|} = \frac{10^6}{\sqrt{1^2 + 200,000^2} \sqrt{1 + (\frac{1}{5})^2} \sqrt{1 + (\frac{1}{20})^2}}$$

$$= \frac{10^6}{(200,000)(1.020)(1.001)}$$

$$= 4.897 \text{ V/V} = 13.80 \text{ dB}$$

$$\angle Ad(10\text{MHz}) = 180^\circ - \arctan\left(\frac{10\text{MHz}}{50\text{Hz}}\right) - \arctan\left(\frac{10\text{MHz}}{50\text{MHz}}\right) - \arctan\left(\frac{10\text{MHz}}{200\text{MHz}}\right)$$

$$= 180^\circ - 90^\circ - 11.31^\circ - 2.86^\circ = -104.17^\circ$$

And repeating for other values (by computer)

f	Ad  - lin	Ad  - dB	Phase
10 MHz	4.897	13.80	-104.17°
20 MHz	2.310	7.21	-117.52°
30 MHz	1.413	3.00	-129.49°
40 MHz	0.9571	-0.38 dB	-139.97°
50 MHz	0.6950	-3.27 dB	-144.04°
70 MHz	0.3919	-8.14 dB	-163.75°
100 MHz	0.2000	-13.98 dB	-180°
200 MHz	0.4284 × 10 <sup>-3</sup>	-27.36 dB	-210.96°
500 MHz	3.696 × 10 <sup>-3</sup>	-48.64 dB	-242.19°
700 MHz	1.398 × 10 <sup>-3</sup>	-57.09 dB	-249.97°
1 GHz	0.489167 × 10 <sup>-3</sup>	-66.20 dB	-255.82°

Loop bandwidth for (a)

Loop bandwidth = unity gain bandwidth for  
loop

From asymptotes, expect 50 MHz  
(1st pole @ 50 Hz,  $T=10^6$ )

Not an ideal estimate due to 2nd pole  
at 50 MHz, using plotting points we  
can get  $T(f)=1$  at  $f=40$  MHz

here phase =  $-139.97^\circ$   
 $\Rightarrow$  Phase margin =  $40.03^\circ$

stable but will see some peaking in  
close loop response.

Also phase of  $180^\circ$  at 100 MHz  
 $\Rightarrow$  gain margin of 13.98 dB

$\therefore$  loop bandwidth = 50 MHz (estimate)  
OR 40 MHz exactly

For 3dB bandwidth of  $V_{out}$   
this is same as estimate of 1st  
closed loop pole

$$f_{CL} = v_{OL}(1+T) = (50 \text{ MHz}) (1+10^6) = 50 \text{ MHz}$$

This is an estimate, higher  
order poles affect this.

esp. since 40 MHz loop bandwidth

For (B)

Same  $A_d(f) \rightarrow$  figures on page 2 apply,

$$\beta = \frac{R_2}{R_1 + R_2} = \frac{1k\Omega}{10k\Omega} = \frac{1}{10}$$

$$\frac{1}{\beta} = 10$$

i.e. with

$$T = A_o \beta \cdot 10^5 \frac{1}{\left(1 + \frac{jf}{50\text{Hz}}\right) \left(1 + \frac{jf}{50 \times 10^6 \text{Hz}}\right) \left(1 + \frac{jf}{200 \times 10^6 \text{Hz}}\right)}$$

$$\text{Loop bandwidth} = 10^5 (50\text{Hz}) = 5\text{MHz}$$

$$\begin{aligned} \text{Phase} &= -\arctan\left(\frac{5\text{MHz}}{50\text{MHz}}\right) - \arctan\left(\frac{5\text{MHz}}{5\text{Hz}}\right) \\ &\quad - \arctan\left(\frac{5\text{MHz}}{200\text{MHz}}\right) \\ &= -97.14^\circ \end{aligned}$$

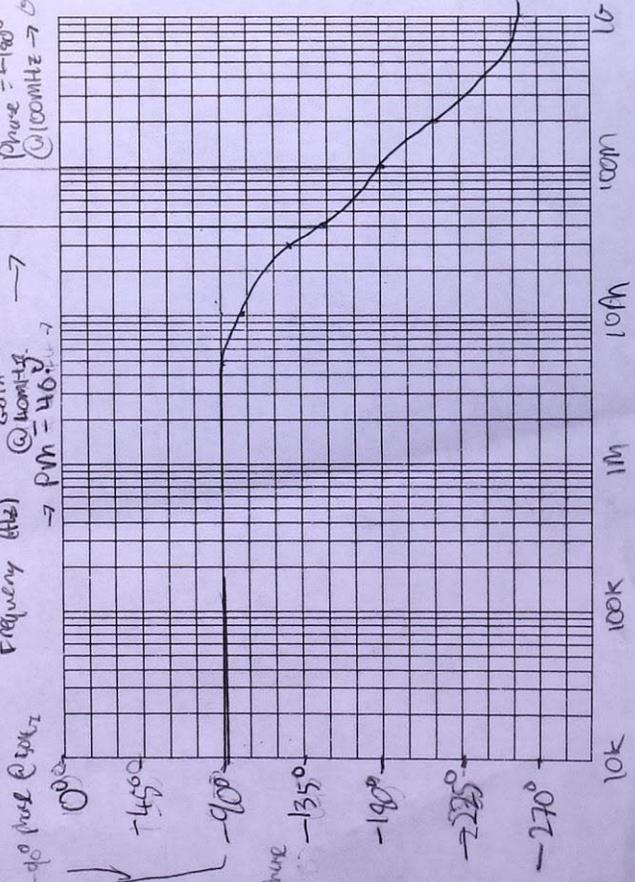
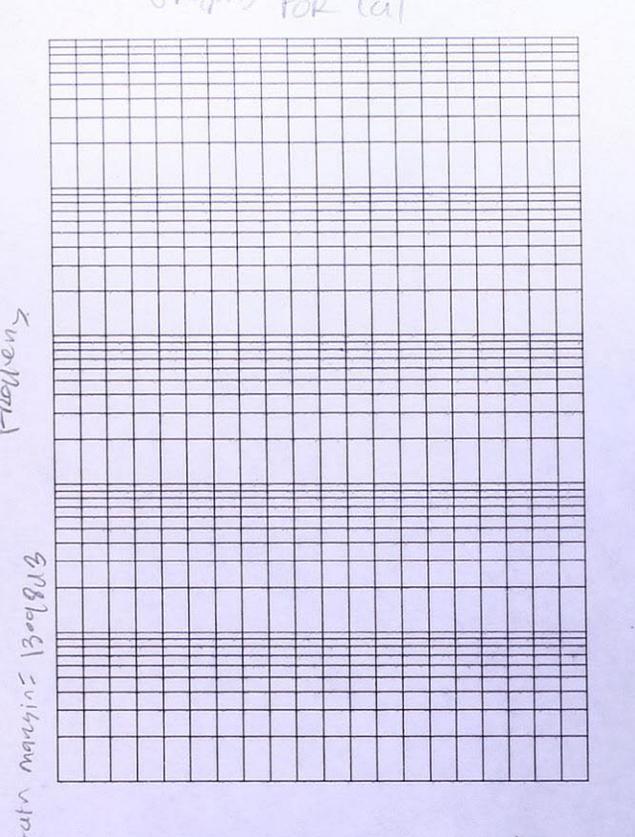
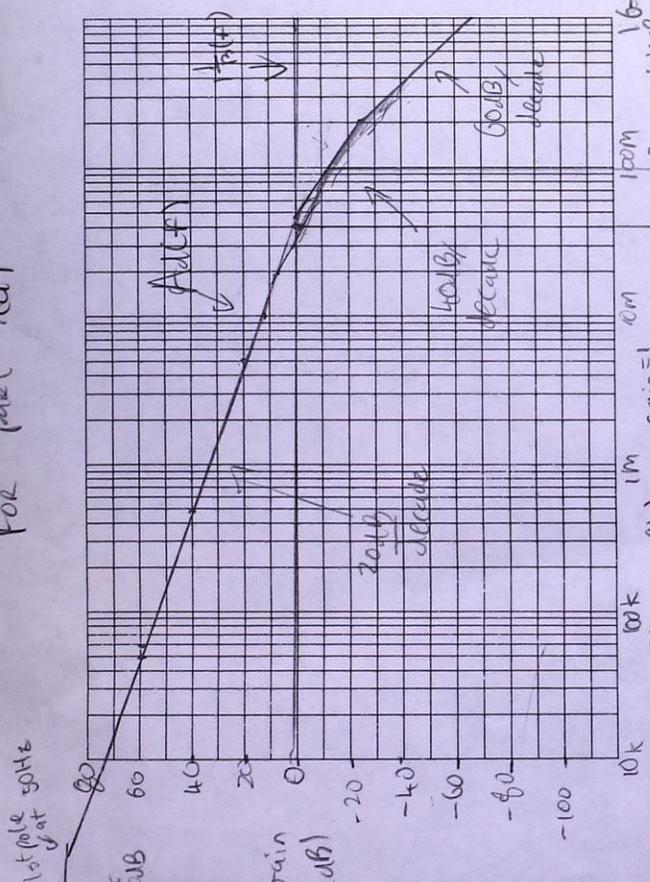
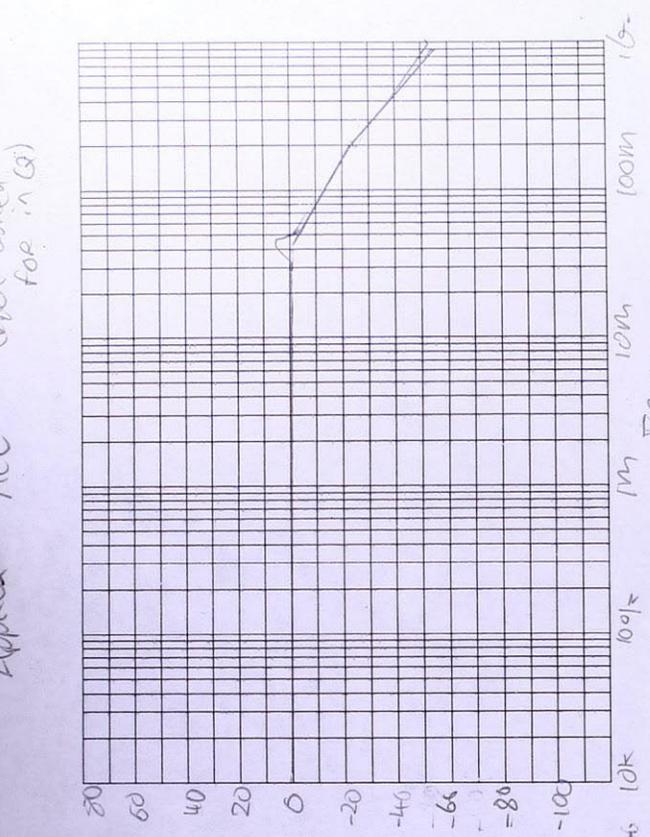
$$\text{Phase margin} = 82.86^\circ$$

