

# Game theoretical approaches to secure and robust routing

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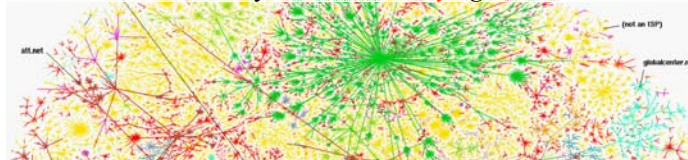
In collaboration with: S. Bohacek (Univ. Delaware), K. Obraczka (UC Santa Cruz)  
J. Lee (Postdoc, UC Santa Barbara), C. Lim (PhD candidate, USC)

## Network Security vs. Fault-Tolerance



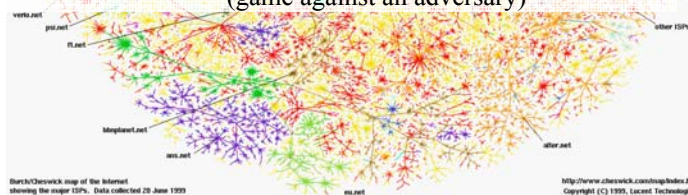
The basic principle behind the design of the Internet was to utilize massive *redundancy* to achieve *fault-tolerance*

but this does not necessarily result in *security* against malicious attacks



Fault tolerance  $\equiv$  robustness with respect to random failures  
(game against chance)

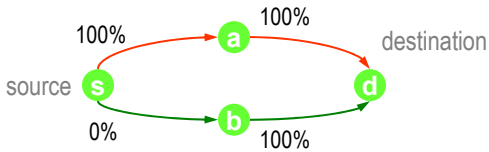
Network security  $\equiv$  robustness with respect to attacks  
(game against an adversary)



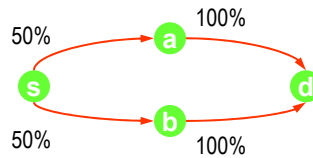
*An adversary can explore weaknesses that chance will not easily find*

# Security vs. Fault-Tolerance in Routing

single-path routing



stochastic multi-path routing



Suppose all links are equally likely to fail, and one of them does fail...

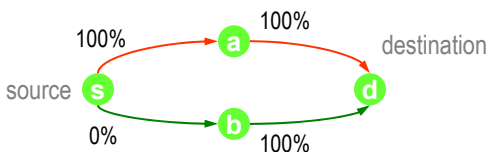
*Which routing strategy results in higher probability that a packet will reach destination?*

link labels refer to probability of forwarding a packet

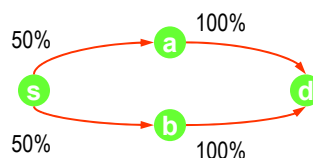
*Both routing schemes result in exactly the same probability (50%)...*

# Security vs. Fault-Tolerance in Routing

single-path routing



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Suppose all links are equally likely to fail, and one of them does fail...

With a transport protocol that guarantees reliable delivery (e.g., TCP):

$$E[\# \text{ transmissions}] = 50\% \times 1 + 50\% \times \infty = \infty$$

$$E[\# \text{ transmissions}] = \sum_{i=1}^{\infty} (1+i)(.50)^i \cdot .50 = 2$$

prob. of failing  $i$  times and succeeding at next attempt

# Security vs. Fault-Tolerance in Routing

Disclaimers:

1. Eventually routing would get fixed but 40% of the path outages take more than 30 minutes to repair [Chandra et al., 2003]
2. If failure is bidirectional, acknowledgement packets can also be dropped and  $E[\# \text{ transmissions}]$  raises to 4

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# Security vs. Fault-Tolerance in Routing



Suppose all links are equally likely to fail, and one of them does fail...

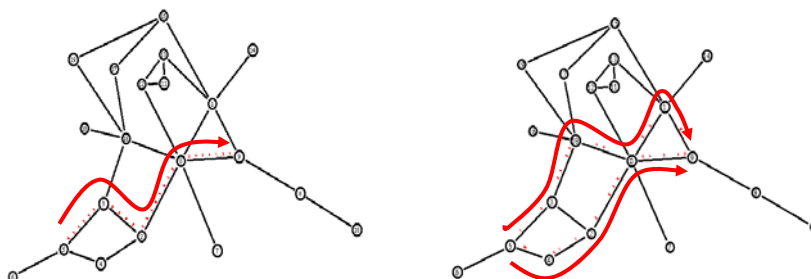
Assume that fail was caused by an attacker that selects the link

*Which routing strategy results in higher probability that a packet will reach destination?*

Attacker can learn routing policy and prevent all communication by compromising a single link

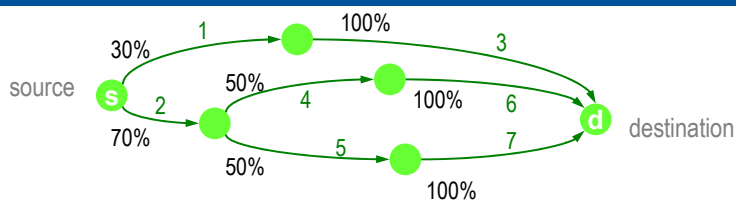
Compromising a single link, probability of intercepting packet is only 50% (assuming stochastic multi-path)

later we will find other reasons why multi-path may be advantageous...



