The Wave Equation in Birefringent Media, Modes in Optical Fiber

Read: Kasap, Chapter 2
Homework#1 due Today

ECE 162C
Lecture #4
Prof. John Bowers
Two primary limits to transmission

• **Loss:** Loss budget for loss limited transmission

• **Dispersion:** Dispersion budget for dispersion limited transmission.
Comparison to cable

**FIGURE 3-11**
A comparison of the attenuation as a function of frequency or data rate of various coaxial cables and several types of high-bandwidth optical fibers.
Loss in early optical fibers  
(now the O-H peaks around 1.4 μm are small)

\[ \tau_R = \frac{C}{\lambda^4} \]

*Figure 3-19* Observed loss spectrum of a germanosilicate single-mode fiber. Estimated loss spectra for various intrinsic materials effects and waveguide imperfections are also shown. (From Reference [20].)
Loss Budget

- $p_{\text{trans}} = \text{transmitter power}$
- $p_{\text{rec}} = \text{sensitivity of receiver}$

\[ P_{\text{rec}} = P_{\text{trans}} e^{-\alpha L} \]

- Take 10 log of each side and express in dBm
- $P_{\text{trans}}, P_{\text{rec}}$

\[ P_{\text{rec}} = P_{\text{trans}} - \alpha L \]

\[ L_{\text{max}} = \frac{P_{\text{trans}} - P_{\text{rec}}}{\alpha} \]

- Example:
- $P_{\text{trans}} = 10 \text{ dBm}$
- $P_{\text{rec}} = -20 \text{ dBm}$
- $L_{\text{max}} = 30 \text{ dB/0.2 dB/km} = 150 \text{ km}$
Two primary limits to transmission

- **Loss**: Loss budget for loss limited transmission
- **Dispersion**: Dispersion budget for dispersion limited transmission.
Dispersion

- Multimode—different modes have different $\beta$
- Intramodal (i.e. group-velocity dispersion)
  - Material dispersion — silica refractive index is a function of wavelength
  - Waveguide dispersion — $V$ parameter is a function of wavelength
- Polarization-Mode Dispersion — bifrebringence induced by perturbations
Multimode Dispersion

- For step index multimode fibers, the fiber bandwidth (in MHz km) is given by

\[ B < \frac{n_2}{n_1^2} \frac{c}{L \Delta} \]

- For graded index fibers, the fiber bandwidth in MHz km is given by

\[ B < \frac{8c}{n_1 L \Delta^2} \]
Fiber-Optic Waveguides

- Step index fiber: Standard for single mode (small core size – 8 micron)
- Graded index fiber: Designed so all multimodes travel at the same velocity.

Figure 2.1: Cross section and refractive-index profile for step-index and graded-index fibers.
Group-Velocity Dispersion

- The index of the mode is dependent on the wavelength (i.e. the fiber is dispersive).
- Two components: material dispersion and waveguide dispersion.
- These contribute to phase index.
- The group index is given by

\[ n_g = n + \omega \frac{\partial n}{\partial \omega} \]

\[ D = -\frac{2\pi c}{\lambda^2} \frac{d^2 \beta}{d\omega^2} = -\frac{2\pi c}{\lambda^2} \beta_2 \]

Units are \( \text{ps/(km·ns)} \)

Figure 2.8: Variation of refractive index \( n \) and group index \( n_g \) with wavelength for fused silica.
Material Dispersion

- Refractive index change of silica with optical frequency is modeled with the Sellmeier Equation:

\[ n^2(\omega) = 1 + \sum_{j=1}^{M} \frac{B_j \omega_j}{\omega^2 - \omega_j^2} \]

- \( B_j \) is the strength of medium resonance \( j \) of the material
- \( \omega_j \) is the frequency of medium resonance \( j \)
Material Dispersion

- Material dispersion $D_M$ is the slope of the $n_g$ vs. $\lambda$ (times 1/c)
- Therefore, looking at the figure we see that the slope hits zero at some wavelength – zero-dispersion wavelength

\[ \lambda_{zD} \sim 1.27-1.29 \, \mu m \]

**Figure 2.8:** Variation of refractive index $n$ and group index $n_g$ with wavelength for fused silica.
Waveguide Dispersion

- Waveguide dispersion $D_W$ comes from the first and second derivatives of $(Vb)$ with respect to $V$.
- For the wavelength range considered, $D_W$ is always negative.
- Therefore, sum of waveguide and material dispersion shifts zero-dispersion wavelength to a slightly longer wavelength.
FIGURE 3-14
The group delay arising from waveguide dispersion as a function of the \( V \) number for a step-index optical fiber. The curve numbers \( jm \) designate the \( \text{LP}_{jm} \) modes. (Reproduced with permission from Glege.)
Dispersion

\[ \tau = D L \sigma \]

\[ \Delta T = D L \Delta \lambda \]

D is dispersion parameter
L is the propagation length
\( \sigma \) is the spectral width
Dispersion (sum of material and waveguide dispersion)

Figure 3-10 Group velocity dispersion of (a) dispersion-unshifted 1.3 μm fiber and (b) dispersion-flattened and dispersion-shifted fibers. (After Reference [1].)
Dispersion Shifted Fiber Designs