$$\text{Phase} = 180^\circ \text{ @ } 100\text{MHz} \text{ (same } A_d \text{ as before)}$$

$$\text{Gain Margin} = 33.98\text{dB}$$

Approx Acl (not asked for in Q)

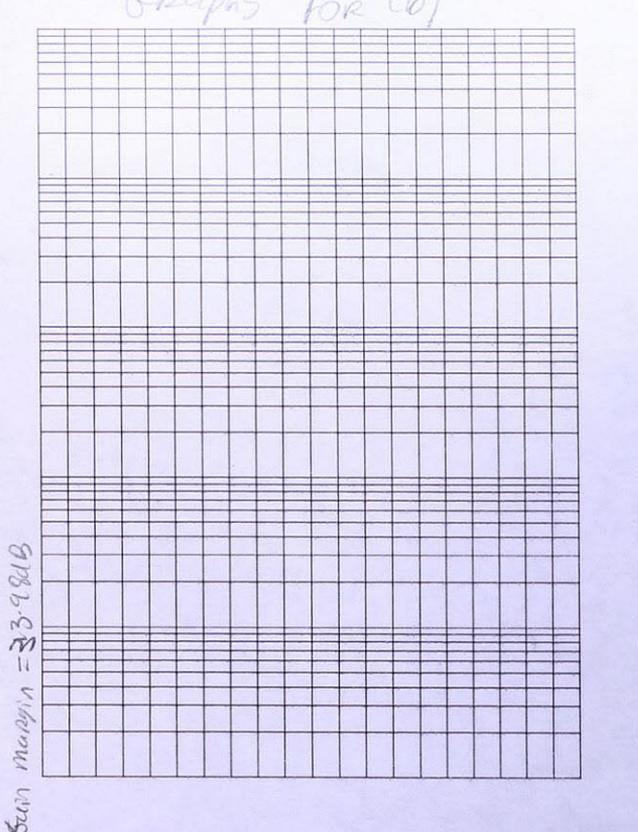
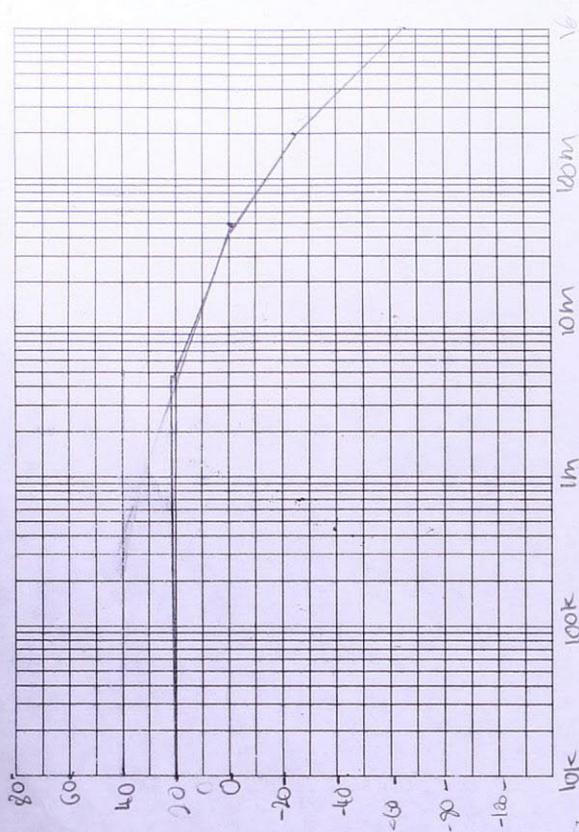
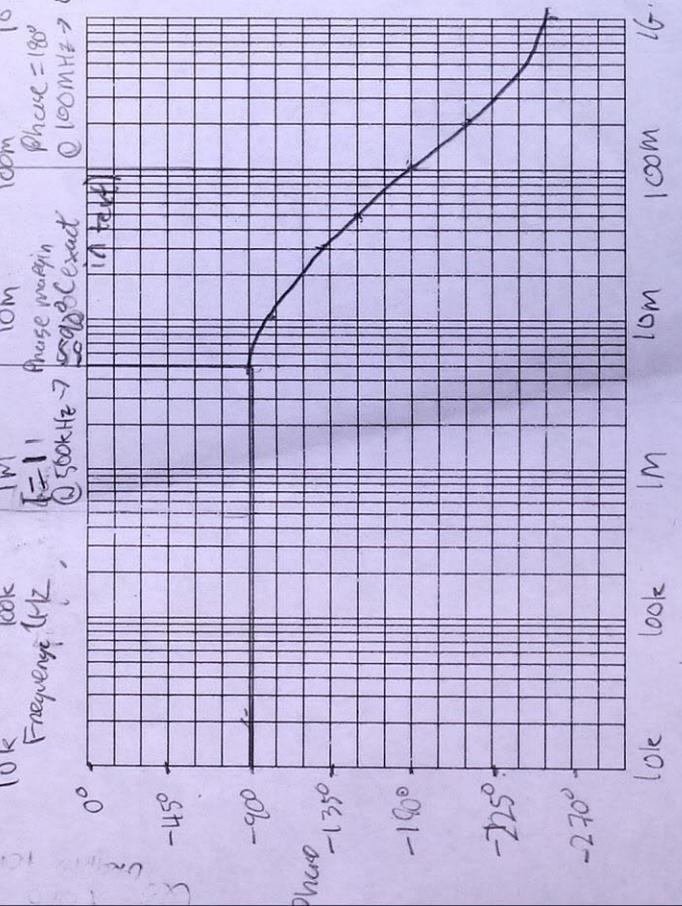
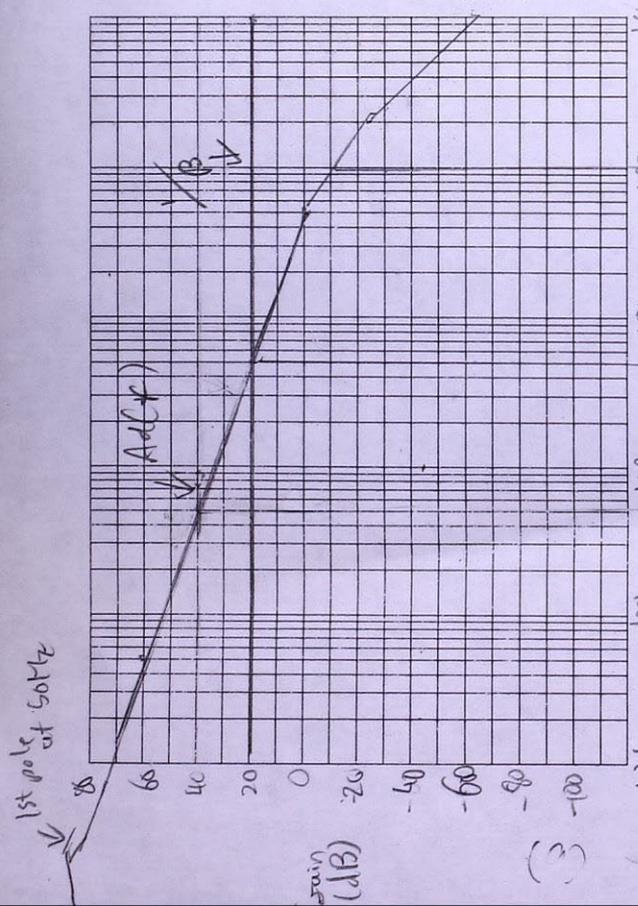
For (part (a))



