State Estimation and Control for Systems with Perspective Outputs[1]

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Abstract

In this paper we consider the problem of estimating the state of a system with perspective outputs. We formulate the problem in a deterministic setting by searching for the value of the state that is "most compatible" with the dynamics, in the sense that it requires the least amount of noise to explain the measured output. We show that, under appropriate observability assumptions, the optimal estimate converges globally to the true value of the state and can be used to design output-feedback controllers by using the estimated state to drive a state-feedback controller. We apply these results to the estimation of position and orientation of a controlled rigid body, using measurements from a charged-coupled-device camera attached to the body.

1 Introduction

This paper deals with dynamical systems with perspective outputs, inspired by the perspective systems introduced in [4]. Ghosh et al. [4] considered systems with linear dynamics and a homogeneous output function, whereas here we consider systems that are affine on the state but possibly nonlinear on the input. We also consider multiple homogeneous output functions. Formally, the systems under consideration are of the form

$$\dot{x} = A(u)x + b(u) + G(u)\mathbf{d}, \tag{1}$$

$$\alpha_j y_j = C_j(u)x + d_j(u) + \mathbf{n}_j, \qquad j \in \{1, 2, \dots, k\}, \tag{2}$$

where $x \in \mathbb{R}^n$ denotes the state of the system, $u \in \mathbb{R}^{n_u}$ its control input, $y_j \in \mathbb{R}^{m_j}$ its jth perspective output, $\mathbf{d} \in \mathbb{R}^{n_d}$ an input disturbance that cannot be measured, and $\mathbf{n}_j \in \mathbb{R}^{n_n}$ measurement noise affecting the jth output. Each $\alpha_j \in \mathbb{R}$, $j \in \{1, 2, \dots, k\}$ denotes a scalar that is determined by a normalization constrain such as

$$\|y_j\| = 1 \qquad \text{or} \qquad v_j' y_j = 1, \tag{3}$$

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where $v_j \in \mathbb{R}^{m_j}$ denotes a constant vector. When the matrices A, b, and all the C_j, d_j are constant, and $\mathbf{d}$ and all the $\mathbf{n}_j$ are zero, we essentially have a *perspective linear system* in the sense of [4]. We call (1)–(2) a *state-affine system with multiple perspective outputs*, or for short simply a *system with perspective outputs.*

Systems with perspective outputs typically arise when charged-coupled-device (CCD) cameras are used to acquire information about the position and orientation of moving rigid bodies. In Section 5 we will consider the specific problem of estimating the position and orientation of a controlled rigid body using measurements from a CCD camera attached to it. The dynamics of this system can be written as (1)–(2). The reader is referred to [4, 5, 14] for several other examples of perspective systems in the context of motion and shape estimation.

This paper addresses state-estimation for systems with perspective outputs. In the last few years, the observability of perspective linear systems has been systematically studied in the literature and [3] provides an elegant algebraic observability test. It should be noted that for perspective linear systems without inputs it is never possible to recover the norm of the state because the system is homogeneous on the initial conditions. Therefore Dayawansa et al. [3] only consider state indistinguishability up a homogeneous scaling of the state. However, as shown in [7], for perspective systems with inputs it is in principle possible to recover the whole state from projective outputs. This is pursued here.

The main contribution of this paper is the design of a state-estimator for (1)–(2) (cf. Section 2). We propose an optimization approach towards state-estimation by defining the optimal state estimate $\hat{x}$ at time t to be the value for the state of (1) that is compatible with the observations (2) collected up to time t and the dynamics (1) for the "smallest" possible noise $\mathbf{n}_j$ and disturbances $\mathbf{d}$. This formulation is purely deterministic but leads to a state-estimator that resembles a Kalman-Bucy filter. In fact, if the same approach was applied to a linear system with linear outputs, one would arrive precisely at the Kalman-Bucy filter that would be obtained in a stochastic setting. The estimator proposed has the desirable property that, under suitable observability assumptions, the state-estimate converges asymptotically to its true value in the absence of noise and is bounded away from it when there is noise (cf. Section 3). We can therefore use this state-estimator to design output-feedback controllers by using the estimated state to drive state-feedback controllers (cf. Section 4).