1. How to compute stochastic multi-path routing tables?
  - Noncooperative game—explore redundancy in an adversarial context
    - Different levels of security/reliability for distinct links/nodes
    - Reduce latency
2. Other issues:
  - Scalable computation for large networks
  - Multi-path routing within the protocol stack

## Stochastic routing policies



probability that a packet arriving at the node where  $\ell$  starts will be routed through link  $\ell$

set of (unidirectional) links

$$\text{stochastic routing policy} \equiv R := \{ r_\ell \geq 0 : \ell \in \mathcal{L} \}$$

for every node  $n$

$$\sum_{\ell \in \mathcal{L}[n]} r_\ell = 1$$

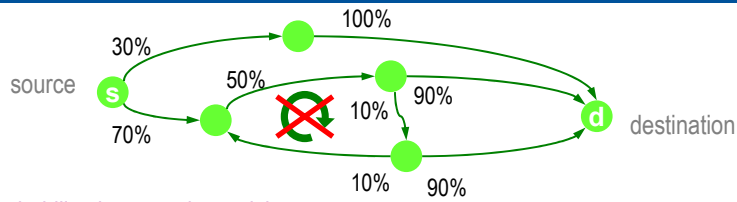
summation over links that exit node  $n$

e.g.,  $R := \{ \underbrace{.3, .7}_1, 1, \underbrace{.5, .5}_1, 1, 1 \}$

add to 1    add to 1

$\mathcal{R}_{\text{stoch}}$   $\equiv$  set of all routing policies

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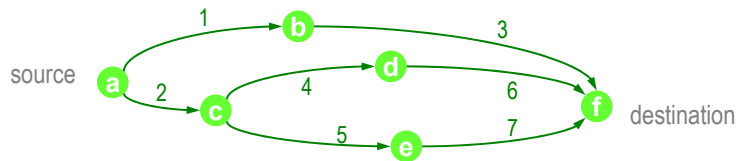
for every node  $n$

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summation over links that exit node  $n$

- $\mathcal{R}_{\text{stoch}}$   $\equiv$  set of all routing policies
- $\mathcal{R}_{\text{no-cycle}}$   $\equiv$  set of all cycle-free policies, i.e., for which there is no closed sequence of links all with positive routing probability

# Attack space



probability that packets in link  $\ell$  are compromised

set of links

$$\text{pure attack} \equiv P := \{ p_\ell : \ell \in \mathcal{L} \}$$

attacker has available a pool of "pure attacks" and will select the one that is more likely to prevent communication

e.g., pure attack at link 3 with 10% probability of success:

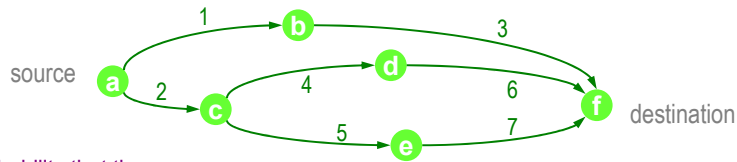
$$P_3 := \{ 0, 0, .1, 0, 0, 0 \}$$

pure attack at node **f** with 20% probability of success:

$$P_f := \{ 0, 0, .2, 0, .2, .2 \}$$

$\mathcal{P} \equiv$  set of all (pure) attacks available to attacker

# Mixed attacks



probability that the attacker will intercept a packet traveling in link  $\ell$

set of links

attacker is allowed to randomize between pure attacks with appropriate probabilities

(pure) attack  $\equiv P := \{ p_\ell : \ell \in \mathcal{L} \}$

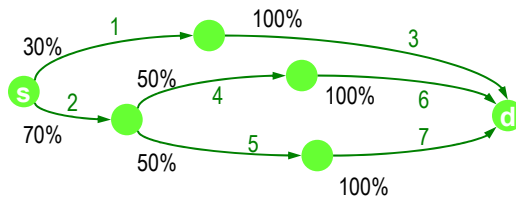
$\mathcal{P} \equiv$  set of all pure attacks available to attacker

mixed attack policy  $\equiv M := \{ m_P : P \in \mathcal{P} \} \in [0,1]^{\mathcal{P}}$

$$\sum_{P \in \mathcal{P}} m_P = 1$$

probability that the attacker select the pure attack  $P$

# Example

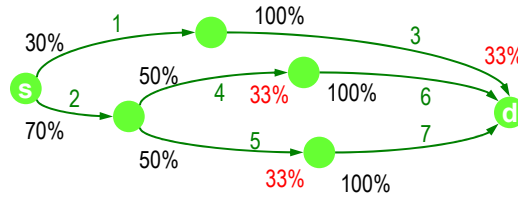


stochastic routing policy  $\equiv R := \{ .3, .7, 1, .5, .5, 1, 1 \}$

pure attacks available to attacker  $\equiv \mathcal{P} := \{ \{1,0,0,0,0,0,0\}, \{0,1,0,0,0,0,0\}, \dots, \{0,0,0,0,0,1,0\}, \{0,0,0,0,0,0,1\} \}$

10% effective link attacks (7 attacks)  
 (attacker can target any link, it will succeed in compromising packet delivery with 10% probability)

# Example



stochastic routing policy  $\equiv R := \{ .3, .7, 1, .5, .5, 1, 1 \}$

pure attacks available to attacker  $\equiv \mathcal{P} := \{ \{1,0,0,0,0,0,0\}, \{0,1,0,0,0,0,0\}, \dots \dots, \{0,0,0,0,0,1,0\}, \{0,0,0,0,0,0,1\} \}$

