Optimal Radar-Communications Spectral Maneuvering for TDOA-based Tracking

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Abstract—In recent literature, three types of radar-communications convergence were identified: coexistence, cooperation, and co-design, with increasingly tight integration between the radar and communication systems. In this paper we discuss a fourth mode of operation, named collaboration, in which the two systems not only share bandwidth but also work together to accomplish a common task, namely the 3D localization of passive, moving objects using time difference of arrival (TDOA). In contrast with previous work’s cooperation mode where spectrum sharing can be posed as a multi-objective optimization problem, with separate performance indices expressing radar (resolution) and communication (bit-rate) concerns, the 3D localization problem naturally leads to a common objective function: the object tracking error. By expressing the steady-state covariance of the track of multiple objects drives the performance objective in [4] in the context of cross-ambiguity processing (CAF) [5], including time difference of arrival (TDOA), frequency difference of arrival (FDOA) frequency rate difference of arrival (FRDOA) and cyclostationary feature extraction. While the track of multiple objects drives the performance objective in [4], in this paper we choose to focus on tracking a single object, which allows one to examine in detail the root of the optimization problem, which is the bandwidth partition between \( B_R \) and \( B_C \). The paper is organized as follows. Section II describes the problem, and the main assumptions. Section III present expressions for the steady-state error covariance of the tracking error. As shown in the paper via computer simulations, the optimal \( B_T \) partition into \( B_R \) and \( B_C \) depends on the geometry of the problem. Section IV closes the paper with our conclusions.

II. PROBLEM STATEMENT

We consider a multistatic apparatus in 3D, formed by one illuminator and four collectors, as depicted in Figure 1. The illuminator includes a radar transmitter, utilized to illuminate objects in its field-of-view, a communications receiver to process information received from the collectors, and a processing unit, which, as shown in Figure 2, computes the 3D position of the object on the basis of the Time Difference of Arrival (TDOA) between the transmitted radar signal and the reflected signals at the four collectors. Assume the total bandwidth available to the apparatus is \( B_T \), to be shared among a radar portion \( B_R \) and a communications portion \( B_C \) with \( B_R + B_C = B_T \). The communications bandwidth \( B_C \) is shared by the channels between the collectors and the illuminator.

A. Variance of TDOA measurement noise

Each collector \( k = 2, 3, 4, 5 \) obtains a noisy TDOA measurement \( \tau_k \):

\[
\tau_k = \frac{1}{c} (\|p_O - p_I\| + \|p_O - p_k\| - \|p_I - p_k\|) + n_{10k} \quad (1)
\]
B. Communications model

Denoting by $T$ the radar’s Pulse Repetition Interval (PRI) and assuming the collector sends time-of-arrival (TOA) information on every pulse, the bit-rate $b$ necessary to transmit $L$ bits of data is given by:

$$bT = L,$$

where $b$ is determined by the communications bandwidth $B_C$ by:

$$b = B_C \log_2 \left(1 + \frac{P_{kl}}{N_0 B_C + \sum_{k \neq k} P_{kl}} \right)$$

(3)

and $P_{kl}$ denotes the power of the communications device used by collector $k$, $f_k$ denotes the carrier frequency used by collector $k$, $G_k$ denotes the antenna gain for communications originated from the collector $k$.

C. Cramér-Rao Lower Bound (CRLB) for position measurements

Assuming the $n_{iok}$ in equation (1) are zero mean independent Gaussian random variables, the Fisher Information Matrix (FIM) $\Theta$ associated with the estimation of $p_O = (x, y, z)$ (a row vector) is given by:

$$\Theta(p_O) = \sum_{k=2}^{5} \frac{1}{\sigma_{iok}^2} \begin{pmatrix} \frac{\partial \tau_k}{\partial x} & \frac{\partial \tau_k}{\partial y} & \frac{\partial \tau_k}{\partial z} \end{pmatrix}^T \begin{pmatrix} \frac{\partial \tau_k}{\partial x} & \frac{\partial \tau_k}{\partial y} & \frac{\partial \tau_k}{\partial z} \end{pmatrix}$$