A fundamental problem in mobile robotics is the determination of the position and orientation of a robot with respect to an inertial coordinate system. A promising solution to this problem is to utilize a camera mounted on the robot that observes the apparent motion on the image of stationary points. The linear and angular velocities of the camera can be assumed known in its own coordinate system (possibly with errors due to noise) but not in the inertial coordinate system. This is quite reasonable in mobile robotics where the motion of the camera is determined by the applied control signals. The problem of estimating the position and orientation of a camera mounted on a rigid body from the apparent motion of point features has a long tradition in the computer vision literature. We are interested here in filtering-like or iterative algorithms that continuously improve the estimates as more data (i.e., images) are acquired and that are robust with respect to measurement noise. Following the lead of other research (cf., e.g., [10, 8, 13, 9, 12, 2] and references therein) we formulate this as a state-estimation problem and utilize the estimator derived in Section 2 to solve it. One of the main contributions of this paper is that—opposite what happens with most previous algorithms—the one proposed here is globally convergent provided that suitable observability assumptions are satisfied. These assumptions are independent of the initialization of the estimator and depend solely on the motion of the camera. Global convergence to the correct position and orientation is achieved in the absence of noise. When there is noise, the magnitude of the estimation error is essentially proportional to the amount of noise (cf. Theorem 2). Another difference with respect to several other algorithms is that we also estimate scale. This can be achieved either through known (scaling) information about the points observed and/or through the knowledge of the camera's linear velocity. We also consider singular configurations for the points under observation, e.g., all points coplanar.

For lack of space we omit here all the proofs. These can be found in [6].

2 State estimation

For appropriate noise and disturbance signals, essentially every value for the state x at a time $t \in \mathbb{R}$ is compatible with any outputs y_j observed on the interval $[0, t)$. However, we will favor estimates for the state that can be made compatible with the measured outputs utilizing "small" noise and disturbance signals. In fact, we formulate state estimation as a deterministic optimization problem in which the estimate $\hat{x}(t)$ of the state at time $t \geq 0$ is the value for which the measured outputs can be made compatible with the system dynamics (1)–(2) for the "smallest" possible noise $\mathbf{n}_j$ and disturbance $\mathbf{d}$. Formally, given an input u and measured outputs y_j defined on an interval $[0, t)$, we define

$$\hat{x}(t) := \arg \min_{z \in \mathbb{R}^n} J(z, t), \tag{4}$$

where

$$\begin{aligned} J(z;t) := \min_{\mathbf{d}, \mathbf{n}_j, \alpha_j} \Big\{ & x(0)' P_0 x(0) \\ & + \int_0^t \Big(\|\mathbf{d}\|^2 + \sum_j \|\mathbf{n}_j\|^2 \Big) d\tau : \\ & x(t) = z,\ \dot{x} = A(u)x + b(u) + G(u)\mathbf{d}, \\ & \alpha_j y_j = C_j(u)x + d_j(u) + \mathbf{n}_j \Big\}, \end{aligned} \tag{5}$$

with $P_0 > 0$. In practice, the term $x(0)' P_0 x(0)$ makes sure that any unobservable component of the state will be set to zero in the state estimate.

The following result (proved in [6]) solves the state-estimation problem defined above.

Theorem 1. *The solution to the state-estimation problem defined by* (4)–(5) *is given by*

$$\dot{Q} = A(u)Q + QA(u)' + G(u)G(u)' - QWQ, \tag{6}$$

$$\dot{\hat{x}} = (A(u) - QW)\hat{x} + b(u) - Qw, \tag{7}$$

with $Q(0) = P_0^{-1}$, $\hat{x}(0) = 0$, *where*

$$W(t) := \sum_j C_j'(u) \Big(I - \frac{y_j y_j'}{\|y_j\|^2} \Big) C_j(u),$$

$$w(t) := \sum_j C_j'(u) \Big(I - \frac{y_j y_j'}{\|y_j\|^2} \Big) d_j(u), \qquad \forall t \geq 0.$$