Step Index

Depressed Cladding

Dispersion Shifted Fiber Designs
Chirp

- Fourier transform of a pulse gives amplitudes and phase of the frequency components. Chirp is a property that describes how the phase changes with time.
- Instantaneous frequency is equal to the slope of the phase (divided by $2\pi$). Therefore chirp can also be described as the frequency modulation.
- If chirp parameter $C$ is zero, the pulse is transform-limited. If the product of the GVD parameter and chirp parameter ($\beta_2 C$) is positive, the pulse broadens faster. If negative, the pulse narrows to a minimum and then broadens.

\[
\delta \omega(t) = -\frac{\partial \phi}{\partial t} = \frac{C}{T_0^2} t
\]

\[
\frac{T_1}{T_0} = \left[ \left(1 + \frac{C \beta_2 z}{T_0^2} \right)^2 + \left( \frac{\beta_2 z}{T_0^2} \right)^2 \right]^{1/2}
\]
Dispersion Budget

- Pulse spreading < Pulse width/4

\[ \tau = DL\sigma < \frac{1}{4B} \]
\[ L < \frac{1}{4BD\sigma} \]

- For Fabry Perot laser, spectral width ~ 2 nm
- So, for conventional fiber at 1.5 micron, D=15 ps/nm/km

\[ L < \frac{1}{4 \cdot 2nm \cdot 15 ps / nm / kmB} = \frac{8.3GHz \cdot km}{B} \]

- Gee, at 10 Gbit/s, this is just 0.8 km.

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Dispersion Limits

• First solution: use a single frequency laser, then spectral width is 0.1 nm and the limit is 167 Gbps km.

• Second solution: Use dispersion shifted fiber, so $D \approx 1 \text{ ps/nm km}$. This was done in Japan. Then, the limit is 2500 Gbps km

$$L < \frac{1}{4 \cdot 0.1 \text{nm} \cdot 1 \text{ps/nm/km} B} = \frac{2.5 \text{THz} \cdot km}{B}$$

• Third solution: Use external modulators. Then

$$\sigma = 0.4B / 17 \text{GHz/nm}$$

$$L < \frac{17 \text{GHz/nm}}{4 \cdot 0.4 \cdot 1 \text{ps/nm/km} B^2}$$

• Note: Nonlinearity and four wave mixing are big problems in fibers with low dispersion

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Fiber-Optic Communication Systems

Table 2.1 Characteristics of several commercial fibers

<table>
<thead>
<tr>
<th>Fiber Type and Trade Name</th>
<th>$A_{\text{eff}}$ ($\mu$m$^2$)</th>
<th>$\lambda_{\text{ZD}}$ (nm)</th>
<th>$D$ (C band) [ps/(km nm)]</th>
<th>Slope $S$ [ps/(km-nm$^2$)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corning SMF-28</td>
<td>80</td>
<td>1302–1322</td>
<td>16 to 19</td>
<td>0.090</td>
</tr>
<tr>
<td>Lucent AllWave</td>
<td>80</td>
<td>1300–1322</td>
<td>17 to 20</td>
<td>0.088</td>
</tr>
<tr>
<td>Alcatel ColorLock</td>
<td>80</td>
<td>1300–1320</td>
<td>16 to 19</td>
<td>0.090</td>
</tr>
<tr>
<td>Corning Vascade</td>
<td>101</td>
<td>1300–1310</td>
<td>18 to 20</td>
<td>0.060</td>
</tr>
<tr>
<td>Lucent TrueWave-RS</td>
<td>50</td>
<td>1470–1490</td>
<td>2.6 to 6</td>
<td>0.050</td>
</tr>
<tr>
<td>Corning LEAF</td>
<td>72</td>
<td>1490–1500</td>
<td>2 to 6</td>
<td>0.060</td>
</tr>
<tr>
<td>Lucent TrueWave-XL</td>
<td>72</td>
<td>1570–1580</td>
<td>$-1.4$ to $-4.6$</td>
<td>0.112</td>
</tr>
<tr>
<td>Alcatel TeraLight</td>
<td>65</td>
<td>1440–1450</td>
<td>5.5 to 10</td>
<td>0.058</td>
</tr>
</tbody>
</table>
Dispersion Limits

- Multimode fiber:
  - Every mode has a different velocity: huge dispersion limit.
  - Solution: Graded index fiber: adjust the index profile so every mode travels at the same velocity. Big improvement-most multimode fiber today is graded index.

- Single mode fiber:
  - Only one mode so the dominant dispersion is due to material dispersion and waveguide dispersion
  - PMD Polarization mode dispersion: real fiber is not perfectly concentric and there is strain in the fiber and cabling so fiber does have birefringence i.e. PMD.
Polarization

- So called single mode fiber is not really single mode. There are two degenerate modes (for example, vertical and horizontal polarization).
- Fiber is in general birefringent due to core ellipticity or strain so the two polarizations travel at different velocities.
- The polarization evolves in time. The distance over which it repeats is the beat length.

Figure 2.6: State of polarization in a birefringent fiber over one beat length. Input beam is linearly polarized at 45° with respect to the slow and fast axes.
Dispersion Summary

• Single mode condition required for high performance
• Multimode fiber used for low cost
• Dispersion is designable.
• 1.3 micron: zero of dispersion
• 1.55 micron: minimum loss
• Zero dispersion is not good because of nonlinearities
Material

- Gain and absorption (1 week)
- Lasers (2 weeks)
- Photodetectors (2 weeks)
- Modulators (2 weeks)