Frequency (Hz)

Approx A<sub>c</sub>L (not asked for in Q)

Q3 p6/6  
graphs for (b)



Frequency

(a)

$$A_d = 10^6 \frac{1}{\left(1 + \frac{j\omega}{1\text{Hz}}\right) \left(1 + \frac{j\omega}{10^6\text{Hz}}\right)}$$

$$= 10^6 \frac{1}{\left(1 + \frac{j\omega}{2\pi \text{ rad s}^{-1}}\right) \left(1 + \frac{j\omega}{2\pi \times 10^6 \text{ rad s}^{-1}}\right)}$$

$$\beta = 1$$

$$T = A_d \beta = 10^6 \frac{1}{\left(1 + \frac{s}{2\pi \text{ rad s}^{-1}}\right) \left(1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right)}$$

$$A_{cl} = \frac{1}{\beta} \frac{T}{1+T} = 1 \frac{10^6 \left[ \frac{1}{\left(1 + \frac{s}{2\pi \text{ rad s}^{-1}}\right) \left(1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right)} \right]}{1 + 10^6 \left[ \frac{1}{\left(1 + \frac{s}{2\pi \text{ rad s}^{-1}}\right) \left(1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right)} \right]}$$

$$= \frac{10^6}{10^6 + \left[ 1 + \frac{s}{2\pi \text{ rad s}^{-1}} \right] \left[ 1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}} \right]}$$

$$= \frac{1}{1 + 10^{-6} + \left[ \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}} + \frac{1}{2\pi \times 10^{12} \text{ rad s}^{-1}} \right] s + \frac{1}{4\pi^2 \times 10^{12} \text{ rad}^2 \text{ s}^{-2}} s^2}$$

$$\approx \frac{1}{1 + \frac{1}{2\pi \times 10^6 \text{ rad s}^{-1}} s + \frac{1}{4\pi^2 \times 10^6 \text{ rad s}^{-1}} s^2}$$

$$= \frac{(2\pi \times 10^6)^2}{s^2 + 2\pi \times 10^6 s + (2\pi \times 10^6)^2}$$

$$= \frac{(2\pi \times 10^6)^2}{s^2 + 2\left(\frac{1}{2}\right)(2\pi \times 10^6)s + (2\pi \times 10^6)^2} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2nd order, underdamped

$$\omega_n = 2\pi \times 10^6 \text{ rad s}^{-1} = 1 \text{ MHz} \quad f_n = 1 \text{ MHz}$$

$$\zeta = 0.5 \rightarrow \text{underdamped, complex poles}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 1 \text{ MHz} \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} \text{ MHz}$$

$$= 0.8660 \text{ MHz} \quad \omega_d = 5.441 \text{ Mrad s}^{-1}$$

$$s = -\omega_n (\zeta \pm j \sqrt{1 - \zeta^2})$$

$$= -1 \text{ MHz} (\frac{1}{2}) \pm j (1 \text{ MHz} (\frac{\sqrt{3}}{2}))$$

$$= -0.5 \text{ MHz} \pm j 0.8660 \text{ MHz}$$

$$\text{Peak amplitude} = \frac{1}{2\zeta} = 1 = 0 \text{ dB}$$

For underdamped response graph

$$X(s) = \frac{1}{s} \cdot \frac{1}{s}$$

$$Y(s) = H(s)X(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\text{Partial fraction} = \frac{1V}{s} - \frac{1V (s + \zeta\omega_n)}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{1V (\zeta\omega_n)}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

$$V_{out}(t) = 1V u(t) \left[ 1 - e^{-\zeta\omega_n t} \left( \cos \omega_d t + \zeta \frac{\omega_n}{\omega_d} \sin \omega_d t \right) \right]$$

$$m_p = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \exp\left(\frac{-\pi(\frac{1}{2})}{\frac{\sqrt{3}}{2}}\right) = \exp\left(\frac{-\pi}{\sqrt{3}}\right)$$

$$= 0.1630$$

$$\text{At } t_p = \frac{\pi}{\omega_d} = 0.5774 \mu\text{s}$$

$$t_r \approx \frac{1.6}{\omega_n} = 0.2865 \mu\text{s}$$

$$\sigma = \zeta\omega_n = \pi \times 10^6 \text{ s}^{-1}$$

$$t_s = \frac{4.6}{\sigma} = 1.464 \mu\text{s}$$

$$\frac{1}{\sigma} = 0.318 \mu\text{s}$$

(b)

$$A\beta = \frac{1}{100}$$

$$T = A\beta = 10^4 \frac{1}{\left(1 + \left(\frac{s}{2\pi \text{ rad s}^{-1}}\right)\right) \left(1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right)}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = A_{cl} = \frac{1}{\beta} \frac{T}{1+T} = 100 \left[ \frac{10^4 \left[ \frac{1}{\left(1 + \frac{s}{2\pi \text{ rad s}^{-1}}\right) \left(1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right)} \right]}{1 + 10^4 \left[ \frac{1}{\left(1 + \frac{s}{2\pi \text{ rad s}^{-1}}\right) \left(1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right)} \right]} \right]$$

$$= 100 \frac{10^4}{\left(1 + \frac{s}{2\pi \text{ rad s}^{-1}}\right) \left(1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right) + 10^4}$$

$$= 100 \frac{1}{1 + 10^{-4} + \left(\frac{1}{2\pi \times 10^4 \text{ rad s}^{-1}} + \frac{1}{2\pi \times 10^6 \text{ rad s}^{-1}}\right) s + \frac{1}{4\pi^2 \times 10^{10} \text{ rad}^2 \text{ s}^{-2}}}$$

$$\approx 100 \frac{1}{1 + \frac{1}{2\pi \times 10^4 \text{ rad s}^{-1}} s + \frac{1}{4\pi^2 \times 10^{10} \text{ rad}^2 \text{ s}^{-2}} s^2}$$

$$= 100 \frac{1}{\left[1 + \left(\frac{1}{2\pi \times 10^4 \text{ rad s}^{-1}}\right) s\right] \left[1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right]}$$

(using the separated pole approximation)

$$= 100 \frac{1}{\left[1 + \frac{jf}{10 \text{ kHz}}\right] \left[1 + \frac{jf}{1 \text{ MHz}}\right]}$$