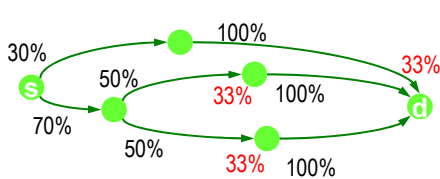
mixed attack policy  $\equiv M := \{ 0, 0, .33, .33, .33, 0, 0 \}$

$$P_{R,M}(\text{capture}) = (30\% \times 33\% + 2 \times 35\% \times 33\%) \times 10\% = 3.3\%$$

probability that packet is captured for routing policy  $R$  and mixed attack policy  $M$

*but not really rational...*

# Example

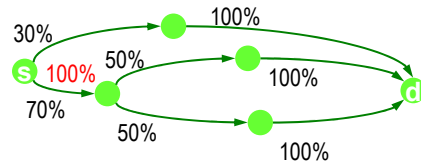


mixed attack policy  $M := \{ 0, 0, .33, .33, .33, 0, 0 \}$

$$P_{R,M}(\text{capture}) = (30\% \times 33\% + 2 \times 35\% \times 33\%) \times 10\% = 3.3\%$$

for this attack policy  $M$  router cannot do better

$$P_{R,M}(\text{capture}) = 3.3\% \quad \forall R$$



stochastic routing policy  $R := \{ .3, .7, 1, .5, .5, 1, 1 \}$

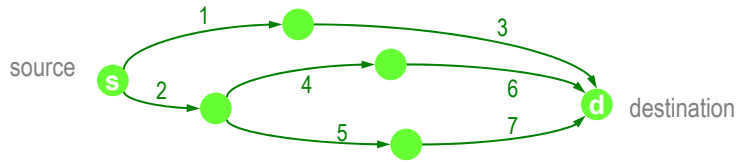
attacker could do better against  $R$  with  $M := \{ 0, 1, 0, 0, 0, 0, 0 \}$

$$P_{R,M}(\text{capture}) = (70\% \times 100\%) \times 10\% = 7\%$$

*but then ...*

*neither of the above policies is an "equilibrium" since at least one player can improve its outcome by changing its policy*

# Routing game



Compute saddle-point equilibrium policies:

$R^* \in \mathcal{R}_{\text{no-cycle}}$  (cycle-free stochastic routing policy)

$M^* \in [0,1]^{\mathcal{P}}$  (mixed attack policy)

for which

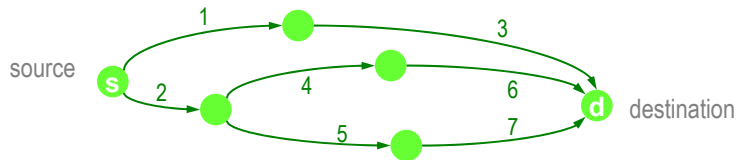
$$P_{R^*, M^*}(\text{capture}) = \min_{R \in \mathcal{R}_{\text{no-cycle}}} \max_{M \in [0,1]^{\mathcal{P}}} P_{R, M}(\text{capture})$$

$$= \max_{M \in [0,1]^{\mathcal{P}}} \min_{R \in \mathcal{R}_{\text{no-cycle}}} P_{R, M}(\text{capture})$$

Existence?  
Computation?

*policies chosen by intelligent opponents to minimize their worst-case losses  
(no player will improve its outcome by deviating from equilibrium)*

# Probability of capture



Given

$R \in \mathcal{R}_{\text{no-cycle}}$  (cycle-free stochastic routing policy)

$M := \{ m_p : P \in \mathcal{P} \} \in [0,1]^{\mathcal{P}}$  (mixed attack policy)

$$P_{R, M}(\text{capture}) = \sum_{P \in \mathcal{P}} m_P \text{row}[P] x_P$$

diagonal matrix with all the elements of  $R$

row vector with all the pure policies  $p_i$

unique solution to

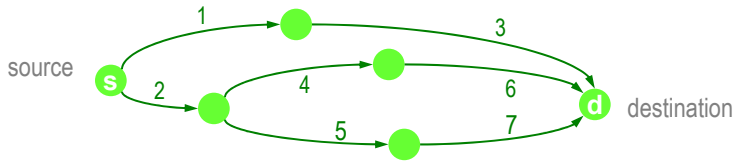
$$x_P = \text{diag}[R]A(I - \text{diag}[P])x_P + \text{diag}[R]c$$

(matrix  $A$  and vector  $c$  only depend on the graph)

*Linear (thus concave) in  $M$  (maximizer)  
but not convex with respect to the routing policy  $R$  (minimizer)  
so mini-max existence theorems do not apply...*



# Probability of capture



Under mild assumptions (\*) on pure attacks

Given  $R \in \mathcal{R}_{\text{no-cycle}}, M := \{m_p : P \in \mathcal{P}\} \in [0,1]^{\mathcal{P}}$

$$P_{R,M}(\text{capture}) = \left( \sum_{P \in \mathcal{P}} m_P \text{row}[P] \right) x$$

row vector with all the pure policies  $p_\ell$

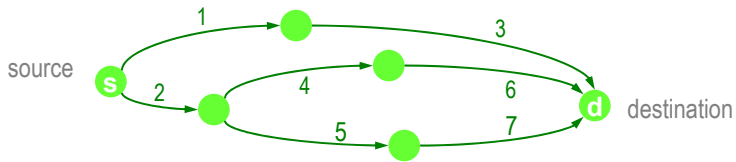
flow vector  $\equiv$  unique solution to

$$x = \text{diag}[R]Ax + \text{diag}[R]c$$

(matrix  $A$  and vector  $c$  only depend on the graph)