Note that we can rewrite the state-estimation equation (7) as

$$\begin{aligned} \dot{\hat{x}} = {} & A(u)\hat{x} + b(u) \\ & + Q \sum_j C_j(u)' \Big(\hat{\alpha}_j y_j - C_j(u)\hat{x} - d_j(u) \Big), \end{aligned}$$

$$\hat{\alpha}_j = \frac{y_j' \big(C_j(u)\hat{x} + d_j(u) \big)}{\|y_j\|^2},$$

which emphasizes the parallel between (7) and a Kalman-Bucy filter for linear systems.

3 Estimator convergence

We are now interested in investigating under what conditions the state estimate $\hat{x}$ provided by Theorem 1 converges to the true state x of the perspective system. The following technical assumption is needed:

Assumption 1. There exist positive constants $\delta, \Delta \in (0, \infty)$ such that $\delta I \leq G(u)G'(u) \leq \Delta I$, $\forall u \in \mathbb{R}^{n_u}$.

This mild assumption essentially guarantees that $G(u)$ is bounded and full-row rank, "uniformly" over all possible inputs. The following result (proved in [6]) establishes the convergence of the state estimate.

Theorem 2. *Assuming that the solution to the process* (1)–(2) *exists globally, the solution to state estimator* (6)–(7) *also exists globally. Moreover, when Assumption 1 holds and Q remains uniformly bounded, there exist positive constants $c, \lambda, \gamma_d, \gamma_1, \ldots, \gamma_k$ such that*

$$\|\tilde{x}(t)\| \leq ce^{-\lambda t}\|\tilde{x}(0)\| + \gamma_d \sup_{\tau \in (0,t)} \|\mathbf{d}(\tau)\| + \sum_j \gamma_j \sup_{\tau \in (0,t)} \|\mathbf{n}_j(\tau)\|, \quad \forall t > 0, \quad (8)$$

where $\tilde{x} := \hat{x} - x$ denotes the state estimation error.

Some condition on the observability[1] of (1)–(2) would be expected to achieve convergence of the estimated state $\hat{x}$ to the process state x. In Theorem 2 this condition appear in the form of the requirement that Q remains bounded. In the remaining of this section we investigate conditions under which this happens.

From (6) it is clear that Q remains bounded if $W(t) \geq \epsilon I > 0$, $\forall t \geq 0$ because in this case the term $-QWQ$ eventually dominates for very large Q. However, this case is not very interesting because, e.g., for the single output case ($k = 1$) the matrix W typically has rank equal to $(\operatorname{rank} C_1) - 1 \leq n - 1$ and therefore cannot be nonsingular. The following Lemma provides a significantly weaker condition for the boundedness of Q.

Lemma 1. *The matrix Q remains bounded along trajectories of the system* (1)–(2) *and state-estimator* (6)–(7)*, provided that there exist positive constants T, ϵ such that*

$$\int_0^T \Phi(t+\tau, t)' W(t+\tau) \Phi(t+\tau, t) d\tau \geq \epsilon I > 0, \quad (9)$$

$\forall t \geq 0$, *where $\Phi(t, \tau)$ denotes the state transition matrix of $\dot{z} = A(u)z$.*

[1] In the present setup, the correct notion is actually constructability because we are attempting to reconstruct the state from past outputs [1, Section 3.3].