2 distinct real poles,

1 at 10 kHz,

1 at 1 MHz

In damped response terms

$$A_{OL} = 100 \frac{(2\pi \times 10^5 \text{ rad s}^{-1}) s}{s^2 + (2\pi \times 10^6 \text{ rad s}^{-1}) s + (2\pi \times 10^5 \text{ rad s}^{-1})^2}$$

$$\omega_n = 2\pi \times 10^5 \text{ rad s}^{-1}$$

$$\zeta = \frac{1}{2\omega_n} 2\pi \times 10^6 \text{ rad s}^{-1} = \frac{1}{2} \frac{2\pi \times 10^6}{2\pi \times 10^5} = 5$$

overdamped  $\Rightarrow$  wd not defined.

$$X(s) = 1V \frac{1}{s}$$

$$Y(s) = A_c(s) X(s)$$

$$= 1V \frac{1}{s} \frac{1}{\left[1 + \frac{s}{2\pi \times 10^4 \text{ rad s}^{-1}}\right] \left[1 + \frac{s}{2\pi \times 10^6 \text{ rad s}^{-1}}\right]}$$

$$= 1V \left[ \frac{k_1}{s} + \frac{k_2}{1 + \left[\frac{1}{2\pi \times 10^4 \text{ rad s}^{-1}}\right] s} + \frac{k_3}{1 + \left[\frac{1}{2\pi \times 10^6 \text{ rad s}^{-1}}\right] s} \right]$$

$$\text{As } s \rightarrow 0 \quad k_1 = 0$$

$$\text{As } s \rightarrow -2\pi \times 10^4 \text{ rad s}^{-1} \quad k_2 = \frac{1}{\left[-2\pi \times 10^4 \text{ rad s}^{-1}\right] \left[1 + \frac{-2\pi \times 10^4 \text{ rad s}^{-1}}{2\pi \times 10^6 \text{ rad s}^{-1}}\right]}$$

$$= \frac{1}{\left[-\pi \times 10^4 \text{ rad s}^{-1}\right] \left[0.99\right]} = \frac{-1.01}{2\pi \times 10^4 \text{ rad s}^{-1}}$$

$$\text{As } s \rightarrow -2\pi \times 10^6 \text{ rad s}^{-1} \quad k_3 = \frac{1}{\left[-2\pi \times 10^6 \text{ rad s}^{-1}\right] \left[1 + \frac{-2\pi \times 10^6 \text{ rad s}^{-1}}{2\pi \times 10^4 \text{ rad s}^{-1}}\right]}$$

$$= \frac{1}{\left[-2\pi \times 10^6 \text{ rad s}^{-1}\right] \left[1 - 100\right]} = \frac{+0.01}{2\pi \times 10^6 \text{ rad s}^{-1}}$$

$$Y(s) = 1V \left[ \frac{1}{s} + \frac{-1.01 \left[\frac{1}{2\pi \times 10^4 \text{ rad s}^{-1}}\right]}{1 + \left[\frac{1}{2\pi \times 10^4 \text{ rad s}^{-1}}\right] s} + \frac{0.01 \left[\frac{1}{2\pi \times 10^6 \text{ rad s}^{-1}}\right]}{1 + \left[\frac{1}{2\pi \times 10^6 \text{ rad s}^{-1}}\right] s} \right]$$

$$y(t) = 1V \left[ 1 - 1.01 e^{-\left(2\pi \times 10^4 \text{ s}^{-1}\right)t} + 0.01 e^{-\left(2\pi \times 10^6 \text{ s}^{-1}\right)t} \right] u(t)$$

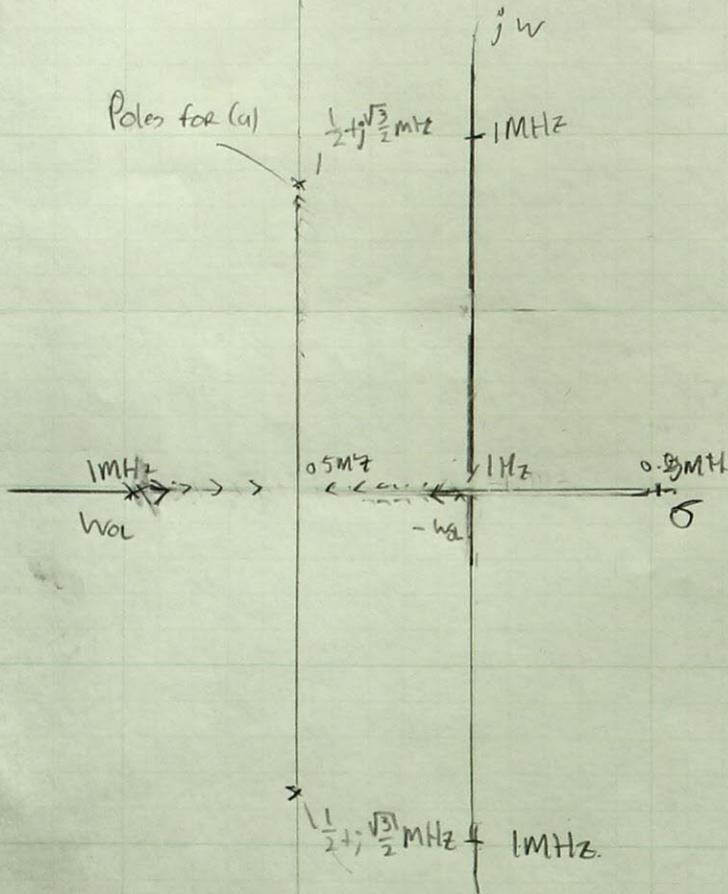
$$\tau_1 = 159.2 \mu\text{s}$$

$$\tau_2 = 1.592 \mu\text{s}$$

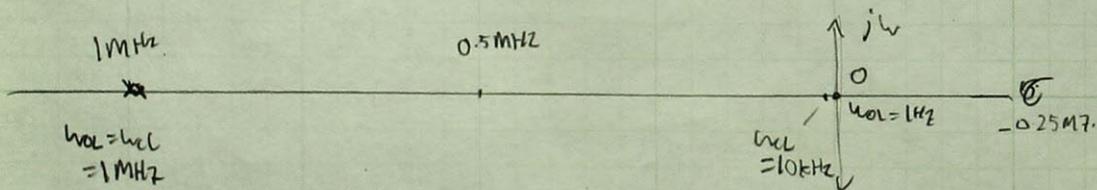
$$y(t) = 1V \left[ 1 - 1.01 e^{-\frac{t}{159.2 \mu\text{s}}} + 0.01 e^{-\frac{t}{1.592 \mu\text{s}}} \right]$$

$$\tau_R = 2.2 \tau = 350 \mu\text{s}$$

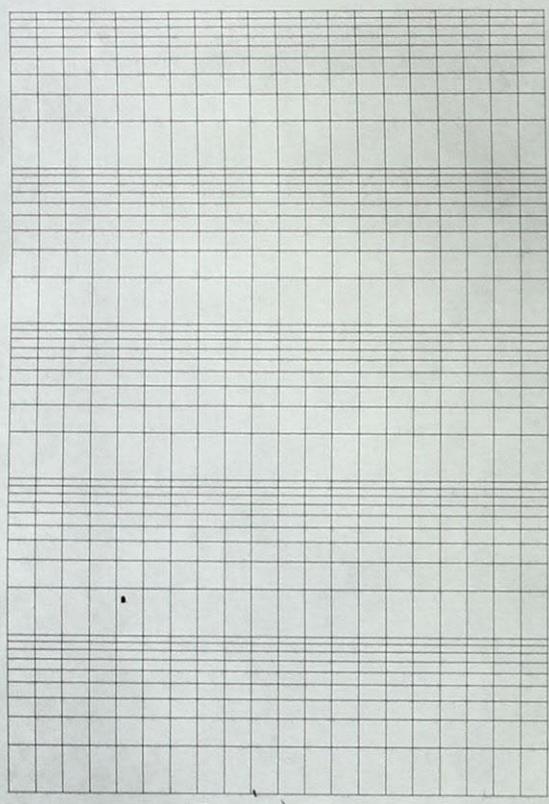
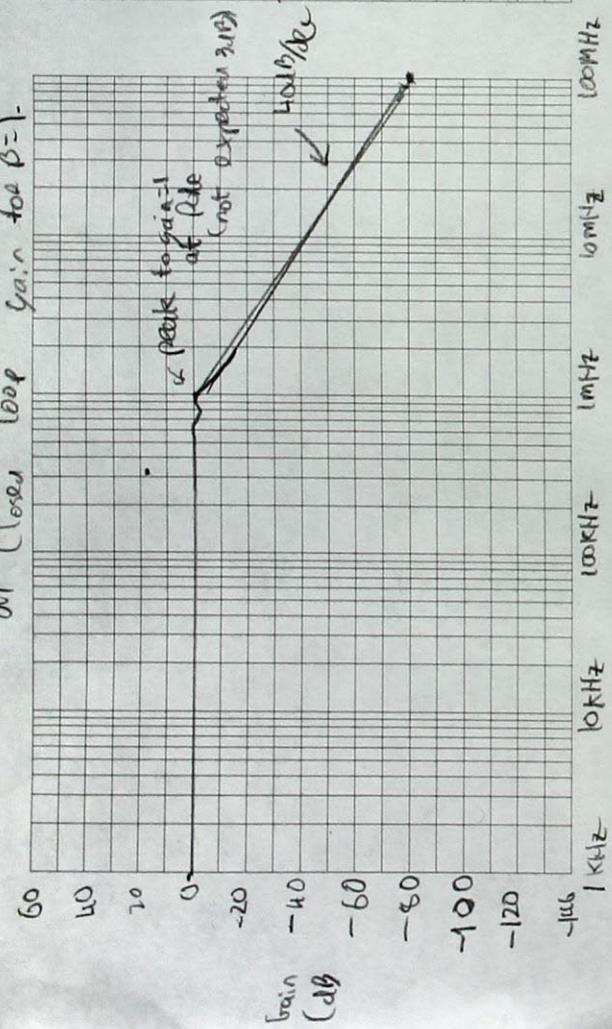
For (a)