(\*) the same pure attack does not simultaneously targets two links in the same path  
(true for every single-link or single-node attacks)

# Probability of capture



Under mild assumptions (\*) on pure attacks

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(matrix  $A$  and vector  $c$  only depend on the graph)

*Not convex with respect to the routing policy  $R$  but linear (convex!) with respect to the vector  $x$ ...  
Key idea: solve game for  $x$  & then compute  $R$*

## Routing policies & Flow vectors

**Theorem:** i) There is a one-to-one correspondence between routing policies  $R$  in  $\mathcal{R}_{\text{stoch}}$  & flow vectors  $x$  in a convex set  $\mathcal{X} \subset \mathbb{R}^L$

ii) For cycle-free  $R \in \mathcal{R}_{\text{no-cycle}}$ , the corresponding flow vector  $x$  satisfies

$$x = \text{diag}[R]Ax + \text{diag}[R]c$$

Therefore

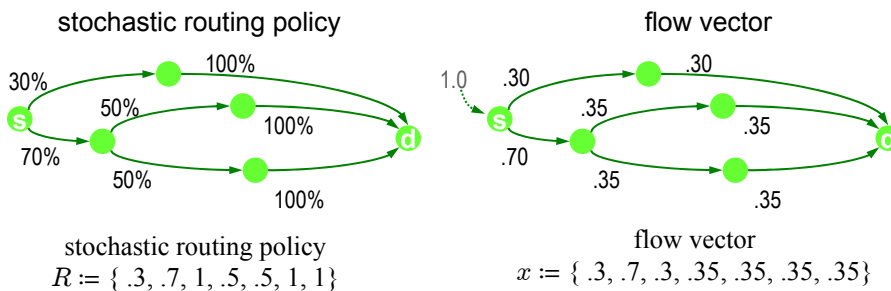
$$P_{R,M}(\text{capture}) = \sum_{P \in \mathcal{P}} m_P \text{row}[P]x$$

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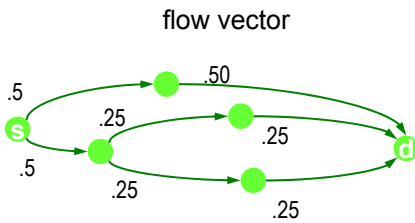
ii) For cycle-free  $R \in \mathcal{R}_{\text{no-cycle}}$ , the corresponding flow vector  $x$  satisfies

$$x = \text{diag}[R]Ax + \text{diag}[R]c$$



the vectors  $x \in \mathcal{X}$  obey a "flow conservation law" at every node, with total unit flow exiting the source node

# Flow game



Compute saddle-point:

$$x^* \in \mathcal{X} \quad (\text{flow vector})$$

$$M^* \in [0,1]^P \quad (\text{mixed attack policy})$$

for which

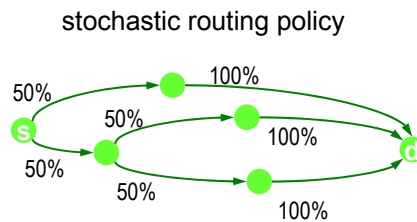
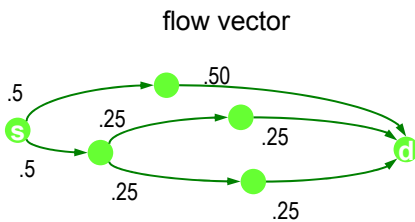
$$\sum_{P \in \mathcal{P}} m_P^* \text{row}[P]x^* = \min_{x \in \mathcal{X}} \max_{M \in [0,1]^P} \sum_{P \in \mathcal{P}} m_P \text{row}[P]x$$

$$= \max_{M \in [0,1]^P} \min_{x \in \mathcal{X}} \sum_{P \in \mathcal{P}} m_P \text{row}[P]x.$$

**Theorem:** Every flow game has a saddle-point  $(x^*, M^*)$  with  $x^*$  cycle-free

by bilinearity of the criterion and  
convexity and (almost) compactness of  $\mathcal{X}$  &  $[0,1]^P$

# Back to routing game...



**Theorem:** The routing game has saddle-point policies.

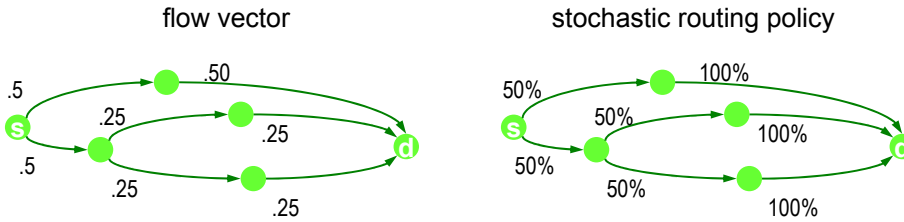
Moreover, for every saddle-point  $(x^*, M^*)$  of the flow game with  $x^*$  cycle-free, the pair  $(R^*, M^*)$  is a saddle-point of the routing game, with  $R^*$  constructed from  $x^*$ :

$$r_\ell^* := \frac{x_\ell^*}{\sum_{\ell' \in \mathcal{L}[\ell]} x_{\ell'}^*} \quad \forall \ell \in \mathcal{L}$$

summation over all links that exit  
from the same node as  $\ell$

*Solving the flow game actually solves the routing game...*

# Solution to the flow & routing games



**Theorem:** The value  $V^*$  of the flow game is given by

$$V^* = \min_{\substack{x \in \mathcal{X} \\ \text{row}[P]x \leq \mu, \forall P}} \mu$$

max-flow problem solvable by linear programming

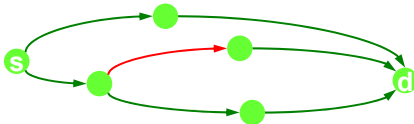
and the saddle-point  $x^*$  is any  $x$  at which the minimum is attained.