To get some intuition for the meaning of (9) note that for $\int_0^T \Phi(t+\tau, t)' W(t+\tau) \Phi(t+\tau, t) d\tau$ to be singular, there would have to be a vector x_0 such that

$$x_0' \Phi(t+\tau, t)' W(t+\tau) \Phi(t+\tau, t) x_0 = 0,$$

$\forall \tau \in (0, T)$, $t \geq 0$, or equivalently, such that

$$\beta_j(t+\tau) y_j(t+\tau) = C_j\big(u(t+\tau)\big) \Phi(t+\tau, t) x_0, \quad (10)$$

$\forall \tau \in (0, T)$, $t \geq 0$, $j \in \{1, \ldots, k\}$, for appropriate scalars $\beta_j(t)$. In essence this means that (9) fails when all the y_j evolve as if u, $\mathbf{d}$, and all the $\mathbf{n}_j$ were zero. In fact, we can view (9) as a persistence of excitation-like condition that requires x to evolve is some interesting way, other than just following the homogeneous dynamics of (1)–(2), along which scaling information could not be recovered.

It is interesting to note the parallel between the integral in (9) and the constructibility Gramian for linear system [1, Section 3.3]. In fact, if W were replaced by $\sum_j C_j' C_j$, the integral in (9) is precisely the constructibility Gramian for the system (1) with *linear* outputs $C_j x + d_j + \mathbf{n}_j$, $j \in \{1, 2, \ldots, k\}$.

4 Output feedback

We start by considering the set-point control of a perspective linear system (1)–(2) using output-feedback. To this extent, we assume that A, G and all the C_j are constant and that

$$b(u) := Bu, \qquad d_j(u) := D_j u, \qquad \forall u \in \mathbb{R}^{n_u},$$

for constant matrices B, D_j. We take a separation approach and assume given a linear state-feedback control law

$$u = Fx + r \quad (11)$$

that would exponentially stabilize (1) around the equilibriums point $x_{\text{eq}} := -(A + BF)^{-1} Br$. We thus assume that $A + BF$ is asymptotically stable and therefore that there exist positive definite matrices R, S such that

$$(A + BF)' R + R(A + BF) \leq -S < 0. \quad (12)$$

Since x cannot be measured we utilize

$$u = F\hat{x} + r, \quad (13)$$

instead of (11), where $\hat{x}$ is generated by the state-estimator (6)–(7). In particular,

$$\dot{\hat{x}} = (A - QW)\hat{x} + (B - Qw)(F\hat{x} + r).$$

We recall that output feedback happens through both W and w that are functions of the outputs y_j. The following result (proved in [6]) establishes the validity of the proposed controller.

Theorem 3. *When Assumption 1 holds and Q remains uniformly bounded, there exist positive constants $c, \lambda, \gamma_d, \gamma_j$ such that*

$$\|x(t)-x_{\text{eq}}\| \leq e^{-\lambda t}\| \begin{bmatrix} x(0)-x_{\text{eq}} \\ \hat{x}(0)-x_{\text{eq}} \end{bmatrix} \| + \gamma_d \sup_{\tau\in(0,t)} \|\mathbf{d}(\tau)\| + \sum_j \gamma_j \sup_{\tau\in(0,t)} \|\mathbf{n}_j(\tau)\|, \quad \forall t>0, \tag{14}$$

along solutions to the closed-loop system (1)–(2)*,* (6)–(7)*,* (13)*.*

It is straightforward to generalize the results above to the nonlinear case, provided that the state-feedback controller used is robust (in the input-to-state stability sense) with respect to state perturbations [6].

5 Rigid body motion estimation using CCD cameras

In this section we show how one can estimate the position and orientation of a mobile robot using a CCD camera mounted on the robot that observes the apparent motion on the image of stationary points. We do this by reducing the problem to the estimation of the state of a system with projective outputs.

Consider a coordinate frame $\{b\}$ attached to a rigid body that moves with respect to an inertial frame $\{i\}$. We denote[2] by $(p_{ib}, R_{ib}) \in \text{SE}(3)$ the configuration of the frame $\{b\}$ with respect to $\{i\}$. Thus, if q_1^i and q_1^b denote the coordinates of a point Q_1 in the frames $\{i\}$ and $\{b\}$, respectively, we have that

$$q_1^i = p_{ib} + R_{ib} q_1^b. \tag{15}$$

Moreover, if q_j^i and q_j^b denote the coordinates of another point Q_j in the frames $\{i\}$ and $\{b\}$, respectively, we conclude that