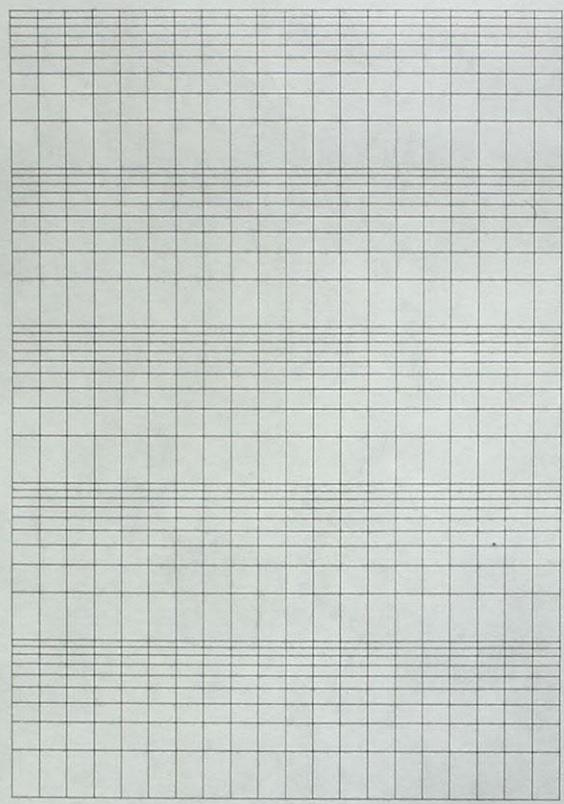
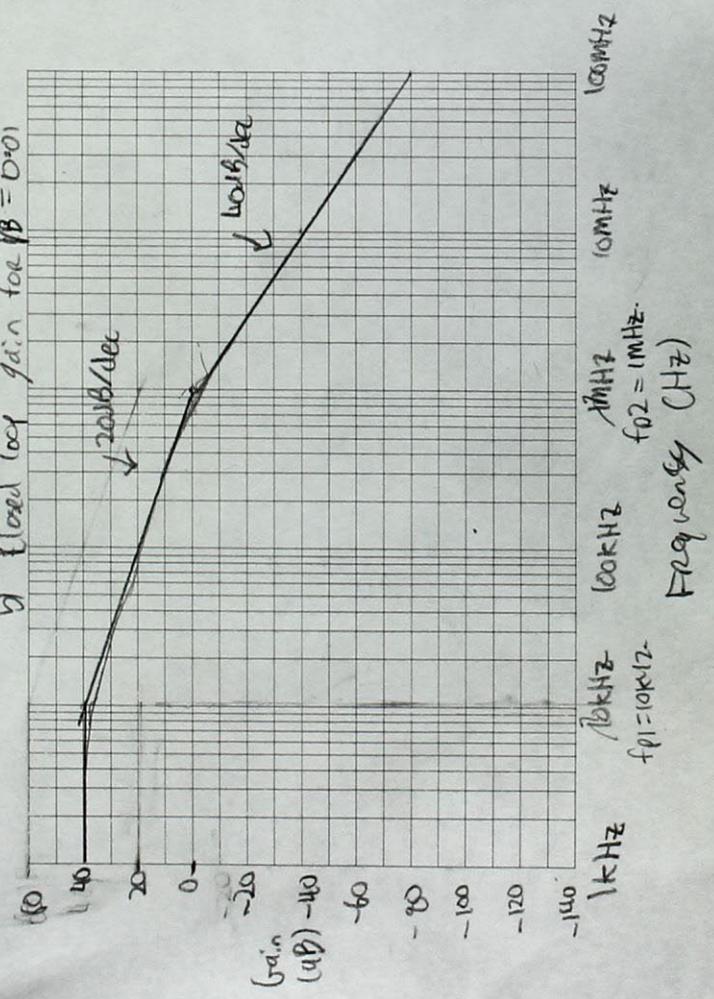
For (b)

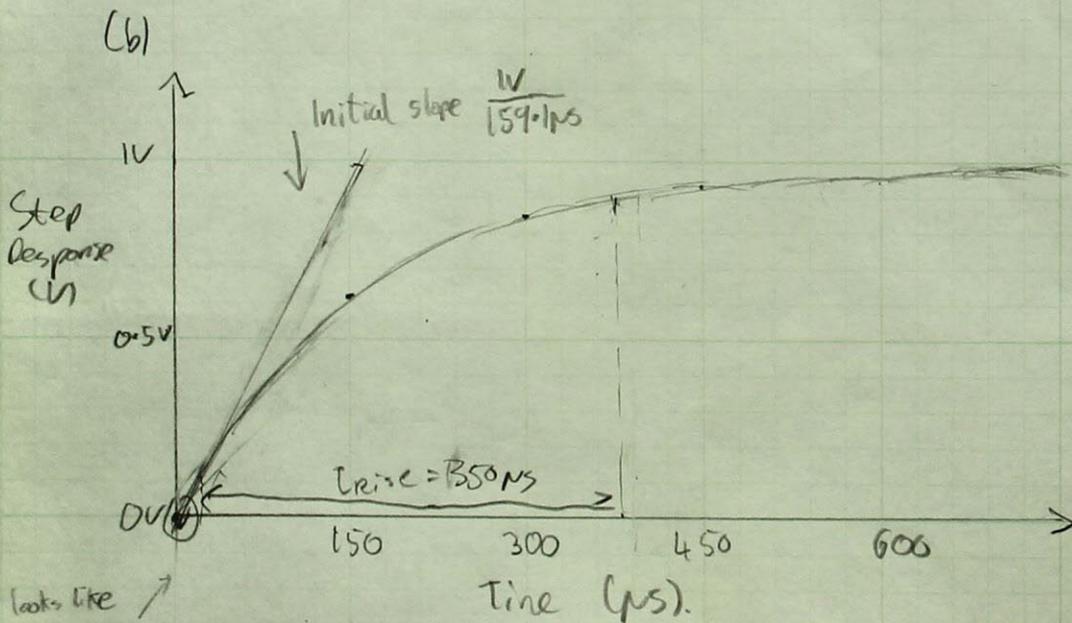
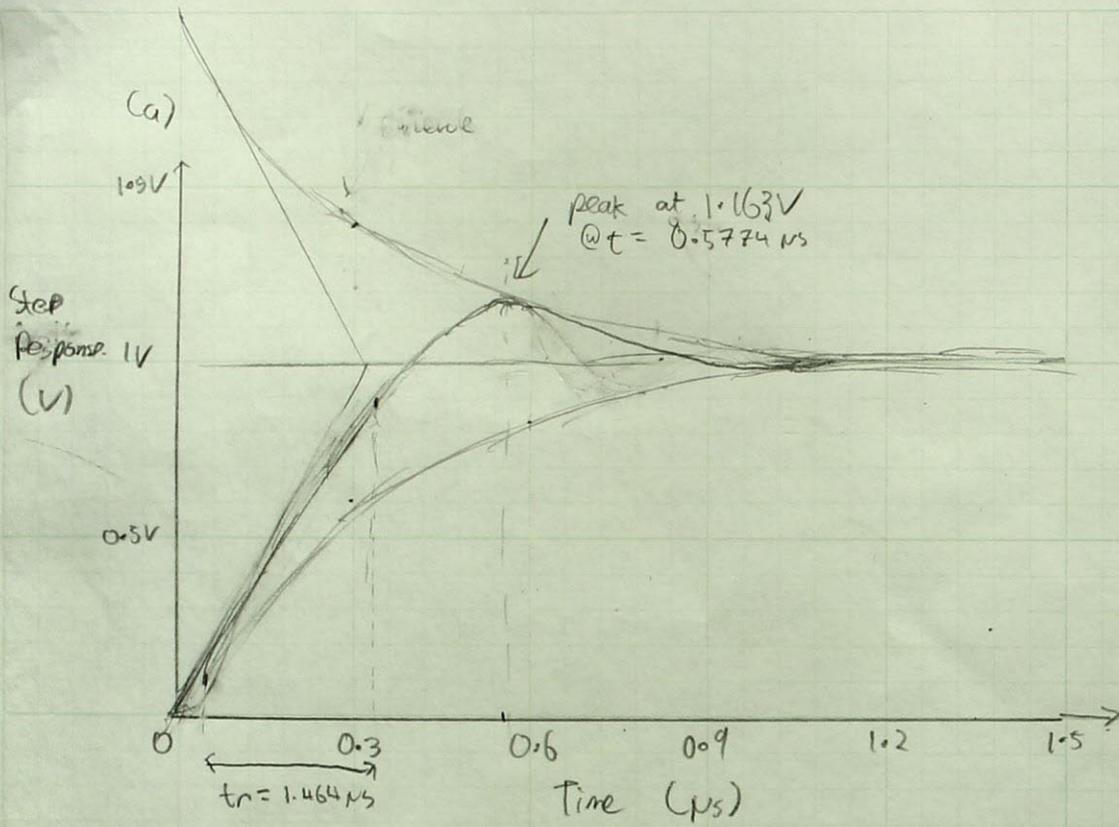


