

# Blind GPS Receiver with a Modified Despreader for Interference Suppression

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**The Global Positioning System (GPS) was designed to provide location estimates for various civilian and military applications using at least four satellites. Since GPS signals have a low signal-to-noise ratio (SNR), they also have a low signal-to-jammer ratio so that the accuracy of location estimates is influenced by cochannel interference and intentional jammers. We propose a low-complexity blind adaptive receiver that is based on a novel modified despreader and the constant modulus (CM) array. This system is capable of nulling directional interference and capturing the GPS signal of interest without requiring explicit angle-of-arrival (AOA) information. We also consider the multiple satellite problem and extend the proposed receiver to capture several GPS signals of interest. Representative computer simulation examples are presented to illustrate the performance of the multicomponent system for the suppression of different jammer types.**

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## I. INTRODUCTION

Twenty-eight GPS satellites operate in six circular orbits for use in location estimation, each of which transmits two types of signals: the  $L_1$  signal modulated by the coarse acquisition (C/A) code and the long precision (P) code, and the  $L_2$  signal modulated only by the P code [1]. The large spreading gain is adequate to compensate for the low received SNR, but high-power jamming signals and cochannel interference are increasingly becoming a potential problem. The jamming signals in the GPS environment can take on a variety of different types, from continuous waveform (CW) and frequency modulated (FM) signals to wideband (WB) noise. Potential interference sources can arise from transmissions at frequencies close to the GPS frequencies, and from signals with a harmonic in the GPS band [2].

The effects of different types of interference, such as coherent CW and broadband signals, as well as pulsed and continuous signals have been studied in [3]. That paper also considers various test statistics such as the correlator output power, the correlator output variance, carrier phase vacillation, and an active gain controller (AGC). The jamming threshold and jamming effectiveness based on different jamming strategies were analyzed and tested in [4]. For narrowband jamming signal rejection in a GPS receiver, a precorrelation temporal filter such as a notch filter was considered in [5] and [6]. In the case of an FM jammer, which is basically a narrowband signal in the GPS environment, a suppression technique based on a subspace projection was recently studied in [7] and [8].

Adaptive space-time processors for WB jammers were examined in [9] and [10]. To generate an approximation of the optimum beamformer in a GPS receiver, an interference-blocking maximum SNR (IB-MSNR) beamformer using a singular value decomposition (SVD) and a Newton-Raphson iterative search algorithm were proposed in [11] and [12], respectively. Space-time GPS signal processing techniques with design criteria for a conventional matched filter (MF), a minimum-variance-distortionless-response (MVDR) beamformer, and an auxiliary-vector (AV) approach were investigated in [13].

For directional WB and narrowband interference suppression, the MVDR beamformer was considered in [14] and [15]. Although these systems provide excellent performance for jammer rejection, the algorithms require knowledge of the GPS signal angles of arrival (AOAs) and a model for the antenna array, and they have a high computational complexity because a correlation matrix is computed from the received signal. In order to overcome their sensitivity to estimation errors in the GPS signal AOA, we

proposed a blind multicomponent system based on the multistage MF and constant modulus (CM) array in [16]. While this system does not require explicit AOA estimation for the GPS signal, it can have a high computational complexity for estimating the AOAs of high-power jamming signals.

In order to reduce this complexity and without the need for AOA estimation for the GPS signal, we recently proposed in [17] a blind interference canceler based on a multistage version of the blind CM array [18]. The multistage CM array operates blindly using stochastic gradient techniques such as the CM algorithm (CMA) [19] and the least-mean-square (LMS) algorithm [20]. However, since this previous receiver must use a multistage version of the CM array for multiple jamming signals, it is not as computationally efficient as a one-stage CM array implementation.

We propose a more efficient and simpler interference rejection system consisting of a novel modified despreader, followed by a one-stage CM array and the GPS decision device, which is a signum-function detector. When the received signal is despread, the modified  $C/A$  code transforms the CM jammers into non-CM signals, while retaining the CM characteristic of the GPS signal [21]. Therefore, only one CM array stage is needed to null the jamming signals and capture the GPS signal of interest after despreading.

For the best location estimation accuracy, GPS requires at least four satellites with three satellites equally spaced on the horizon at low elevation angles, and with the fourth satellite directly overhead [22]. In the work presented here, we also explore an extension of the modified despreader system to handle multiple GPS signals [23]. The multistage CM array is used to null the jamming signals and capture the GPS signals after despreading. The MVDR beamformer for the multiple satellite problem was previously considered in [14]. However, that approach has a dramatic increase in the computational complexity compared with the single satellite case, and requires multiple despanders for the satellite signals. The proposed system here requires only one despreader and the multistage CM array, so it is considerably more efficient than previously proposed systems for capturing multiple GPS signals.

The rest of this paper is organized as follows. In Section II, we define the received signal model for GPS and the interference signals in additive white Gaussian noise (AWGN). In Section III, the multicomponent interference rejection system based on the modified despreader and one-stage CM array is described. The extended system for detecting multiple GPS signals is considered in Section IV. Computer simulations are provided in Section V to demonstrate the performance of the proposed system for the

suppression of different jammer types. Finally, the conclusions of this work are outlined in Section VI.

## II. SIGNAL MODEL

GPS employs the  $C/A$  code to modulate the  $L_1$  signal or the long precision ( $P$ ) code to modulate the  $L_2$  signal [22]. In this work, we only consider detection of the  $C/A$  code, which employs binary phase-shift keying (BPSK) and thus has the CM characteristic. Assuming that  $M$  elements are employed in the receiver and the  $i$ th satellite signal is of interest, the received signal vector at sample instant  $k$  can be modeled as