Optimal routing policy  $R^*$  can be computed using:

$$r_\ell^* := \frac{x_\ell^*}{\sum_{\ell' \in \mathcal{L}[\ell]} x_{\ell'}^*} \quad \forall \ell \in \mathcal{L}$$

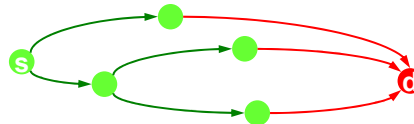
# Max-flow interpretations

for pure attacks at individual links



- Optimal routing minimizes the maximum link flow (subject to constraints that depend on the link reliability)
- In practice, maximizes throughput subject to link bandwidth constraints

for pure attacks at individual nodes



- Optimal routing minimizes the maximum node load (subject to constraints that depend on node reliability)
- In practice, balances the load between nodes (useful for energy-starved nodes)

## Several reasons to use multi-path routing

**increase security**

- Hespanha, Bohacek. Preliminary Results in Routing Games, 2001.
- Bohacek, Hespanha, Lee, Obraczka, Lim, Enhancing security via stochastic routing, 2002
- Papadimitratos, Haas, Secure message transmission in mobile ad hoc networks, 2003
- Lee, Misra, Rubenstein, Distributed Algorithms for Secure Multipath Routing, 2005

**improve robustness**

- Ganesan, Govindan, Shenker, Estrin, Highly Resilient, Energy Efficient Multipath Routing in Wireless Sensor Networks, 2002
- Wei, Zakhor, Robust Multipath Source Routing Protocol (RMPSR) for Video Communication over Wireless Ad Hoc Networks, 2004
- Tang, McKinley, A distributed multipath computation framework for overlay network applications, 2004

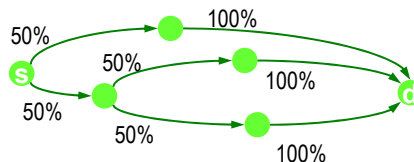
**increase throughput**

- Chen, Chan, Li, Multipath routing for video delivery over bandwidth-limited networks, 2004

**maximize network utilization**

- Elwalid, Jin, Low, Widjaja, MATE: MPLS adaptive traffic engineering, 2001
- Lee, Gerla, Split multipath routing with maximally disjoint paths in ad hoc networks, 2001
- Mirrokni, Thottan, Uzunalioglu, Paul, Simple polynomial time frameworks for reduced-path decomposition in multi-path routing, 2004

## Latency-aware stochastic routing



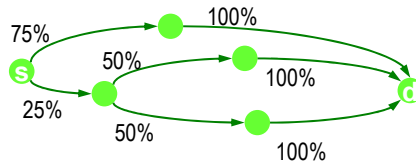
So far...

$$\text{Cost} = P(\text{capture})$$

probability that packet will be captured

*cost does not favor shorter paths*

# Latency-aware stochastic routing



So far...

$$\text{Cost} = \sum_{t=1}^{\infty} P(\text{capture at } t \text{ hop})$$

probability that packet will be captured

*cost does not favor shorter paths...*

One can bias routing towards shorter paths:

$$\text{Cost} = \sum_{t=1}^{\infty} (1 + \epsilon)^{t-1} P(\text{capture at } t \text{ hop}) \quad \epsilon > 0$$

more hops result in larger costs  
since  $(1 + \epsilon)^{t-1}$  increases with  $t$

larger  $\epsilon$  leads to larger penalty  
incurred by extra hop

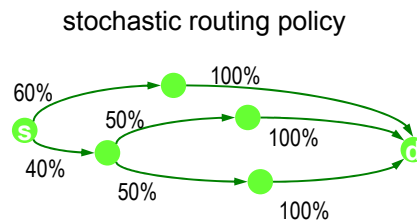
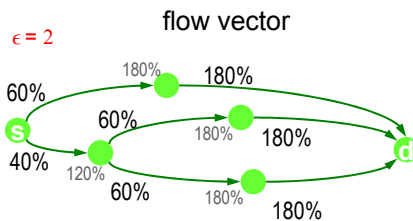
# Latency-aware routing game

$$\text{Cost} = \sum_{t=1}^{\infty} (1 + \epsilon)^{t-1} P(\text{capture at } t \text{ hop}) \quad \epsilon > 0$$

**Theorem** The latency-aware routing game has saddle-point policies.

These can be computed from the saddle-points of a flow game, where flow gets amplified by  $(1 + \epsilon)$  at every link.

still solvable by linear programming

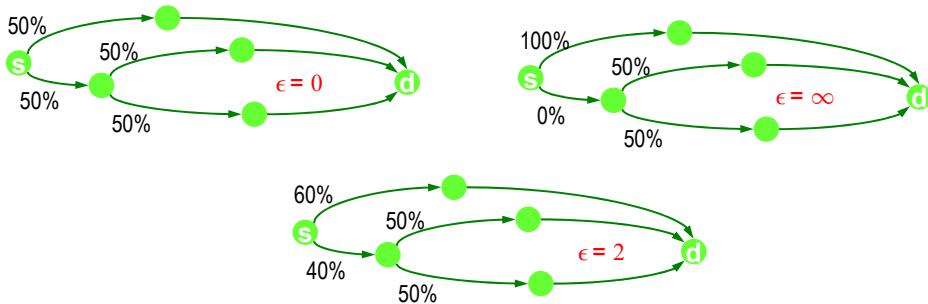


# Latency-aware routing game

$$\text{Cost} = \sum_{t=1}^{\infty} (1 + \epsilon)^{t-1} P(\text{capture at } t \text{ hop}) \quad \epsilon > 0$$

