Lecture 4
Material and Homework

Read:
Chapter 6 Yariv and Yeh
Appendices 1 and 2 in Coldren (on reserve)

Homework #2
1. Derive the expression for mode spacing in a Fabry-Perot laser.

2. Problem 6.1 Yariv

3. Problem 6.2 Yariv

4. Consider a 1.55 $\mu$m InGaAsP /InP bulk laser 400 $\mu$m in length with confinement factor $\Gamma = 0.05$, internal quantum efficiency of 80% and internal loss of 10 $\text{cm}^{-1}$. cleaved facets, 0.2 mm thick active region, 0.1 ps lifetime for the linewidth calculation.

- Plot the gain versus wavelength
- Plot the peak gain versus carrier density
- What is the mirror loss?
- What is the threshold modal gain?
- What is the differential quantum efficiency?
- What is the axial mode spacing?
A heterostructure is a p-n junction between materials with dissimilar bandgaps.

Used to confine: Carriers (efficiency) and Photons (waveguide)

\[ \Gamma_a = \%\ overlap\ between\ optical\ mode\ and\ active\ waveguide \]
Define the laser output power $P(t)$, the current $I(t)$, the active gain volume $V$, and the carrier and photon densities $N(t)$ and $S(t)$ respectively.

The dynamics of carrier and photon density in the semiconductor laser cavity is governed by couple rate equations:

\[
\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_n} - G(N) \cdot (1 - \varepsilon \cdot S) \cdot S
\]

\[
\frac{dS}{dt} = \Gamma_a \cdot G(N) \cdot (1 - \varepsilon \cdot S) \cdot S - \frac{S}{\tau_p} + \frac{\Gamma_a \beta_{sp} N}{\tau_n}
\]
Gain saturation (recovery) is governed by the rate that carriers can be replenished.

- Band to band is governed by \( \tau_r \).
- Carriers cool from levels within band to band-edge by giving up energy to phonons \( \tau_{DCH} \).
- Carriers scatter off one another, changing momentum and energy along the band-edge \( \tau_{SHB} \).

\[
\varepsilon = \frac{1}{\tau_r} + \frac{1}{\tau_{DCH}} + \frac{1}{\tau_{SHB}}
\]

\( \tau_r \approx 1 \text{ ns} \), \( \tau_{DCH} \approx 650 \text{ fs} \), \( \tau_{SHB} \approx 50 \text{ fs} \)
Steady State Solutions to Rate Equations

- Setting the rate equations equal to zero

\[
\frac{dN}{dt} = \frac{dS}{dt} = 0
\]

- and the cavity photon density \( S = 0 \), we get the below (at) threshold condition

\[
I_{th} = \frac{N_{th}}{qV} \tau_n
\]

\[
I_{th} = \frac{qVN_{th}}{\tau_n} = qV R(N_{th})
\]

- using the linear gain relation, and at threshold gain = total losses

\[
N = N_{tr} + \frac{g_p(N)}{a}
\]

\[
N_{th} = N_{tr} + \frac{g_p(N_{th})}{a} = N_{tr} + \frac{\alpha_{total}}{a}
\]

- At and above threshold, with non-zero \( S \) and assuming linear operation \((\epsilon = 0)\)

\[
S = \frac{I}{qVG(N_{th})} - \frac{R(N_{th})}{G(N_{th})} = \frac{I - qVR(N_{th})}{qV \alpha_{total}} = \frac{I - I_{th}}{qV \alpha_{total}}
\]
Fabry-Perot Cavity

- The equivalent of an electronic comb filter, but for optical frequencies, the Fabry-Perot (FP) cavity is used for feedback in lasers and as optical filters.

The propagation constant for a plane wave propagating in the cavity is

\[ k'(\omega) = k_0 n_{\text{eff}} = k_0 \sqrt{n^2 + \chi' - i \chi''} - i \frac{\alpha}{2} \approx k + k \frac{\chi'(\omega)}{2 n^2} - i k \frac{\chi''(\omega)}{2 n^2} - i \frac{\alpha}{2} \]

The output wave \( P_{\text{out}} \) can be written as

- \( E_{\text{out}} = E_1 + E_2 + E_3 + \cdots \)
- \( E_1 = t_z e^{-i k(l-z)} E_i \)
- \( E_2 = (r_1 r_2 e^{-i 2k l}) t_2 e^{-i k(l-z)} E_i \)
- \( E_3 = (r_1 r_2 e^{-i 2k l})^2 t_2 e^{-i k(l-z)} E_i \)
- \( E_4 = (r_1 r_2 e^{-i 2k l})^3 t_2 e^{-i k(l-z)} E_i \)
Fabry Perot Cavities

⇒ Summing up the fields

\[ E_{out} = t_2 e^{-i k'(l-a)} E_i \left\{ 1 + (r_1 r_2 e^{-i 2k'l}) + (r_1 r_2 e^{-i 2k'l})^2 + (r_1 r_2 e^{-i 2k'l})^3 + \cdots \right\} \]

⇒ Rewriting \( k' \)

\[ k' = k + \Delta k + i (\gamma - \alpha) \]

⇒ Model the gain as (we will look at this in more detail next class)

\[ \gamma = -k \frac{\chi''(\omega)}{n^2} = (N_2 - N_1) \frac{\lambda^2}{8 \pi n^2 t_{spont}} g(\nu) \]
Fabry Perot Cavities

_locating the optical source at the input (z=0)_

\[ E_{\text{out}} = \frac{t_1 t_2 e^{-i(k+\Delta k)l} e^{(\gamma-\alpha)l/2}}{1 - r_1 r_2 e^{-i2(k+\Delta k)l} e^{(\gamma-\alpha)l}} (E_i)_{\text{outside}} \]

_locating the optical source at the input (z=0)_

\[ r_1 r_2 e^{-i2(k+\Delta k)l} e^{(\gamma-\alpha)l} = 1 \]

\[ r_1 r_2 e^{(\gamma(\omega)-\alpha)l} = 1 \]

\[ 2(k + \Delta k(\omega))l = 2m\pi, \ m=1,2,3... \]

_locating the optical source at the input (z=0)_

The amplitude condition can be written as

\[ \gamma_i(\omega) = \alpha - \frac{1}{l} \ln r_1 r_2 \]
Fabry-Perot Cavities

For end mirror loss \( \alpha_m = \frac{1}{2L} \ln \frac{1}{R_1R_2} \)

The oscillation condition becomes \( 2j\beta L + \alpha_m L = 2j\pi N \), where N is an integer denoting longitudinal mode number.

Defining cavity roundtrip gain \( g_c = g_{net} - \alpha_m \)

The real part of oscillation condition gives \( g_c = 0 \)

And \( \Gamma g_m - \alpha_i - \alpha_m = 0 \)

Defining the total cavity loss as \( \alpha_{total} = \alpha_i + \alpha_m \)

Then \( g_c = g_{eff} - \alpha_{total} = 0 \)

And the resonance condition for the cavity is

\[
\lambda_n = \frac{2n_{eff}' \left( \lambda_n' \right) L}{N}
\]

\[
\Delta \lambda_m = \lambda_n - \lambda_{n+1} \quad \frac{\lambda_N^2}{2n_{g,eff} L}
\]

\( n_{g,eff} = \) effective group index
Multimode Fabry-Perot SC Lasers

\[ n \approx 3.5 \]

\[ I_{\text{bias}} \]

FP cavity modes  SC optical gain

Relative Output Power

ECE162C, Spring 2010, Professor Blumenthal
FP Laser Output Characteristics

- 1.3 µm multimode lasers are good for bit rates < 2Gbs and distances up to 100 km.