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{a}_c c_i(k) b(k) + \mathbf{A} \mathbf{j}(k) + \mathbf{v}(k) \\ &= \mathbf{a}_c c_i(k) b(k) + \mathbf{A}_{\text{CM}} \mathbf{j}_{\text{CM}}(k) + \mathbf{A}_{\text{WB}} \mathbf{j}_{\text{WB}}(k) + \mathbf{v}(k) \end{aligned} \quad (1)$$

where  $\mathbf{a}_c$  is the array response vector (size  $M$ ) for the GPS signal,  $c_i(k)$  is an element of the cyclostationary pseudorandom noise (PRN) code with length  $N = 20 \times 1023$  for the  $i$ th satellite, and  $b(k)$  is the GPS data bit which remains constant over the length of one cycle of the PRN code.  $\mathbf{A}$  is the  $M \times L$  array response matrix,  $L$  is the number of interferers,  $\mathbf{j}(k)$  is the vector (size  $L$ ) of jammers, and  $\mathbf{v}(k)$  is an AWGN vector (size  $M$ ) with independent and identically distributed components, each with zero mean and variance  $\sigma^2$ .  $\mathbf{A}_{\text{CM}}$  and  $\mathbf{A}_{\text{WB}}$  are the  $M \times L_{\text{CM}}$  and  $M \times L_{\text{WB}}$  array response matrices for the CM jammers and the WB noise jammers, respectively,  $L_{\text{CM}}$  and  $L_{\text{WB}}$  are the numbers of CM and WB noise jammers, respectively, and the vectors  $\mathbf{j}_{\text{CM}}$  (size  $L_{\text{CM}}$ ) and  $\mathbf{j}_{\text{WB}}$  (size  $L_{\text{WB}}$ ) contain the CM and WB jammers, respectively. In this paper, we assume that all non-CM jamming signals are WB noise jammers.

For a receiver with a grid antenna array of size  $P \times Q$  ( $M = PQ$ ), the  $l$ th column of the array response matrix is given by [24]

$$\mathbf{a}_l \triangleq \begin{bmatrix} 1 \\ e^{-j\zeta_l} \\ \vdots \\ e^{-j(Q-1)\zeta_l} \\ e^{-j\eta_l} \\ e^{-j(\zeta_l + \eta_l)} \\ \vdots \\ e^{-j((Q-1)\zeta_l + (P-1)\eta_l)} \end{bmatrix} \quad (2)$$

where  $\zeta_l \triangleq (d/\lambda)2\pi \sin \theta_l \cos \phi_l$ ,  $\eta_l \triangleq (d/\lambda)2\pi \sin \theta_l \sin \phi_l$ ,  $d$  is the interelement spacing,  $\lambda = d/2$  is the

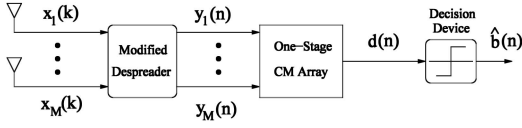


Fig. 1. System architecture of GPS receiver for single satellite.

wavelength of the signals,  $\theta_l$  is the elevation angle, and  $\phi_l$  is the azimuth angle, all for the  $l$ th signal. The wavelength of the signals is twice the interelement spacing, which is a common assumption for an adaptive array. The components of the vector  $\mathbf{j}_{\text{CM}}(k) \triangleq [j_1(k), \dots, j_{L_{\text{CM}}}(k)]^T$  comprise a set of  $L_{\text{CM}}$  CM jammers, and each signal can be modeled as one of two types: a CW signal  $j_{l_{\text{CM}}}(k) = \exp j(2\pi f_{l_{\text{CM}}} k + \psi_{l_{\text{CM}}})$  where  $f_{l_{\text{CM}}}$  and  $\psi_{l_{\text{CM}}}$  are the frequency and phase of the  $l_{\text{CM}}$ th signal, respectively, or an FM signal  $j_{l_{\text{CM}}}(k) = \exp j(2\pi f_{l_{\text{CM}}} k + \beta \sin(2\pi f_m k))$  where  $\beta$  and  $f_m$  are the modulation index and frequency, respectively. Note that for convenience here we utilize a cosine as the message signal for the FM jammers. A chirp or frequency-hopping jammer, where the center frequency of a CW signal varies over time, can be classified as a CM jammer for our purposes because its magnitude is constant. The components of the vector  $\mathbf{j}_{\text{WB}}(k) \triangleq [j_1(k), \dots, j_{L_{\text{WB}}}(k)]^T$  comprise a set of  $L_{\text{WB}}$  WB jammers, and each signal can be modeled as additive Gaussian noise (AGN) with a bandwidth not exceeding that of the GPS signal. Finally, we consider pulsed jammers, which are periodic on/off signals and classified as WB jammers [16].

### III. INTERFERENCE REJECTION SYSTEM

The proposed system consists of the modified despreader, a one-stage CM array, and the GPS decision device, with the architecture shown in Fig. 1. The modified despreader is designed to transform all jammers into non-CM signals, while the GPS signal of interest retains the CM property. The output of the modified despreader is processed by a one-stage CM array that nulls the transformed jammers and extracts the GPS signal.

#### A. Modified Despreader

Each satellite has a unique PRN code consisting of twenty identical  $C/A$  codes. For the  $i$ th satellite, the PRN code can be written as [25]

$$\mathbf{c}_i = [\mathbf{ca}_i, \dots, \mathbf{ca}_i]^T \quad (3)$$

where  $\mathbf{ca}_i$  is the  $C/A$  code (length-1023 row vector). Since  $\mathbf{c}_i^T \mathbf{c}_i = N$ , the output of the despreader based on  $\mathbf{c}_i$  can be expressed as

$$\mathbf{x}(n) = \mathbf{a}_c N b(n) + \mathbf{A}_{\text{CM}} \mathbf{j}_{\text{CM}}(n) + \mathbf{A}_{\text{WB}} \mathbf{j}_{\text{WB}}(n) + \mathbf{v}(n) \quad (4)$$

where  $\mathbf{j}_{\text{CM}}(n) \triangleq \mathbf{J}_{\text{CM}}(n) \mathbf{c}_i$ ,  $\mathbf{J}_{\text{CM}}(n) \triangleq [\mathbf{j}_{\text{CM}}(k), \dots, \mathbf{j}_{\text{CM}}(k + N - 1)]$ ,  $\mathbf{j}_{\text{WB}}(n) \triangleq \mathbf{J}_{\text{WB}}(n) \mathbf{c}_i$ ,  $\mathbf{J}_{\text{WB}}(n) \triangleq [\mathbf{j}_{\text{WB}}(k), \dots, \mathbf{j}_{\text{WB}}(k + N - 1)]$ ,  $\mathbf{v}(n) \triangleq \mathbf{V}(n) \mathbf{c}_i$ , and  $\mathbf{V}(n) \triangleq [\mathbf{v}(k), \dots, \mathbf{v}(k + N - 1)]$ . Note that  $Nb(n)$  and  $\mathbf{j}_{\text{CM}}(n)$  have the CM characteristic.