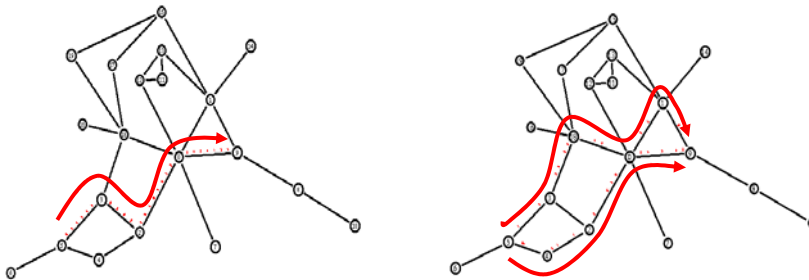
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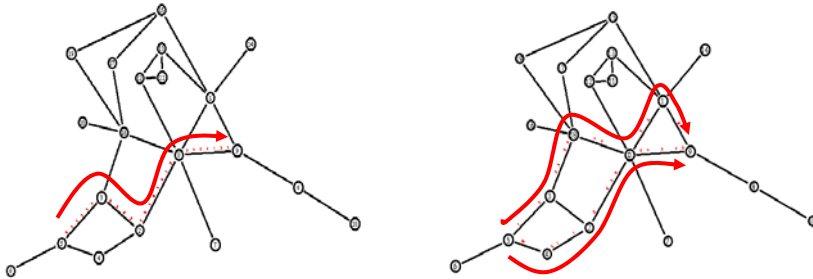
*As  $\epsilon \rightarrow \infty$ , saddle-point converges to minimum hop routing  
As  $\epsilon \rightarrow 0$ , saddle-point converges to max-flow routing*

# Outline



1. How to compute stochastic multi-path routing tables?
  - Noncooperative game—explore redundancy in an adversarial context
    - Different levels of security/reliability for distinct links/nodes
    - Reduce latency
2. Other issues:
  - Scalable computation for large networks
  - Multi-path routing within the protocol stack

## Computational complexity



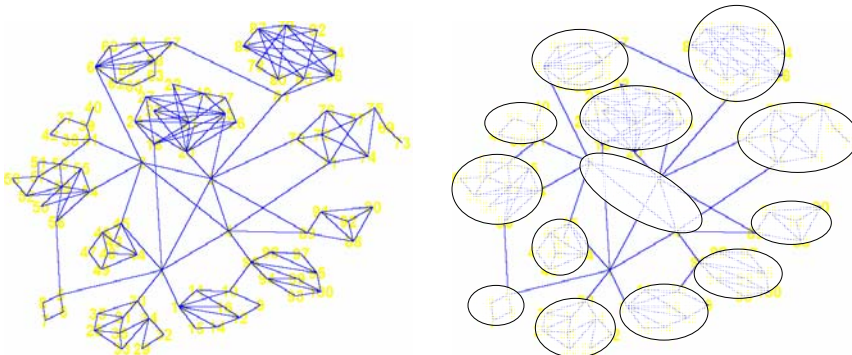
$n \equiv \#$  of nodes

$n^2 \equiv \#$  source/destination pairs

$O(n^5) \equiv$  max-flow routing total computation  
( Goldberg & Tarjan pre-push algorithm, assuming dense worst-case dense graph. For Internet-like graphs more like  $O(n^4 \log n)$  )

*How to improve this for large networks?*

## Hierarchical computation



1. Overall network graph is partitioned into several clusters of nodes (metanodes)
2. Routing is computed in two steps:

Option I: first inter- and then intra-metanode

S. Bohacek, JH, C. Lim, K. Obraczka. Hierarchical Max-Flow Routing. Feb. 2005. Submitted.

Option II: first intra- and then inter-metanode

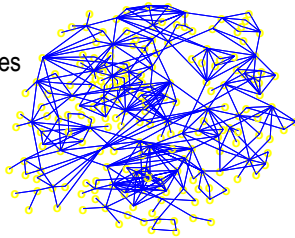
J. Riehl, JH. Fractal Graph Optimization Algorithms. Mar. 2005. Submitted.

*Either option reduces computation at the expense of a getting a suboptimal solution...*

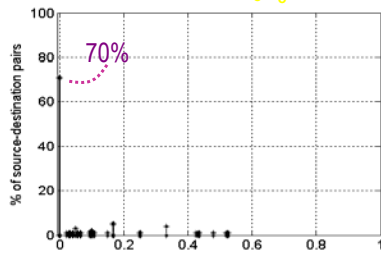
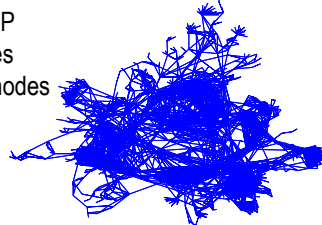


