Game Theory
Lecture #7

Focus of Lecture:

- Cost Sharing Games
- Reasonable Axioms
- Marginal Contribution
- Shapley Value

1 Recap

The last lecture introduced the general cost sharing problem. The central focus of that lecture was on the concept of the core, which identified all allocations (i.e., cost shares) such that strategic users would pursue the most efficient joint venture. It is well-known that the core may in fact be empty, so we began to pursue the question of whether there are interesting classes of cost sharing games that are guaranteed to have a non-empty core. To that end, we considered the framework of minimum spanning tree games which focuses on the problem of distributing the infrastructure costs incurred by connecting a set of customers to a common source. We proposed a mechanism for generating an allocation that is always in the core, hence proving that the core is non-empty in any minimum spanning tree game.

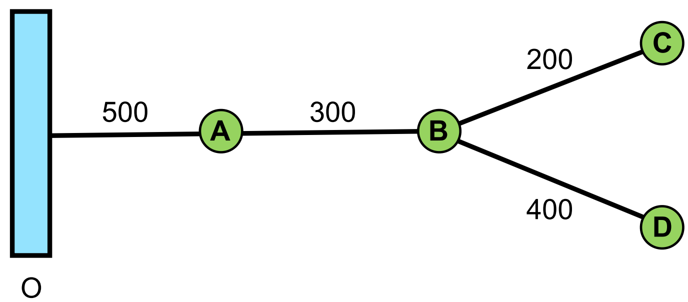
In this lecture, we shift focus to defining desirable properties of cost sharing mechanisms, and then investigating specific cost sharing mechanisms in light of these properties. This approach will be reminiscent of our study of social choice, in which we defined axioms which could be stated independently of any particular social choice rule. At a high level, a cost sharing mechanism is a methodology which specifies how to derive cost shares from a given cost sharing problem (N, c), as shown in the following figure.

The ultimate goal would be to establish a cost sharing mechanism that always produces an allocation in the core whenever the core is non-empty. That objective is unfortunately beyond our reach, and rather we will seek to design cost shares that are reasonable, fair and are in the core for certain classes of problems. To that end, we draw inspiration from the following example to establish the axioms that any reasonable cost sharing mechanism should satisfy.

2 The Decomposition Principle

In the following we consider a simple example of an electricity distribution network with decomposable costs.

Example 2.1 (Electricity Distribution Network) *Consider the problem of developing an electricity distribution network to meet the demands of four towns* $\{A, B, C, D\}$*. Power is generated at location* O *and the four towns are geographically distributed as highlighted in the following figure.*



Each town is required to connect to the source O *and the only viable power lines are between towns* (O, A)*,* (A, B)*,* (B, C)*, and* (B, D)*, each with an accompanying cost as highlighted. Each town can directly construct power lines connecting their town to the source, which would lead to the opportunity costs of* $c(\{A\}) = 500$*,* $c(\{B\}) = 500 + 300 = 800$*,* $c(\{C\}) = 500 + 300 + 200 = 1000$*, and* $c(\{D\}) = 500 + 300 + 400 = 1200$*. However, towns can also pursue joint ventures that utilize common power lines. Accordingly, towns* A *and* B *can build a distribution network that serves both towns for a cost* $c(\{A, B\}) = 500 + 300 = 800$ *or towns* A *and* D *can build a distribution network that serves both towns for a cost* $c(\{A, D\}) = 500 + 300 + 400 = 1200$*. Clearly, the most economically advantageous option in the above example is for all towns to pursue a joint venture with an opportunity cost* $c(\{A, B, C, D\}) = 500 + 300 + 200 + 400 = 1400$*. However, are there any allocations that promote such a venture? In other words, are there any allocations in the core?*

Last lecture we identified a specific allocation that was in the core for any minimum spanning tree game, which is related to the electricity distribution network scenario provided above. In particular, the specific allocation assigned to each individual $i \in N$ a cost share that is equal to the cost associated with the outgoing edge from i in the minimum cost spanning tree. For this specific example, these cost shares result in the following allocation:

$$\begin{aligned} CS(A, \{A, B, C, D\}) &= 500, \\ CS(B, \{A, B, C, D\}) &= 300, \\ CS(C, \{A, B, C, D\}) &= 200, \\ CS(D, \{A, B, C, D\}) &= 400. \end{aligned}$$

While this allocation is indeed in the core, it appears to lack a degree a fairness. To see this, consider A who is asked to pay the full cost of \$500 for the edge (A, O). The other towns

B, C, and D are then able to use this edge equally without paying any of the cost. Hence, it is hard to consider this allocation scheme fair for this particular problem. Another way to see this is that A has bargaining power in this interaction: since A could build a line all by themself for \$500, A could plausibly threaten to defect from the coalition in exchange for a discounted cost share. Are there other allocation in the core that exhibit a more equitable division of costs?