Next, we define the modified PRN code

$$\tilde{\mathbf{c}}_i \triangleq [\mathbf{ca}_{i,1}, \dots, \mathbf{ca}_{i,20}]^T \quad (5)$$

where  $\mathbf{ca}_{i,l}$  ( $l = 1, \dots, 20$ ) is randomly chosen from the  $C/A$  codes, excluding  $\mathbf{ca}_i$ . Since  $\mathbf{c}_i^T \tilde{\mathbf{c}}_i = -20$ , the output of a despreader using  $\tilde{\mathbf{c}}_i$  can be written as

$$\tilde{\mathbf{x}}(n) = -20 \mathbf{a}_c b(n) + \mathbf{A}_{\text{CM}} \tilde{\mathbf{j}}_{\text{CM}}(n) + \mathbf{A}_{\text{WB}} \tilde{\mathbf{j}}_{\text{WB}}(n) + \tilde{\mathbf{v}}(n) \quad (6)$$

where  $\tilde{\mathbf{j}}_{\text{CM}}(n) \triangleq \mathbf{J}_{\text{CM}}(n) \tilde{\mathbf{c}}_i$ ,  $\tilde{\mathbf{j}}_{\text{WB}}(n) \triangleq \mathbf{J}_{\text{WB}}(n) \tilde{\mathbf{c}}_i$ , and  $\tilde{\mathbf{v}}(n) \triangleq \mathbf{V}(n) \tilde{\mathbf{c}}_i$ . Note that only the term  $-20b(n)$  in (6) has the CM property.

Defining  $\mathbf{y}(n) \triangleq \mathbf{x}(n) - \tilde{\mathbf{x}}(n)$  and  $\bar{\mathbf{c}}_i \triangleq \mathbf{c}_i - \tilde{\mathbf{c}}_i$ , we can write

$$\mathbf{y}(n) = \mathbf{a}_c (N + 20) b(n) + \mathbf{A}_{\text{CM}} \bar{\mathbf{j}}_{\text{CM}}(n) + \mathbf{A}_{\text{WB}} \bar{\mathbf{j}}_{\text{WB}}(n) + \bar{\mathbf{v}}(n) \quad (7)$$

where  $\bar{\mathbf{j}}_{\text{CM}}(n) \triangleq \mathbf{J}_{\text{CM}}(n) \bar{\mathbf{c}}_i$ ,  $\bar{\mathbf{j}}_{\text{WB}}(n) \triangleq \mathbf{J}_{\text{WB}}(n) \bar{\mathbf{c}}_i$ , and  $\bar{\mathbf{v}}(n) \triangleq \mathbf{V}(n) \bar{\mathbf{c}}_i$ . Since only  $(N + 20)b(n)$  in (7) has the CM characteristic, the GPS signal can be captured by a one-stage CM array. The third term in (7) obviously contains non-CM signals, and the second term is investigated below. Note that even if we utilize an arbitrary number of identical  $C/A$  codes instead of twenty, the modified despreader transforms the CM jammers into non-CM signals because the modified PRN code varies with the time index  $n$ . The CM characteristic of the GPS signal is retained for the other modified despreaders.

#### B. Operation of Modified Despreader

Next, we explore how the modified despreader transforms the CM jammers into non-CM signals while retaining the CM property of the GPS signal of interest. Consider the  $l$ th CM jamming signal of the second term in (7):

$$\mathbf{a}_{\text{CM},l} \bar{\mathbf{j}}_{\text{CM},l}(n) = \mathbf{a}_{\text{CM},l} \mathbf{j}_{\text{CM},l}^T(n) \bar{\mathbf{c}}_i \quad (8)$$

where  $\mathbf{a}_{\text{CM},l}$  is the array response vector for the  $l$ th CM jamming signal,  $\mathbf{j}_{\text{CM},l}(n) \triangleq [j_{\text{CM},l}(k), \dots, j_{\text{CM},l}(k + N - 1)]^T$ , and  $j_{\text{CM},l}(k)$  is the  $l$ th CM jamming signal. The magnitude squared of (8) (ignoring

the array response vector) is given by

$$\begin{aligned}
\bar{\mathbf{j}}_{CM,l}^*(n)\bar{\mathbf{j}}_{CM,l}(n) &= \bar{\mathbf{c}}_i^T \mathbf{j}_{CM,l}^*(n) \mathbf{j}_{CM,l}^T(n) \bar{\mathbf{c}}_i \\
&= \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} \bar{\mathbf{j}}_{CM,l}^*(k) \bar{\mathbf{j}}_{CM,l}(k') \bar{c}_i(k) \bar{c}_i(k') \\
&= \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} \bar{\mathbf{j}}_{CM,l}^*(k) \bar{\mathbf{j}}_{CM,l}(k') c_i(k) c_i(k') \\
&\quad + \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} \bar{\mathbf{j}}_{CM,l}^*(k) \bar{\mathbf{j}}_{CM,l}(k') \check{c}_i(k, k')
\end{aligned} \tag{9}$$

where  $\check{c}_i(k, k') \triangleq \bar{c}_i(k) \bar{c}_i(k') - c_i(k) c_i(k') - \bar{c}_i(k) c_i(k')$ , and the superscript \* denotes complex conjugation. We assume the worst case scenario where the sample period  $N$  of  $\bar{\mathbf{j}}_{CM,l}(k)$  is the same as that of the PRN code of the GPS signals. Since  $\bar{\mathbf{c}}_i$  is fixed for all  $n$  and the component  $c_i(k)$  is a periodic sequence with period 1023, the first summation is constant for all  $n$ . However, the second summation is not constant because  $\bar{\mathbf{c}}_i$  is randomly chosen from the  $C/A$  codes and thus is different for each  $n$  so that its component  $\bar{c}_i(k)$  is an aperiodic sequence. Therefore, (9) is not constant and the CM jamming signals after the modified despreader no longer have the CM property.