a) (open loop gain for  $\beta=1$ )

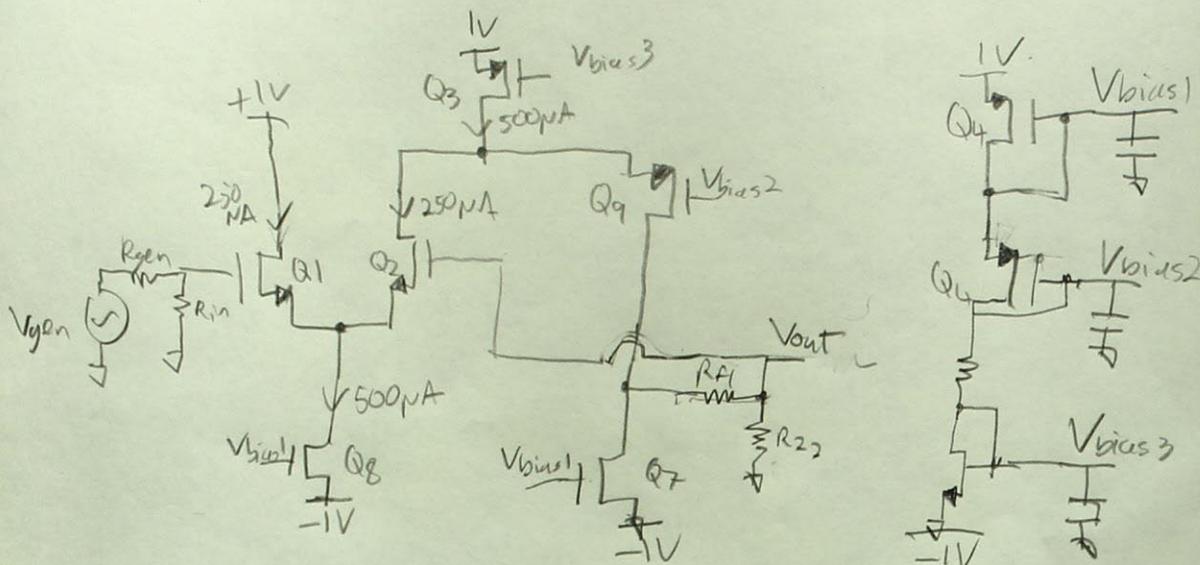


b) (closed loop gain for  $\beta=0.01$ )





looks like  $\nearrow$   
on much smaller timescale



Bias network

$$V_{bias1} = V_{dd} - V_{gs4} = 1V - 0.3V = 0.7V$$

$$V_{bias2} = V_{bias1} - V_{gs5} = 1V - 0.3V = 0.4V$$

$$V_{bias3} = V_{ss} + V_{gs6} = -1V + 0.3V = -0.7V$$

$$R_{bias} = \frac{V_{bias2} - V_{bias3}}{I(Q6)} = \frac{0.4V - (-0.7V)}{250nA} = \frac{1.1V}{0.25mA} = 4.4k\Omega$$

Given that Q1, Q2, Q4, Q5, Q6, Q7, Q8 all run at 250 nA  
 $\Rightarrow$  Q3 and Q8 run at 500 nA.

$$\text{Also } V_{gs3} = V_{dd} - V_{bias1} = 1V - 0.7V = 0.3V$$

$$V_{gs3} = V_{bias3} - V_{ss} = -0.7 - (-1V) = 0.3V$$

so same gate-source voltage as other transistors too

$$\Delta V = \frac{L_g v_{sat}}{\mu_n} = \frac{(22 \text{ nm})(10^7 \text{ cm/s})}{200 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}} = \frac{(22 \times 10^{-7} \text{ cm})(10^7 \text{ cm/s})}{200 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}}$$

$$= 0.11 \text{ V}$$

Not given separate parameters for P  
 $\rightarrow$  same values apply

$$V_{gs} = 0.3 \text{ V for all transistors}$$

$$V_{gs} - V_{th} = 0.3 \text{ V} - 0.25 \text{ V} = 0.05 \text{ V} < \Delta V_N = 0.11 \text{ V}$$

$\rightarrow$  all transistors mobility saturated

For  $Q1, Q2, Q6, Q7, Q8$

$$I_{ds} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_{th})^2$$

$$\text{where } C_{ox} = \frac{\epsilon_r \epsilon_0}{T_{ox}} = \frac{(3.8)(8.854 \times 10^{-12} \text{ F m}^{-1})}{0.8 \text{ nm}}$$

$$= 42.06 \times 10^{-3} \text{ F m}^{-2}$$

$$= 42.06 \text{ fF } (\mu\text{m})^{-2}$$

$$W = \frac{2 I_{ds}}{\mu_n C_{ox} (V_{gs} - V_{th})^2} L = \frac{2(250 \times 10^{-6} \text{ A})(22 \text{ nm})}{(0.02 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1})(42.06 \times 10^{-3} \text{ F m}^{-1})(0.05 \text{ V})^2}$$

$$= 5231 \text{ nm} = 5.231 \mu\text{m}$$

$$C_{gd} = (0.5 \text{ fF}/\mu\text{m}) = 2.616 \text{ fF}$$

$$C_{gs} = \frac{\epsilon_r \epsilon_0}{T_{ox}} W_g L_g + (0.5 \text{ fF}/\mu\text{m}) W_g$$

$$= C_{ox} W_g L_g + (0.5 \text{ fF}/\mu\text{m}) W_g$$

$$= 7.456 \text{ fF}$$

$$g_m = \mu_n C_{ox} \frac{W_g}{L_g} (V_{gs} - V_{th}) = \frac{2 I_{d}}{V_{gs} - V_{th}} = \frac{2(250 \mu\text{A})}{0.05} = 10 \text{ mS}$$

For pmos  $\rightarrow$  same physical values  
 $Q_4, Q_5, Q_9$  implies same number