$$q_j^b = R'_{ib} q_j^i - R'_{ib} p_{ib} = R'_{ib}(q_j^i - q^i) + q_1^b.$$

We denote by $(v_{ib}^b, \Omega_{ib}^b) \in \text{se}(3)$ the twist that defines the velocity of frame $\{b\}$ with respect to $\{i\}$, expressed in the frame $\{b\}$, i.e.,

$$v_{ib}^b = R'_{ib}\dot{p}_{ib}, \qquad \Omega_{ib}^b = R'_{ib}\dot{R}_{ib}.$$

From this and (15), we obtain

$$\dot{q}_1^b = -\Omega_{ib}^b q_1^b - v_{ib}^b + R'_{ib}\dot{q}_1^i, \qquad \dot{R}_{ib} = R_{ib}\Omega_{ib}^b.$$

Suppose now that a camera attached to the body frame $\{b\}$ sees k points $Q_1, Q_2, \ldots, Q_k$ rigidly attached to the inertial frame $\{i\}$. Denoting by $y_j \in \mathbb{R}^3$ the homogeneous image coordinates of the point Q_j, the dynamics of the system can be described by the following dynamical system with k perspective outputs:

$$\dot{q}_1^b = -\Omega_{ib}^b q_1^b - v_{ib}^b, \tag{16}$$

$$\dot{R}'_{ib} = -\Omega_{ib}^b R'_{ib}, \tag{17}$$

$$\alpha_j y_j = q_1^b + R'_{ib}(q_j^i - q_1^i), \quad j \in \{1,2,\ldots,k\}. \tag{18}$$

To put this system in the form of (1)–(2) we simply need to define x to be a 12-dimensional vector whose first 3 entries are the entries of q_1^b and the remaining 9 entries are the columns of R_{ib} stacked on top of each other. Note that once we have estimates $\hat{R}_{ib}$ and $\hat{q}_1^b$ of R_{ib} and q_1^b, respectively, we can also estimate p_{ib} using

$$\hat{p}_{ib} = q_1^i - \hat{R}_{ib}\hat{q}_1^b.$$

Figure 5 shows simulation results obtained for a robot following a circular path on the $x-y$ plane, carrying a camera whose optical axis is aligned with the z axis and looks up at four non-coplanar points. The results presented were obtained with and without measurement noise. In the presence of noise, the error in the z coordinate actually does not fully converge to zero. This can be explained because the motion of the robot does not probe the z direction and therefore depth recovery is more sensitive to measurement noise.

5.1 Singular configurations

Depending on the configurations of the points $Q_1, Q_2, \ldots, Q_k$, the state of (16)–(18) may not be observable. However, even in this case it may still be possible to recover it by using the fact that R_{ib} is a rotation matrix. To this effect let $M \in \mathbb{R}^{3\times m}$ be a matrix whose columns are a basis for the vector space generated by the $k-1$ vectors $\{q_j^i - q_1^i : j = 2, \ldots, k\}$ and let $\tilde{q}_j$, $j \in \{2, \ldots, k\}$ be such that

$$q_j^i - q_1^i = M\tilde{q}_j.$$

In this case the system (16)–(18) can be re-written as

$$\dot{q}_1^b = -\Omega_{ib}^b q_1^b - v_{ib}^b, \tag{19}$$

$$\dot{N} = -\Omega_{ib}^b N \tag{20}$$

$$\alpha_j y_j = q_1^b + N\tilde{q}_j, \quad j \in \{1,2,\ldots,k\}. \tag{21}$$

where $N := R'_{ib}M \in \mathbb{R}^{3\times m}$. Note that when $m = \operatorname{rank} M < 3$, the system (16)–(18) is not observable because its input-output map is consistent with that of the lower-dimensional model (19)–(21).

To compute an estimate $\hat{R}_{ib}$ of R_{ib} from the estimate $\hat{N}$ of N, the following two cases should be considered separately. For simplicity we assume that M was chosen orthonormal, i.e., that $M'M = I$.