## Flat vs. hierarchical (option I)

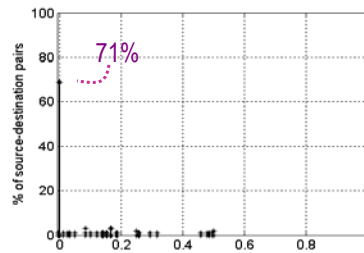
Exodus ISP  
200 nodes  
18 metanodes



Verios ISP  
960 nodes  
41 metanodes



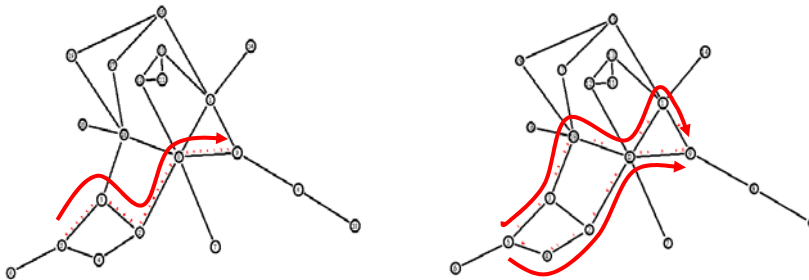
(topologies from Rocketfuel dataset)



increase in % of packets that go most "congested" link

*routing computation down to  $O(n^{3.2})$   
over 70% of the source/destination pairs see no difference in the  
maximum number of packets that go through any link (measure of fragility)*

## Outline

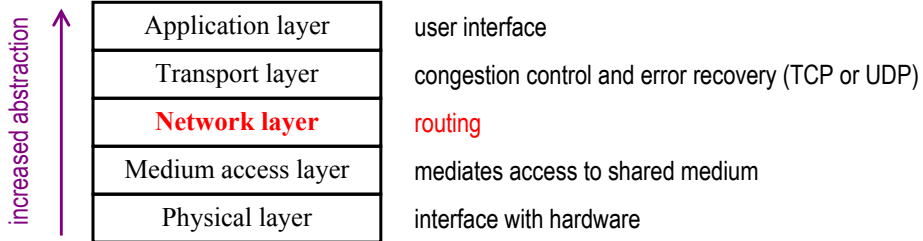


1. How to compute multi-path routing tables?
  - Explores redundancy in an adversarial context
  - Different levels of security/reliability for distinct links/nodes
2. Other issues:
  - Scalable computation for larger networks
  - Multi-path routing within the protocol stack

## Protocol stack



TCP/IP protocol stack:



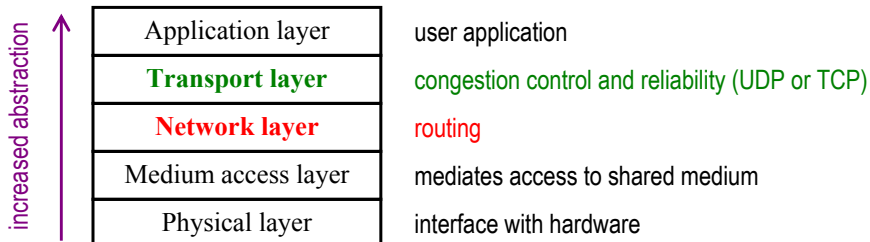
*Key principles behind layered architectures:*

1. Each layer provides a specific service to the layers above
2. The implementation details of one layer should be “transparent” to all layers above, as long as it provides its designated service.

## Multi-path routing & the protocol stack



TCP/IP protocol stack:



### **Transport layer**

*Assumes*

**network layer** forwards packets from source to destination, possibly losing and/or re-ordering some packets

*Guarantees*

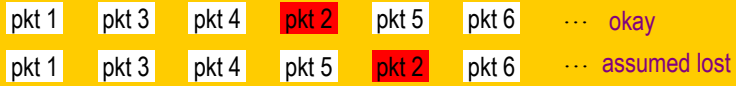
UDP – nothing!

TCP – network will not be overloaded with traffic (congestion control), all packets eventually arrive at dest. in the correct order (reliability)

# Multi-path routing & the protocol stack

However, most versions of TCP (NewReno, SACK) also assumes that...

- re-ordering never exceeds two packets



- all losses are due to congestion and therefore trigger a reduction in the sending rate
  - persistent re-ordering leads to very small sending rates

Assumes

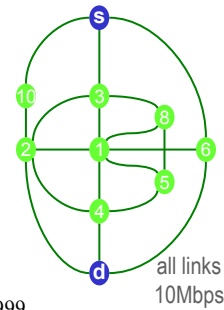
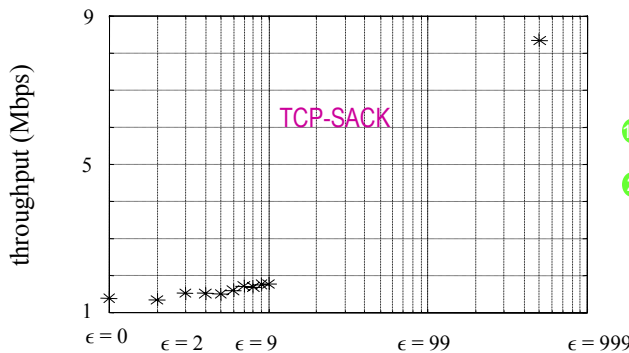
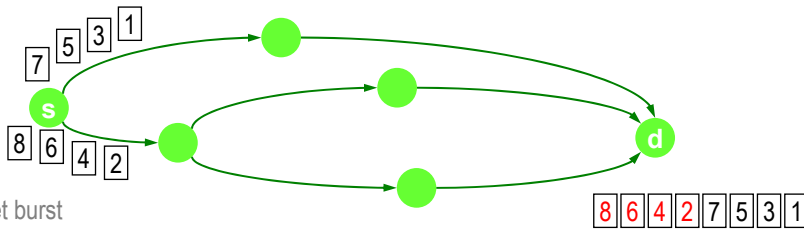
network layer forwards packets from source to destination, possibly losing and/or re-ordering some packets

Guarantees

UDP – nothing!