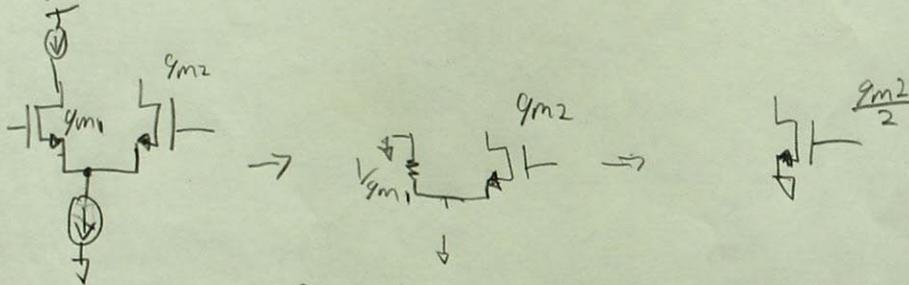
For  $Q_3, Q_8$   $2 \times$   $l_d$  at same  $V_{gs}$  implies double  $W_g$   
 $\rightarrow$  double  $C_{gd}, C_{gs}, g_m$

Mosfet	Type	$(V_{gs})$	$(I_d)$	$W_g$	$g_m$	$C_{gd}$	$C_{gs}$
Q1	N	0.3V	0.25 $\mu$ A	5.231 $\mu$ m	10 mS	2.616 fF	7.456 fF
Q2	N	0.3V	0.25 nA	5.231 $\mu$ m	10 mS	5.231 fF	7.456 fF
Q3	P	0.3V	0.50 nA	10.462 $\mu$ m	20 mS	2.616 fF	14.912 fF
Q4	P	0.3V	0.25 nA	5.231 $\mu$ m	10 mS	2.616 fF	7.456 fF
Q5	P	0.3V	0.25 nA	5.231 $\mu$ m	10 mS	2.616 fF	7.456 fF
Q6	N	0.3V	0.25 nA	5.231 $\mu$ m	10 mS	2.616 fF	7.456 fF
Q7	N	0.3V	0.25 nA	5.231 $\mu$ m	10 mS	2.616 fF	7.456 fF
Q8	N	0.3V	0.50 nA	10.462 $\mu$ m	20 mS	5.231 fF	14.912 fF
Q9	P	0.3V	0.25 nA	5.231 $\mu$ m	10 mS	2.616 fF	7.456 fF

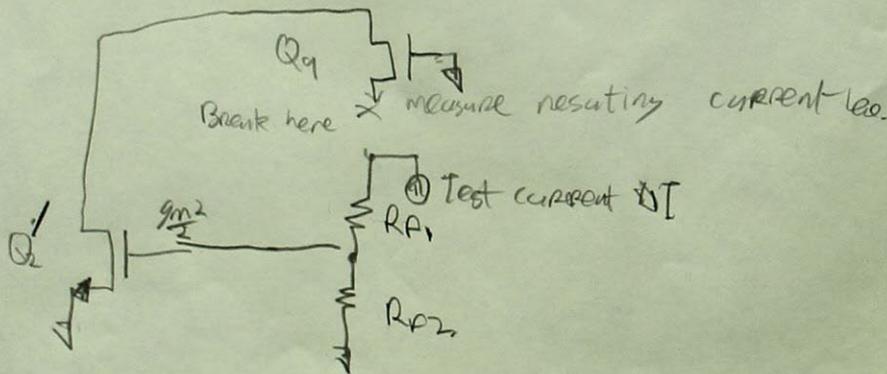
(b) For small signal  $T$ , want a convenient loop break to make solution easy

$Q_2$ 's output current <sup>output</sup> is a good choice, ~~and~~ since it's at common drain, thus no caps across the stage.

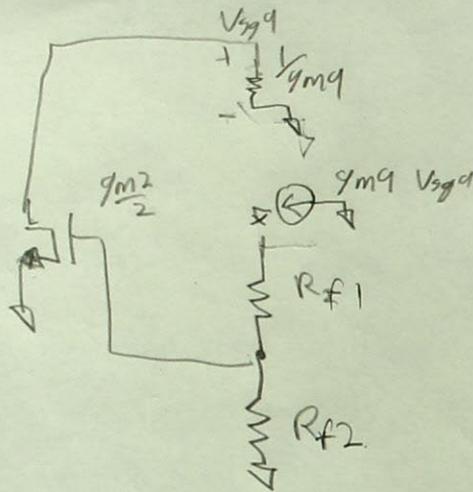
Also for  $T$ ,  $V_{GS}$  is shorted so  $Q_1$  acts like a  $\frac{1}{g_m}$  resistor from  $Q_2$ 's point of view.



$$g_{m2,eff} = \frac{g_{m2}}{1 + g_{m2}(\frac{1}{g_{m1}})} = \frac{g_{m2}}{1+1} = \frac{g_{m2}}{2}$$



$$\frac{I_T}{2} = I_o$$



$$I_{IT} = (R_{f1} + R_{f2}) \frac{R_{f2}}{(R_{f1} + R_{f2})} \cdot \frac{g_{m2}}{2} \left( \frac{1}{g_{m4}} \right) \cdot g_{m4}$$

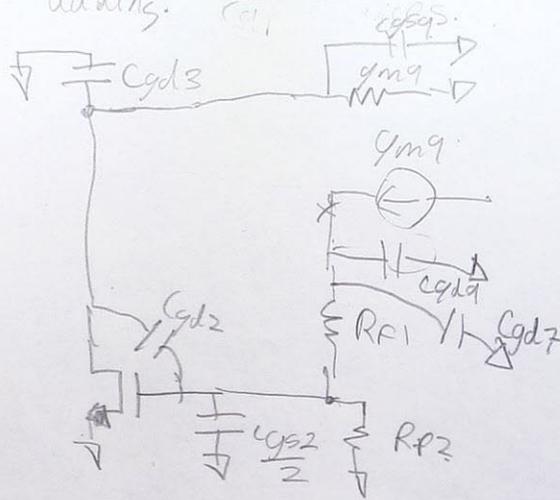
$$= \frac{g_{m2}}{2} R_{f2} = \frac{10 \text{ mS}}{2} (100 \text{ k}\Omega) = 500$$

$$\beta = \frac{R_{f2}}{R_{f1} + R_{f2}} = \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 1000 \text{ k}\Omega} = \frac{1}{11}$$

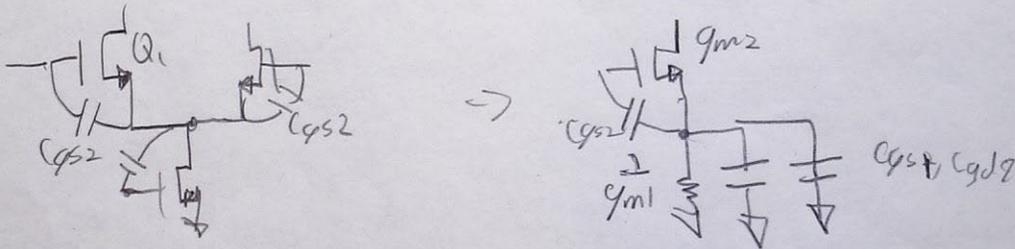
$$\rightarrow A_{\infty} = \frac{1}{\beta} = 11$$

$$A_{CL} = \frac{1}{\beta} \frac{I}{I+T} = 11 \frac{500}{1+500} = 10998$$

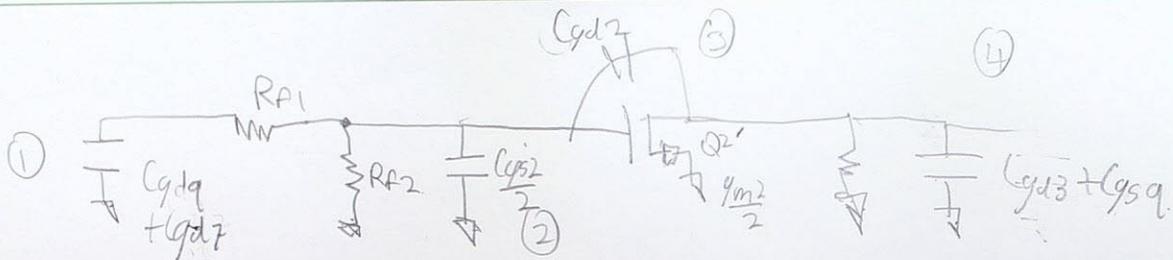
(C) Now adding Capacitances



Note we still make the degeneration approximation here at AC.