1. $\operatorname{rank} M = 3$, which corresponds to the existence of 4 non-coplanar points. In this case $\hat{R}_{ib}$ can be

[2] We denote by SE(3) the Cartesian product of $\mathbb{R}^3$ with the group SO(3) of 3×3 rotation matrices; and by se(3) the Cartesian product of $\mathbb{R}^3$ with the space so(3) of 3×3 skew-symmetric matrices (cf.,e.g., [11]).

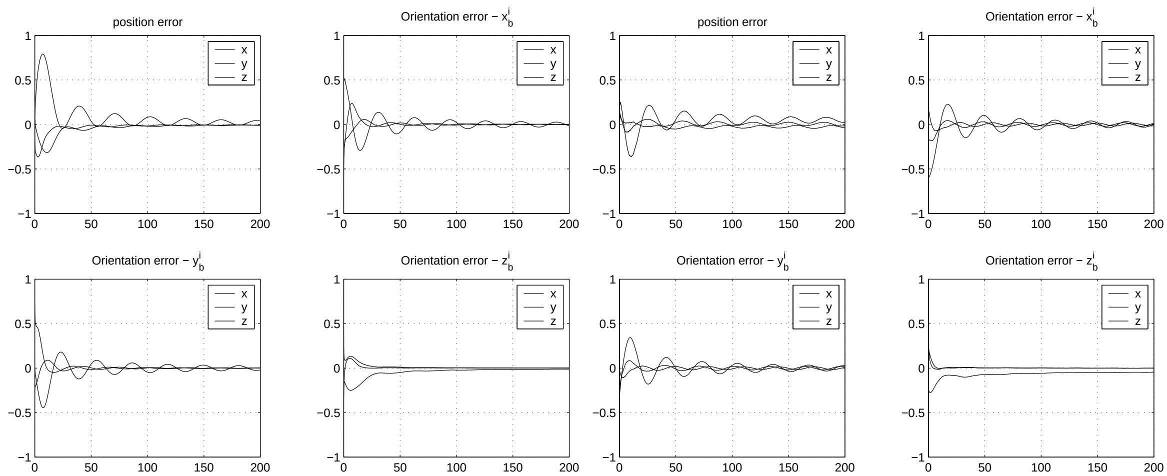

Figure 1: Estimation errors for a simulation in which a robot follows a circular path with a camera looking up at four non-coplanar points. The four plots on the left correspond to no measurement noise, whereas the ones on the right correspond to Gaussian measurement noise with standard deviation equal to roughly 5% of the measurements. The orientation errors labeled x_b^i, y_b^i, and z_b^i correspond to the estimation errors for the first, second, and third columns of R_{ib}, respectively.

recovered directly from $\hat{N}$ using

$$\hat{R}'_{ib} = \hat{N} M^{-1}.$$

2. $\operatorname{rank} M < 3$, which corresponds to all points being coplanar ($\operatorname{rank} M \leq 2$) or even collinear ($\operatorname{rank} M = 1$). Denoting by $M^{\perp} \in \mathbb{R}^{3\times(3-m)}$ a matrix whose columns form an orthonormal basis for the orthogonal complement of the image of M (i.e., a full rank matrix such that $M^{\perp\prime} M^{\perp} = I$ and $M' M^{\perp} = 0$), the general solution to $M'\hat{R}_{ib} = \hat{N}'$, is of the form

$$\hat{R}_{ib} = M\hat{N}' + M^{\perp}\mu',$$

for some vector $\mu \in \mathbb{R}^{3\times(3-m)}$. This vector needs to be determined from the fact that $\hat{R}'_{ib}$ is orthonormal. Since

$$\begin{aligned}\hat{R}'_{ib}\hat{R}_{ib} &= (\hat{N}M' + \mu M^{\perp\prime})(M\hat{N}' + M^{\perp}\mu') \\ &= \hat{N}\hat{N}' + \mu\mu',\end{aligned}$$

we conclude that μ needs to be chosen to make

$$\hat{N}\hat{N}' + \mu\mu'$$

as close to the identity as possible.