### C. One-Stage CM Array

A one-stage CM array is used to extract the GPS signal of interest while nulling all the (modified) interference signals. The CM array output can be written as

$$d(n) = \mathbf{w}_c^H(n) \mathbf{y}(n) \tag{10}$$

with weight vector  $\mathbf{w}_c(n)$  updated by CMA as follows [19]:

$$\mathbf{w}_c(n+1) = \mathbf{w}_c(n) + 2\mu \mathbf{y}(n) \varepsilon^*(n) \tag{11}$$

where

$$\varepsilon(n) \triangleq d(n)/|d(n)| - d(n) \tag{12}$$

and  $\mu > 0$  is a step-size parameter that controls the convergence rate.

### D. CM Array Wiener Weight Vector

Next, we derive the Wiener weight vector of this system in order to investigate its steady-state (optimal) performance. The output of the CM array after the modified despreader is given by

$$d_o(n) = \mathbf{w}_o^H \mathbf{y}(n) \tag{13}$$

where  $\mathbf{w}_o$  is the Wiener weight vector for extracting the GPS signal of interest. The corresponding

estimation error can be written as

$$\begin{aligned}
\varepsilon_o(n) &= b(n) \bar{\mathbf{c}}_i^T \bar{\mathbf{c}}_i - d_o(n) \\
&= b(n)(N+20) - d_o(n).
\end{aligned} \tag{14}$$

Applying the orthogonality principle [20]  $E[\mathbf{y}(n) \varepsilon_o^*(n)] = E[\mathbf{y}(n)(b(n)(N+20) - d_o(n))^*] = \mathbf{0}$ , we obtain

$$E[\mathbf{y}(n) b(n)(N+20)] = E[\mathbf{y}(n) \mathbf{y}^H(n)] \mathbf{w}_o \tag{15}$$

which yields the Wiener weight vector

$$\mathbf{w}_o = (N+20)^2 \mathbf{R}_y^{-1} \mathbf{a}_c \tag{16}$$

where  $\mathbf{R}_y \triangleq E[\mathbf{y}(n) \mathbf{y}^H(n)]$  is the autocorrelation matrix of the modified despreader output. In order to obtain (16), we have used the fact that the  $\{b(n)\}$  have zero mean and are independent of the jammers and the noise. Note that when the Wiener weight vector is used in (10), the output  $d(n)$  contains the GPS signal of interest, AGN, and residual interference signals.

### E. Decision Device

After the one-stage CM array, a signum-function detector is used to estimate the data because the transmitted GPS signal is real and binary. Thus, the final output of the multicomponent receiver is given by

$$\hat{b}(n) = \text{sgn}[\text{real}[d(n)]]. \tag{17}$$

### F. Computational Complexity

The overall required number of multiplications/divisions and additions/subtractions for estimating a GPS data bit using the modified despreading system is approximately  $NM + 2M + 2 \approx NM$  and  $(N-1)M + 2M \approx NM$ , respectively. This computational complexity is much lower than that of previously proposed systems as described in [16]. (Note that we ignore the computational complexity of the decision device.)

## IV. INTERFERENCE REJECTION SYSTEM FOR MULTIPLE GPS SIGNALS

In this section, we consider using the modified despreading approach to extract multiple GPS signals. Fig. 2 shows the system architecture consisting of the modified despreader for multiple GPS signals, a multistage CM array, and a binary decision device. Using the modified despreader, all CM interference signals are transformed into non-CM signals, while the GPS signals retain the CM characteristic. The output of the modified despreader is processed by the multistage CM array to null all

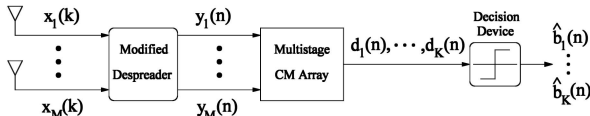


Fig. 2. System architecture of modified despreader and multistage CM array for multiple GPS signals.

jamming signals and sequentially extract the GPS signals of interest.

#### A. Signal Model for Multiple GPS Signals

For multiple GPS signals, (1) can be rewritten as

$$\mathbf{x}(k) = \mathbf{A}_c \begin{bmatrix} c_1(k)b_1(k) \\ \vdots \\ c_K(k)b_K(k) \end{bmatrix} + \mathbf{A}_{\text{CM}}\mathbf{j}_{\text{CM}}(k) + \mathbf{A}_{\text{WB}}\mathbf{j}_{\text{WB}}(k) + \mathbf{v}(k) \quad (18)$$

where each column of  $\mathbf{A}_c$  (size  $M \times K$  where  $K$  is the number of satellites) is the AOA array response vector for a GPS satellite, and  $c_i(k)$  and  $b_i(k)$  ( $i = 1, \dots, K$ ) are the elements of the cyclostationary PRN code and GPS data bits, respectively, for a satellite.

#### B. Modified Despreader for Multiple GPS Signals

Assuming that the satellites labeled  $i = 1, \dots, K$  are transmitting the GPS signals of interest, the PRN code for the  $i$ th satellite is given by

$$\mathbf{c}_i = [\mathbf{c}_{a_i}, \dots, \mathbf{c}_{a_i}]^T, \quad i = 1, \dots, K \quad (19)$$

where  $\mathbf{c}_{a_i}$  is the  $C/A$  code (length-1023 row vector) for the  $i$ th satellite, which is repeated 20 times. In this system, a new PRN code  $\tilde{\mathbf{c}}_i$  is defined as

$$\tilde{\mathbf{c}}_i = [\mathbf{c}_{a_{i,1}}, \dots, \mathbf{c}_{a_{i,20}}]^T \quad (20)$$

where  $\mathbf{c}_{a_{i,l}}$  ( $l = 1, \dots, 20$ ) is randomly chosen from all possible  $C/A$  codes, excluding  $\mathbf{c}_{a_i}$  ( $i = 1, \dots, K$ ). For multiple satellites, we define the modified PRN code  $\tilde{\mathbf{c}}$

$$\tilde{\mathbf{c}} \triangleq \sum_{i=1}^K \mathbf{c}_i - \tilde{\mathbf{c}}_i \quad (21)$$