(5)

(6)

where:

$$\begin{pmatrix} \frac{\partial \tau_k}{\partial x} & \frac{\partial \tau_k}{\partial y} & \frac{\partial \tau_k}{\partial z} \end{pmatrix} = \frac{1}{2c} \left( \begin{pmatrix} p_O - p_1 \\ \|p_O - p_1\| \end{pmatrix} + \begin{pmatrix} p_O - p_k \\ \|p_O - p_k\| \end{pmatrix} \right)$$

D. Motion model of the object

We assume the object follows a discrete white noise acceleration model along the three dimensions – [7], p. 273 – where the process noise is white:

$$\xi_x(t+1) = F \xi_x(t) + w_x(t), \quad \xi_y(t+1) = F \xi_y(t) + w_y(t), \quad \xi_z(t+1) = F \xi_z(t) + w_z(t),$$

with

$$\mathbb{E}[w_x(t)w_y^*(t)] = \mathbb{E}[w_y(t)w_x^*(t)] = \mathbb{E}[w_z(t)w_y^*(t)] = Q.$$

$$F = \begin{bmatrix} 1 & T & T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix},$$

$$Q = \sigma_a^2 \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} & \frac{T^2}{2} \\ \frac{T^2}{2} & \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & \frac{T^2}{2} & \frac{T^3}{3} \end{bmatrix}. $$

$T$ denotes the update time for the tracker, coinciding in this case with the radar’s PRI. $\sigma_a T^2/2$ is the position error caused by a constant acceleration of $\sigma_a$.  

\[ \text{Fig. 1. Multistatic apparatus for object tracking.} \]

\[ \text{Fig. 2. TDOA-based tracking.} \]
III. Steady-State Tracking Error Covariance for Position and Velocity

We assume the tracker utilizes a Kalman filter in three dimensions, with process model described in Section II-D and measurement noise matrix $R$. The a priori (just before measurement) steady state error in position ($e_p$) and in velocity ($e_v$) for one-dimensional motion for the process in Section II-D has been derived in [8]-[9] as follows:

$$e_p = \frac{\sigma^2_a}{\sigma_m^2} \sqrt{1 + 2r(\sqrt{1 + 2r + 1})^2}, \quad \text{where} \quad r = \frac{4\sigma_e}{\sigma_m T^2} \quad (7)$$

$$e_v = \frac{\sigma^2_a T^2}{2} (\sqrt{1 + 2r + 1}) \quad (8)$$

A. Computer simulations

For nominal choices of the various parameters, we consider two 1D configurations:

- Configuration 1: $p_0 = 0$, $p_2 = 50e3$, $p_3 = 60e3$, $p_4 = 70e3$, $p_5 = 80e3$, $p_1 = 100e3$.
- Configuration 2: $p_0 = 0$, $p_1 = 50e3$, $p_2 = 70e3$, $p_3 = 80e3$, $p_4 = 90e3$, $p_5 = 100e3$.

The first row in Figure 3 depicts the results corresponding to Configuration 1, while the second row depicts the results corresponding to Configuration 2.

![Figure 3](image)

Fig. 3. Steady-state errors as a function of $B_R$.

IV. Conclusions

This paper introduces a new mode of interaction between radar and communications systems named collaboration, in which the two systems not only share bandwidth but also work together to accomplish a common task, namely the 3D localization of passive, moving objects using time difference of arrival (TDOA). In contrast with previous work’s cooperation mode where spectrum sharing can be posed as a multi-objective optimization problem, with separate performance indices expressing radar (resolution) and communication (bit-rate) concerns, the 3D localization problem naturally leads to a common objective function: the object tracking error. By expressing the steady-state covariance of the tracking error in position and velocity in terms of the bandwidth devoted to the radar portion, it is shown by computer simulations that the functions relating the radar bandwidth with the tracking error covariance have distinct global minima, corresponding to the optimal partitioning of a total fixed bandwidth among the radar and the communication portions. As shown in the paper, the optimal radar-communications partition depends on the geometry of the problem.

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REFERENCES