For the case, $\operatorname{rank} M = 2$ (points coplanar but not collinear), it is straightforward to compute the vector μ that minimizes the Frobenius norm $\|\cdot\|_F$ of $\hat{N}\hat{N}' + \mu'\mu - I$:

$$\begin{aligned}\|\hat{N}\hat{N}' + &\mu\mu' - I\|_F^2 \\ &:= \operatorname{trace}(\hat{N}\hat{N}' + \mu\mu' - I)'(\hat{N}\hat{N}' + \mu\mu' - I) \\ &= \|\mu\|^4 + 2\mu'(\hat{N}\hat{N}' - I)\mu + \operatorname{trace}(\hat{N}\hat{N}' - I)^2.\end{aligned}$$

Denoting by λ the most negative eigenvalue of $\hat{N}\hat{N}' - I$ (which must be negative since any vector in the kernel of $\hat{N}'$ is an eigenvector corresponding to the eigenvalue -1) and by v the corresponding unit-norm eigenvector, the previous expression has a minimum at $\mu' = \alpha v$, which is equal to

$$\alpha^4 + 2\alpha^2\lambda + \operatorname{trace}(\hat{N}\hat{N}' - I)^2,$$

where α is a scalar. The minimum is then obtained for $\alpha^2 = -\lambda$ and therefore $\mu = \pm\sqrt{-\lambda}\, v$,

$$\hat{R}_{ib} = M\hat{N}' \pm \sqrt{-\lambda}\, M^{\perp} v'.$$

The sign for the square root can be determined from the constrain that the determinant of $\hat{R}_{ib}$ be positive. Note that μ is unique as long as $\hat{N}\hat{N}' - I$ does not have more than one eigenvector associated with the most negative eigenvalue.

5.2 Unknown inertial coordinates

Consider now the case in which the inertial coordinates of the points $Q_1, Q_2, \dots, Q_k$ are not known. In this case, one can still estimate x by using three of the points to define the inertial coordinate system. To this effect, let

$$S := R'_{ib}\begin{bmatrix} q_2^i - q_1^i & q_3^i - q_1^i & \cdots & q_k^i - q_1^i \end{bmatrix},$$

We can re-write (16)–(18) as

$$\dot{q}_1^b = -\Omega_{ib}^b q_1^b - v_{ib}^b, \tag{22}$$

$$\dot{S} = -\Omega_{ib}^b S \tag{23}$$

$$\alpha_j y_j = q_1^b + S e_j, \quad j \in \{1, 2, \dots, k\}, \tag{24}$$

where e_j denotes the jth column of the $(k-1)\times(k-1)$ identity matrix.

To recover an estimate $\hat{R}_{ib}$ of R_{ib} from the estimate $\hat{S}$ of S, we can use the QR decomposition to obtain a rotation matrix $\hat{R}_{ib}$ and an upper triangular matrix $\hat{U}$ such that

$$\hat{S} = \hat{R}'_{ib}\hat{U},$$

and then defining $q_1^i := 0$ and $q_j^i,\ j \in \{2, 3, \dots, k\}$ equal to the $(j-1)$th column of $\hat{U}$. This corresponds to the following convention to construct the inertial coordinate system: the origin of $\{i\}$ is the point Q_1; its first axis is defined by the direction from Q_1 to Q_2; its second axis is orthogonal to the first one and lies on the plane defines by Q_1, Q_2, and Q_3; and its third axis is defined by the cross product of the first two.

6 Conclusions

In this paper we considered the problem of estimating the state of a system with perspective outputs. We designed an estimator that is globally convergent under appropriate observability assumptions and can therefore be used to design output-feedback controllers. We applied these results to estimate the position and orientation of a mobile robot using measurements from an attached CCD camera. The estimator proposed requires the robot's linear and angular velocities. Adaptive estimation techniques can probably be used to estimate these parameters. This is the subject of future research. Another topic for future research is to incorporate algebraic constrains on the state in the estimation algorithm.

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