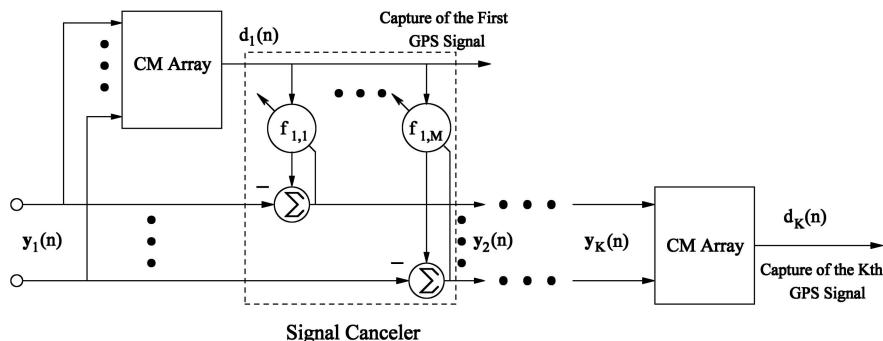


Fig. 3. Multistage CM array architecture for capturing multiple GPS signals.

and obtain the output of the modified despreader as

$$\mathbf{y}(n) = \mathbf{A}_c(N - 20(K - 1)) \begin{bmatrix} b_1(k) \\ \vdots \\ b_K(k) \end{bmatrix} + \mathbf{A}_{\text{CM}}\bar{\mathbf{j}}_{\text{CM}}(n) + \mathbf{A}_{\text{WB}}\bar{\mathbf{j}}_{\text{WB}} + \bar{\mathbf{v}}(n). \quad (22)$$

Since only the term  $(N - 20(K - 1))[b_1(k), \dots, b_K(k)]^T$  has the CM characteristic in (22), multiple GPS signals can be recovered using the multistage CM array [26].

#### C. Multistage CM Array

The multistage CM array shown in Fig. 3 operates such that the output of each stage contains a GPS signal from a different satellite. A captured GPS signal is removed by the signal canceler whose output becomes the input of the next stage. This process continues in a sequential manner until all GPS signals are captured.

The output of the  $i$ th CM array stage is

$$d_i(n) = \mathbf{w}_i^H(n)\mathbf{y}_i(n) \quad (23)$$

where  $\mathbf{y}_i(n)$  is the input signal vector (size  $M$ ) (note that  $\mathbf{y}_1(n) \equiv \mathbf{y}(n)$ ), and  $\mathbf{w}_i(n)$  is the adaptive weight vector updated by CMA as follows:

$$\mathbf{w}_i(n + 1) = \mathbf{w}_i(n) + 2\mu\mathbf{y}_i(n)\varepsilon_i^*(n) \quad (24)$$

with error signal

$$\varepsilon_i(n) = d_i(n)/|d_i(n)| - d_i(n). \quad (25)$$

The output of the CM array, which contains one GPS signal, is applied to an adaptive signal canceler to generate the following output vector:

$$\mathbf{y}_{i+1}(n) = \mathbf{y}_i(n) - \mathbf{f}_i(n)d_i(n) \quad (26)$$

where  $\mathbf{f}_i(n)$  is the canceler weight vector (size  $M$ ) updated by the LMS algorithm:

$$\mathbf{f}_i(n + 1) = \mathbf{f}_i(n) + 2\mu_{\text{LMS}}\mathbf{y}_i(n)d_i^*(n) \quad (27)$$

with step-size parameter  $\mu_{\text{LMS}} > 0$ .

#### D. CM Array and Signal Canceler Wiener Weight Vectors

Using the orthogonality principle (as was done in Section III D) for the  $i$ th stage of the multistage CM array, the beamformer Wiener weight vector is

$$\mathbf{w}_{i,o} = (N - 20(K - 1))^2 \mathbf{R}_i^+ \tilde{\mathbf{a}}_i \quad (28)$$

where  $\tilde{\mathbf{a}}_i = \mathbf{T}_{i-1} \times \cdots \times \mathbf{T}_1 \mathbf{a}_i$ ,  $\mathbf{a}_i$  is the AOA vector of the GPS signal captured in the  $i$ th stage,  $\mathbf{R}_i = \mathbf{T}_{i-1} \times \cdots \times \mathbf{T}_1 \mathbf{R}_y \mathbf{T}_1^H \times \cdots \times \mathbf{T}_{i-1}^H$ , and

$$\mathbf{T}_i \triangleq \mathbf{I}_M - \mathbf{f}_{i,o} \mathbf{w}_{i,o}^H \quad (29)$$

is a signal transfer matrix where  $\mathbf{I}_M$  is the size- $M$  identity matrix. Since the correlation matrix  $\mathbf{R}_i$  is singular, we utilize the pseudoinverse  $\mathbf{R}_i^+$  [27] in (28). The corresponding canceler Wiener weight vector is

$$\mathbf{f}_{i,o} = [(N - 20(K - 1))^2 / \sigma_{s_i}^2] \tilde{\mathbf{a}}_i \quad (30)$$

where  $\sigma_{s_i}^2 \triangleq \mathbf{w}_{i,o}^H \mathbf{R}_i \mathbf{w}_{i,o}$  is the signal variance (power) at the output of the  $i$ th stage.

#### E. Decision Device

For the  $i$ th satellite, the estimated GPS data bits from the decision device can be written as

$$\hat{b}_i(n) = \text{sign}[\text{real}[d_i(n)]], \quad i = 1, \dots, K. \quad (31)$$

#### F. Computational Complexity

Since this system requires only one despreader and the multistage CM array, it is more efficient than previously proposed systems for capturing multiple GPS signals. The number of multiplications/divisions and additions/subtractions for estimating multiple GPS data bits at each sample instant  $n$  using the modified despreading system is approximately  $NM + 4MK - 2M + 3K - 1 \approx NM + (4M + 3)K$  and  $(N - 1)M + 4MK - 2M + K - 1 \approx NM + 4MK$ , respectively. For multiple GPS signals, the computational complexity of previously proposed systems can be expressed as  $K \times N_o$  where  $N_o$  is the number of operations described in [16] for a system with a single GPS signal. For example, for the generalized sidelobe canceler (GSC) considered in [16], which has the lowest computational complexity of those algorithms, there are  $NM(M + 1) + K_{\text{GSC}}M$  multiplication/divisions and  $NM^2 + K_{\text{GSC}}(M - 1)$  additions/subtractions, where we assume that the adaptive weights converge by the  $K_{\text{GSC}}$ th sample.