If only had  $C_{gs1}$  (no  $C_{gd3}$ )  
 then cap divider same as resistor divider,  
 could perfectly degenerate to  $\frac{C_{gs2}}{2}$ . As it is,  
 its an approximation to do this but an  
 Ok one since the effects here are  
 close to transistor FT



Method of time constants

$$R_{11}^{\circ} C_1 = (C_{gd1} + C_{gd2}) (R_{A1} + R_{E2}) = (2.616 \text{ fF} + 2.616 \text{ fF}) (1100 \text{ k}\Omega) = 5.755 \text{ ps} = 5.755 \text{ ns}$$

$$R_{22}^{\circ} C_2 = \frac{C_{gs2}}{2} R_{E2} = \left( \frac{7.456 \text{ fF}}{2} \right) (100 \text{ k}\Omega) = 372.8 \text{ ps} = 0.3728 \text{ ns}$$

$$\begin{aligned} R_{33}^{\circ} C_3 &= C_{gd2} \left[ R_{E2} \left( 1 + \frac{g_{m2}}{g_{m1}} \right) + \frac{1}{g_{m1}} \right] \quad g_{m2} = g_{m1} \\ &= C_{gd2} \left[ \frac{3}{2} R_{E2} + \frac{1}{g_{m1}} \right] \\ &= (2.616 \text{ fF}) \left[ \frac{3}{2} (100 \text{ k}\Omega) + \frac{1}{10 \text{ mS}} \right] \\ &= (2.616 \text{ fF}) (150 \text{ k}\Omega + 0.1 \text{ k}\Omega) = 392.7 \text{ ps} \end{aligned}$$

$$\begin{aligned} R_{44}^{\circ} C_4 &= (C_{gd3} + C_{gsq}) \left[ \frac{1}{g_{m1}} \right] = (5.231 \text{ fF} + 7.456 \text{ fF}) \left( \frac{1}{10 \text{ mS}} \right) \\ &= (12.687 \text{ fF}) (0.1 \text{ k}\Omega) = 1.269 \text{ ps} \end{aligned}$$

$$\begin{aligned} a_1 &= R_{11}^{\circ} C_1 + R_{22}^{\circ} C_2 + R_{33}^{\circ} C_3 + R_{44}^{\circ} C_4 = 6.522 \text{ ps} \\ &= 6.522 \text{ ns} \end{aligned}$$

$$R_{22}^1 = R_{f2} \parallel R_{f1} = 90.91 \text{ k}\Omega$$

$$R_{11}^0 R_{22}^1 C_1 C_2 = (1100 \text{ k}\Omega)(90.91 \text{ k}\Omega)(5.232 \text{ nF})(3.728 \text{ nF}) \\ = 1.951 \times 10^6 \text{ (ps)}^2$$

$$R_{33}^1 = [R_{f2} \parallel R_{f1}] \left(\frac{3}{2}\right) + \frac{1}{g_{mq}} = 136.4 + 0.1 = 136.5 \text{ k}\Omega$$

$$R_{44}^0 R_{33}^1 C_1 C_3 = (1100 \text{ k}\Omega)(136.5 \text{ k}\Omega)(5.232 \text{ nF})(2.616 \text{ nF}) \\ = 2.055 \times 10^6 \text{ (ps)}^2$$

$$R_{44}^1 = \frac{1}{g_{mq}} = \frac{1}{10 \text{ mS}} = 0.1 \text{ k}\Omega$$

$$R_{11}^0 R_{44}^1 C_1 C_4 = (1100 \text{ k}\Omega)(0.1 \text{ k}\Omega)(5.232 \text{ nF})(12.687 \text{ nF}) \\ = 7.302 \times 10^5 \text{ (ps)}^2$$

$$R_{33}^2 = \frac{1}{g_{mq}} = \frac{1}{10 \text{ mS}} = 0.1 \text{ k}\Omega$$

$$R_{22}^0 R_{33}^2 C_2 C_3 = (100 \text{ k}\Omega)(0.1 \text{ k}\Omega)(3.728 \text{ nF})(2.616 \text{ nF}) \\ = 97.52 \text{ (ps)}^2$$

$$R_{44}^2 = \frac{1}{g_{mq}} = 0.1 \text{ k}\Omega$$

$$R_{21}^0 R_{44}^2 C_2 C_4 = (100 \text{ k}\Omega)(0.1 \text{ k}\Omega)(3.728 \text{ nF})(12.687 \text{ nF}) \\ = 472.97 \text{ (ps)}^2$$

$$R_{33}^4 = R_{f2} = 100 \text{ k}\Omega$$

$$R_{33}^4 R_{44}^0 C_3 C_4 = (100 \text{ k}\Omega)(0.1 \text{ k}\Omega)(2.616 \text{ nF})(3.728 \text{ nF}) \\ = 97.52 \text{ nF}$$

$$\begin{aligned}
 q_2 &= R_{11}^0 R_{22}^1 C_1 C_3 + R_{11}^0 R_{33}^1 C_1 C_3 + R_4^0 R_{44}^1 C_1 C_4 \\
 &+ R_{22}^0 R_{33}^2 C_2 C_3 + R_{22}^0 R_{44}^2 C_2 C_4 + R_{33}^4 R_{44}^0 C_3 C_4 \\
 &= 4.014 \times 10^6 (\text{ps})^2 = 4.014 (\text{ns})^2
 \end{aligned}$$

Try separated poles approximation

$$\frac{q_2}{q_1} = \frac{4.014 (\text{ns})^2}{6.522 \text{ ns}} = 0.6154 \text{ ns}$$

10x separation - SPA (just) holds

$$\tau_{p1} = 6.522 \text{ ns}$$

$$\tau_{p2} = 0.6154 \text{ ns}$$

Also -  $C_{gd2}$  is a cap across a CS stage.  
 $\Rightarrow$  set usual RHP zero

$$\begin{aligned}
 \tau_{\text{zero}} &= -\frac{C_{gd2}}{\left(\frac{g_{m2}}{2}\right)} = -0.5232 \text{ ns} \\
 &= -523.2 \times 10^{-6} \text{ ns}
 \end{aligned}$$

$$\text{Thus } T(s) = 500 \frac{1 - (523 \times 10^{-6} \text{ ns})s}{[1 + (6.522 \text{ ns})s][1 + (0.6154 \text{ ns})s]}$$

$$f_{p1} = \frac{1}{2\pi (6.522 \text{ ns})} = 24.40 \text{ MHz}$$

$$f_{p2} = 258.6 \text{ MHz}$$

$$f_{\text{zero}} = 304.2 \text{ GHz}$$

(a) Bode plot

By asymptotes

10 MHz - 24.40 MHz - flat at  $T_{OL} = 500 \text{ V/V} = 53.98 \text{ dB}$ 

24.40 MHz - 258.6 MHz - fall off at 20 dB/dec

@ 258.6 MHz  $|T(j\omega)|_{\text{asymptote}} = 47.19 \text{ V/V} = 33.48 \text{ dB}$ 

258.6 MHz - 304.2 GHz - fall off at 40 dB/dec

@ 304.2 GHz asymptote =  $34.10 \times 10^{-6} = -89.34 \text{ dB}$ 

304.2 GHz - 1 THz - fall off at 20 dB/dec

@ 1 THz, asymptote =  $10.37 \times 10^{-6} = -99.68 \text{ dB}$ 

Estimating unit gain from asymptotes

$$f = (258.6 \text{ MHz} \sqrt{47.19}) = 1.776 \text{ GHz}$$

$$\text{Phase} = \arctan\left(\frac{1.776 \text{ MHz}}{24.40 \text{ MHz}}\right) + \arctan\left(\frac{1.776 \text{ MHz}}{258.6 \text{ MHz}}\right) \\ + \arctan\left(\frac{1.776 \text{ MHz}}{304.2 \times 10^3 \text{ MHz}}\right)$$

$$= 89.21^\circ + 81.72^\circ + 0.33^\circ$$

$$= 171.26^\circ$$

$$\text{Phase margin} = 180^\circ - 171.26^\circ = 8.74^\circ$$

∴ stable, but just about

Q5 p11/11