TCP – network will not be overloaded with traffic (congestion control), all packets eventually arrive at dest. in the correct order (reliability)

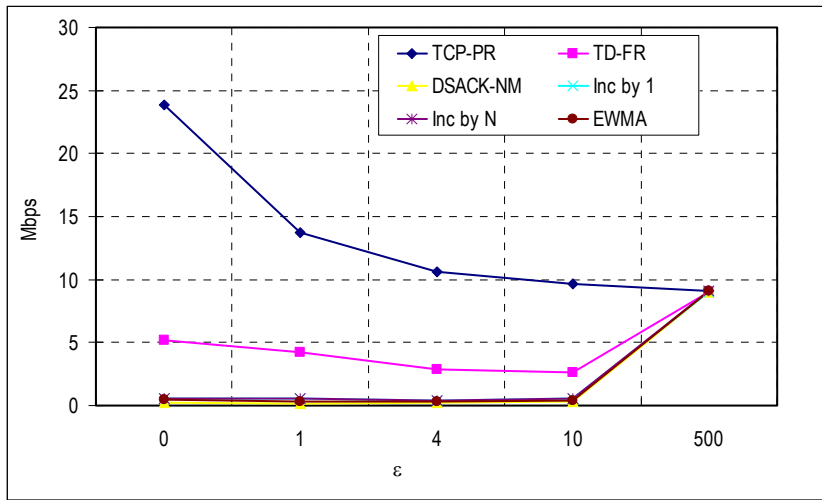
# Packet re-ordering in multi-path routing



max flow

single-path

## TCP versions for persistent re-ordering UCSB



TCP-PR [Bohacek, JH, Lee, Lim, Obraczka 03]  
 TD-FR [Paxson 97]  
 remaining (DSACK option) [Blanton, Allman 02]

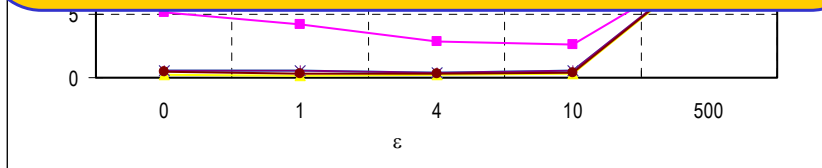
ordering not used at all to detect drops  
 does not use the DSACK option so  
 only needs changes in server side

## TCP versions for persistent re-ordering UCSB

However...

For real-time applications (such as control) one generally does not want to rely on a "general-purpose" reliable transport protocol like TCP (retransmits old data until arrival can be confirmed)

Instead, one probably wants to use UDP and develop protocols (at the application layer) that are more adequate for networked control systems...



TCP-PR [Bohacek, JH, Lee, Lim, Obraczka 03]  
 TD-FR [Paxson 97]  
 remaining (DSACK option) [Blanton, Allman 02]

ordering not used at all to detect drops  
 does use DSACK option  
 only needs changes in server side

## Conclusions



- **Communication networks** are extremely **vulnerable** components to critical systems
  - multitude of individual components, spatially distributed, difficult to protect
  - especially true for wireless networks (jamming, eavesdropping, battery drainage due to overuse, etc.)
- **Game theory** is a natural framework for network security
  - redundancy, by itself, will not solve the problem
  - attacks are not random events: very unlikely events can be prompted by an attacker
- Determined **routing policies** that are robust with respect to attacks
  - formulation as a **zero-sum game** between router and attacker
  - saddle-point solutions found by reducing problem to a flow-game (solvable by **linear programming**)
  - policies found also have applications to **throughput maximization** and **load balancing**
  - other formulations are possible (Markov games, leading to distance vector algorithms)
- **Other challenges**
  - **scalability** (addressable by hierarchical or distributed computation)
  - compatibility with **transport layer** (use newest TCP versions or UDP)

## References



### Stochastic routing

- S. Bohacek, J. Hespanha, K. Obraczka. Saddle Policies for Secure Routing in Communication Networks. In *Proc. 41st CDC*, Dec. 2002.
- S. Bohacek, J. Hespanha, J. Lee, C. Lim, K. Obraczka. Enhancing security via stochastic routing. In *Proc. of the 11th IEEE ICCCN*, May 2002.
- J. Hespanha, S. Bohacek. Preliminary Results in Routing Games. In *Proc. ACC*, June 2001.

### TCP under persistent packet re-ordering

- S. Bohacek, J. Hespanha, J. Lee, C. Lim, K. Obraczka. A New TCP for Persistent Packet Reordering-TCP-PR. Oct. 2003. Submitted to the *IEEE/ACM Trans. on Networking*.
- S. Bohacek, J. Hespanha, J. Lee, C. Lim, K. Obraczka. TCP-PR: TCP for Persistent Packet Reordering. In *Proc. of the IEEE ICDCS*, May 2003.

### Hierarchical routing

- S. Bohacek, J. Hespanha, C. Lim, K. Obraczka. Hierarchical Max-Flow Routing. Feb. 2005. Submitted.
- J. Riehl, J. Hespanha. Fractal Graph Optimization Algorithms. Mar. 2005. Submitted.

<http://www.ece.ucsb.edu/~hespanha>