### V. COMPUTER SIMULATIONS

In this section, we present computer simulation examples that demonstrate the performance of the

TABLE I  
First Scenario for Single GPS Signal

Signal	Azimuth (°)	Elevation (°)	Center Frequency
GPS	67	-28	—
CW	52, -41	71, 71	0.05, 0.1
FM	-72, 38	71, 71	0.17, 0.22
WB	10, -12	71, 71	0.33, 0.43

TABLE II  
Second Scenario for Single GPS Signal

Signal	Azimuth (°)	Elevation (°)	Center Frequency
GPS	-58	27	—
CW	-27	-62	0.35
FM	41, 55	-62, -62	0.20, 0.45
WB	-77	-62	0.1
Pulsed	75, -14	-62, -62	—

modified despreading system for a single GPS signal and then multiple GPS signals using two scenarios with  $M = 8$  antenna array elements.

#### A. Computer Simulations for Single GPS Signal

For a single GPS signal, we consider two scenarios as described in Tables I and II. In the first scenario, the received signal consists of one GPS signal of interest, two CW jammers and two FM jammers as the CM interference signals, two WB noise jammers, and AWGN. In the second scenario, the received signal consists of one GPS signal of interest, one CW jammer and two FM jammers as the CM jammers, one WB noise jammer, two pulsed jammers, and AWGN. The two pulsed jamming signals are periodic on/off signals with periods of 100 and 1,000 samples. The SNR of the GPS signal is -30 dB, and the jammer-to-signal ratio (JSR) for each jammer is 60 dB. In both scenarios, the FM jammers have modulation index  $\beta = 0.05$  and normalized modulation frequency  $f_m = 0.001$ . For these simulations, we arbitrarily chose the center frequencies of the jamming signals.

Note that we cannot directly compare the spectrum of the one-stage CM array output  $d(n)$  after despreading to that of the received signal  $\mathbf{x}(k)$  because the time indexes are different. For simulation purposes only, we reconfigured the multicomponent architecture similar to that in [16] where the modified despreader follows the CM array with weights copied from the original adaptive configuration.

Figs. 4 and 5 show the performance of the modified despreader system by comparing the spectra of the received signal and the output of the receiver for the first and second scenario, respectively. From these results, we observe that all jammers are almost perfectly suppressed by the proposed system for a single GPS signal. The beampatterns of the

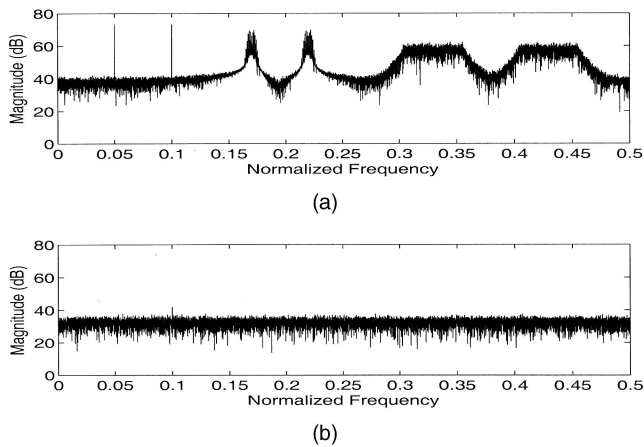


Fig. 4. Signal spectra for first scenario for single GPS signal. (a) Received signal. (b) Output of modified despreader.

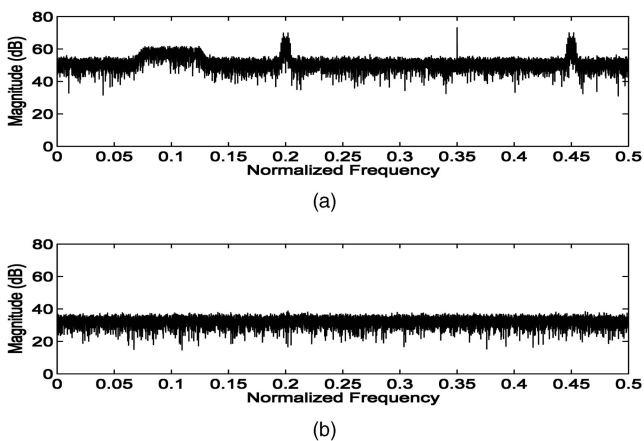


Fig. 5. Signal spectra for second scenario for single GPS signal. (a) Received signal. (b) Output of modified despreader.

MVDR beamformer and the modified despreader system are shown in Figs. 6 and 7 for the first scenario, and in Figs. 8 and 9 for the second scenario, which demonstrate that both systems have similar performance regarding the null depth of the jamming signals, even though the MVDR beamformer has far greater complexity.

## B. Computer Simulations for Multiple GPS Signals

For multiple GPS signals, we consider the two scenarios described in Tables III and IV. In the first scenario, the received signal consists of four GPS signals of interest, one CW jammer and one FM jammer as the CM jamming signals, one WB noise jammer, and AWGN. In the second scenario, the received signal consists of four GPS signals with different SNR levels, one FM jammer as the CM jammer, one WB noise jammer, one pulsed jammer with an on/off period of 1,000 samples, and AWGN. The JSR for each jammer is 60 dB, and the FM jammer has modulation index  $\beta = 0.05$  and normalized modulation frequency  $f_m = 0.001$ .

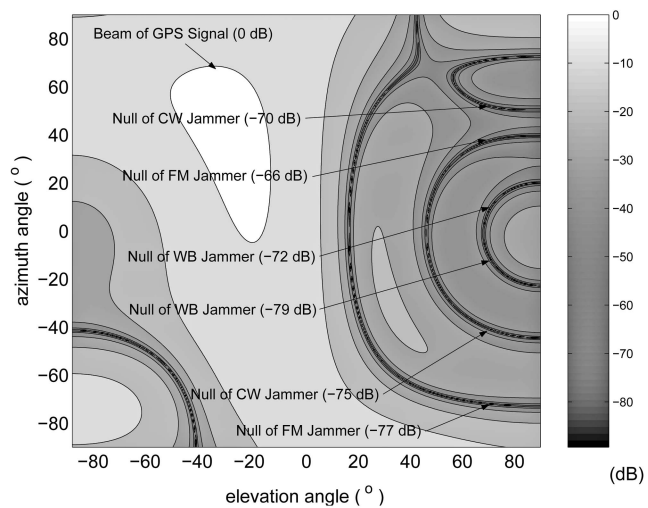


Fig. 6. Beampattern of MVDR beamformer for first scenario for single GPS signal.

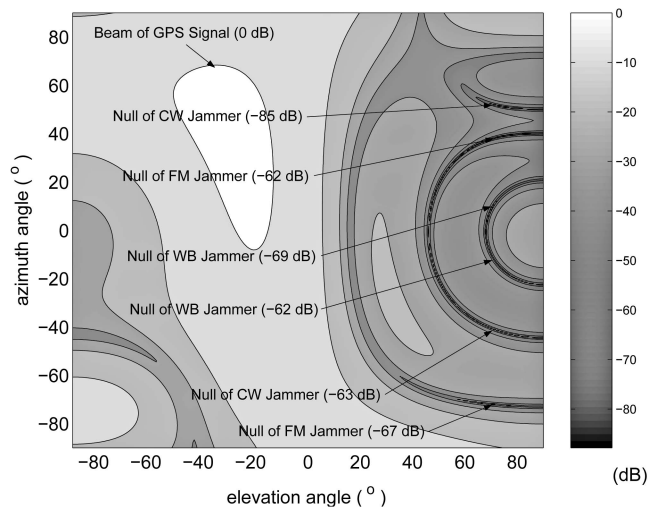


Fig. 7. Beampattern of modified despreader system for first scenario for single GPS signal.

Figs. 10 and 11 show the performance of the modified despreader system for  $\text{SNR} = -28$  dB by comparing the spectra of the received signal and the output of the receiver. The results of the modified despreader system in the first stage of the CM array are shown in Figs. 10(b) and 11(b), respectively, for both scenarios. Observe that all interferers are almost perfectly suppressed by the proposed system. Simulated signal-to-interference-plus-noise ratio (SINR) curves of the MVDR beamformer and the modified despreader system for multiple GPS signals are shown in Figs. 12 and 13, respectively, for both scenarios. Figs. 12(a)–(d) and Figs. 13(a)–(d) show the output SINRs for the first, second, third, and fourth GPS signals for both scenarios. They demonstrate that both systems have similar performance (the maximum SINR difference of both systems is about 5 dB), even though the proposed receiver is blind and has far less complexity than

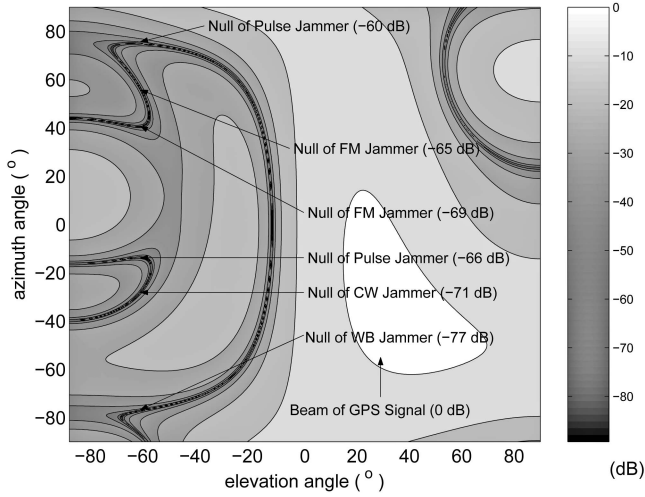


Fig. 8. Beampattern of MVDR beamformer for second scenario for single GPS signal.

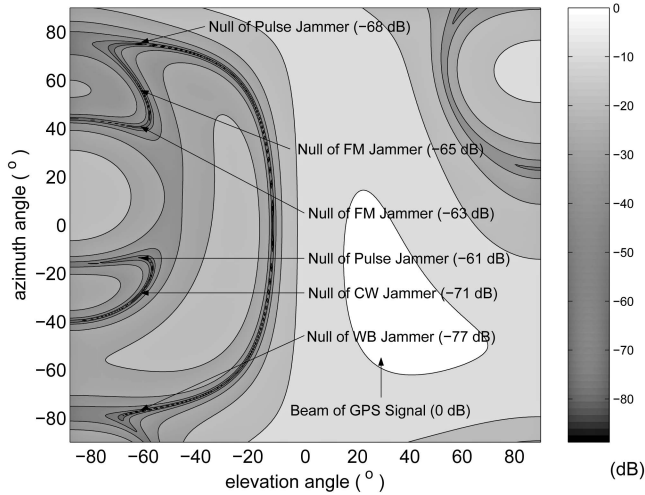


Fig. 9. Beampattern of modified despreader system for second scenario for single GPS signal.

TABLE III  
First Scenario for Multiple GPS Signals

Signal	Azimuth (°)	Elevation (°)	Center Frequency
GPS	12, 105, 167, 301	11, 11, 11, 81	—
CW	57	47	0.25
FM	136	47	0.4
WB	251	47	0.1

TABLE IV  
Second Scenario for Multiple GPS Signals

Signal	Azimuth (°)	Elevation (°)	Center Frequency
GPS	-46, 37, 82, -65	-14, 77, -14, -14	—
FM	67	-51	0.2
WB	21	-51	0.35
Pulsed	-31	-51	—

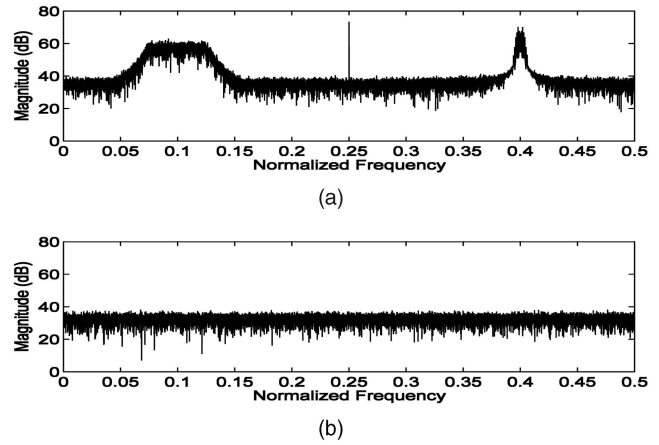


Fig. 10. Signal spectra for first scenario for multiple GPS signals. (a) Received signal. (b) Output of modified despreader system.

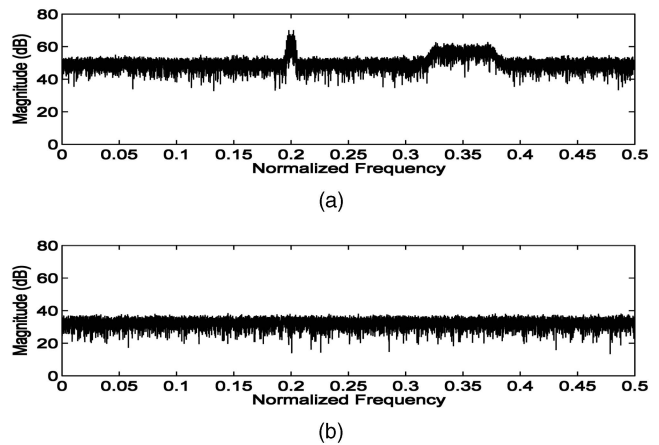


Fig. 11. Signal spectra for second scenario for multiple GPS signals. (a) Received signal. (b) Output of modified despreader system.

the MVDR beamformer. The SINRs of the modified despreading system are slightly lower than those of the MVDR beamformer because the weights of the modified despreading system may not have converged to the optimal values.

## VI. CONCLUSION

GPS is particularly susceptible to high-power interference because it uses low-power spread-spectrum signals. In order to overcome this problem, we proposed an efficient and simple blind multicomponent interference rejection system consisting of a novel modified despreader, followed by a one-stage CM array and the GPS decision device. The modified despreader transforms CM jamming signals into non-CM signals so that a one-stage CM array can null these interferers while extracting the GPS signal of interest, which has its CM characteristic preserved. This system does not require explicit AOA estimation and it has a low computational complexity.



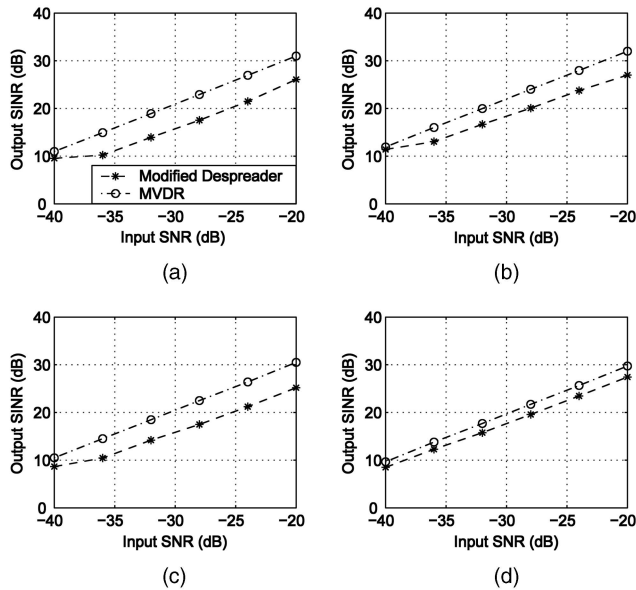


Fig. 12. Output SINR performance of modified despreader system and MVDR beamformer for multiple GPS signals for first scenario. (a) First GPS signal. (b) Second GPS signal. (c) Third GPS signal. (d) Fourth GPS signal.

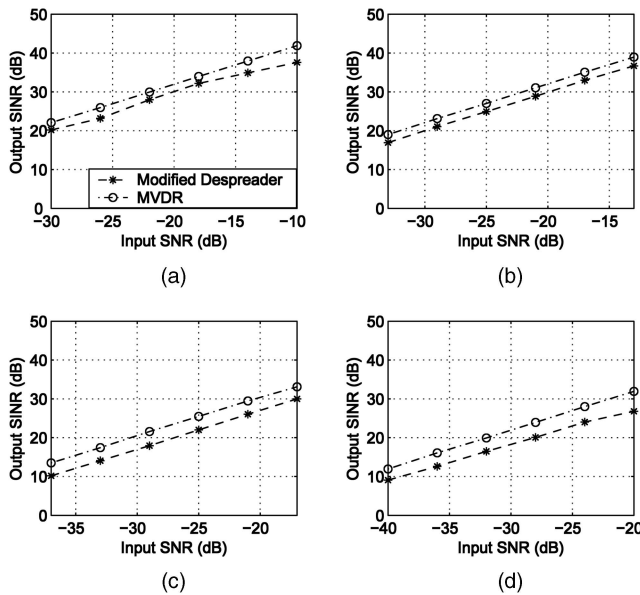


Fig. 13. Output SINR performance of modified despreader system and MVDR beamformer for multiple GPS signals with different SNR levels ( $SNR_1 = SNR_2 + 3 = SNR_3 + 7 = SNR_4 + 10$  dB, where  $SNR_i$  corresponds to  $i$ th GPS signal) for second scenario. (a) First GPS signal. (b) Second GPS signal. (c) Third GPS signal. (d) Fourth GPS signal.

For capturing multiple GPS signals, we utilize a multistage CM array in place of the one-stage CM array. Since this receiver requires only one despreader for multiple GPS signals, it is more efficient than previously proposed systems. The performance of the system for interference suppression was illustrated via example computer simulations and for a range of jammer types.

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