This simplified problem has a decomposable structure where the opportunity cost associated with a given collection of towns can be broken down by the costs associated with a collection of elements, i.e., the power lines, that are needed to meet the coalition's demands. We refer to these elements as cost elements, as the opportunity costs are defined as the sum of the costs of the elements needed to meet the coalition's demand. Accordingly, the following basic principles should hold for any reasonable cost sharing mechanism:

(i) Those who do not use a cost element, i.e., a power line segment, should not be charged for it.

(ii) Everyone who uses a given cost element equally should be charged equally for it, i.e., equal usage gives rise to equal charge.

(iii) The resulting allocation should be additive over the allocations associated with each cost element.

Given these guiding principles any reasonable cost sharing rule should yield the following allocation for the above joint venture involving all towns $\{A, B, C, D\}$ are of the form

$$\begin{aligned} CS(A, \{A, B, C, D\}) &= 500/4, \\ CS(B, \{A, B, C, D\}) &= 500/4 + 300/3, \\ CS(C, \{A, B, C, D\}) &= 500/4 + 300/3 + 200, \\ CS(D, \{A, B, C, D\}) &= 500/4 + 300/3 + 400. \end{aligned}$$

It is straightforward to verify that this is a valid cost sharing rule and also yields an allocation in the core. Furthermore, the cost shares associated with other ventures could be derived in a similar fashion. These guiding principles are known as the *Decomposition principle*.

3 An axiomatic view of reasonable cost sharing

The previous section highlights some guiding principles for the design of a cost sharing rule $CS(\cdot)$ from a given opportunity cost function $c(\cdot)$. In general, our goal is to find a mechanism for determining a cost sharing mechanism $CS(\cdot)$ that always provides an allocation in the core. Since that is not necessarily possible, our goal is rather to find a cost sharing mechanism that meets the criteria set forth in the decomposition principles. While the decomposition principle was motivated with regards to a simple problem with decomposable elements, here we characterize these guiding principles for any cost sharing problem with a given opportunity cost function $c : 2^N \to \mathbb{R}$. Recall the definition of a cost sharing rule:

Definition 3.1 (Cost Sharing Rule) *A cost sharing rule is a function* $CS : 2^N \to \mathbb{R}^n$ *that allocates the total cost of a venture among the members of a group for every possible group of players* $S \subseteq N$*, i.e., for any set of players* $S \subseteq N$ *the cost sharing rule satisfies*

$$\sum_{i \in S} CS(i, S) = c(S)$$

where $CS(i, S)$ *represents the cost share of player* i *in group* S*. We refer to the cost shares of all players* $i \in S$ *as an allocation.*

Our first guiding principle is that those who do not use a cost element should not be charged for it. We capture this ideology in our first property, termed the *Dummy Property*, which states that if an individual contributes no additional cost to any coalition (set of individuals), then the individual is charged nothing. A formal statement of this property is as follows:

Property #1 (Dummy) *If for any coalition* $S \subseteq N$ *the opportunity costs satisfy* $c(S) = c(S \cup \{i\})$*, then for any coalition* S *such that* $i \in S$ *the cost sharing rule satisfies*

$$CS(i, S) = 0. \tag{1}$$

Consider the opportunity costs associated with the power line example depicted above. Here, we can decompose the opportunity costs $c : 2^N \to \mathbb{R}$ as the sum of the opportunity costs for each edge, e.g., $c(\cdot) = c_{0A}(\cdot) + c_{AB}(\cdot) + c_{BC}(\cdot) + c_{BD}(\cdot)$. Focusing on the structure of the opportunity costs c_{AB}, we have $c_{AB}(\emptyset) = c_{AB}(\{A\}) = 0$ as town A does not requires power line (A, B) to meet its demand. Further, all coalitions $S \subseteq N$ where $|S| \geq 1$ and $S \neq \{A\}$ satisfies $c_{AB}(S) = 300$ since any such coalition S requires the power line (A, B) and must absorb the cost of 300. Given this definition, it is straightforward to show that town A is a dummy as $c_{AB}(S) = c_{AB}(S \cup \{A\})$ for any $S \subseteq N$. For example, if $S = \emptyset$, we have

$$c_{AB}(S \cup \{A\}) - c_{AB}(S) = c_{AB}(\{A\}) - c_{AB}(\emptyset) = 0 - 0 = 0.$$

Hence, this property requires that for any coalition $S \subseteq N$ such that $A \in S$, we have $CS_{AB}(A, S) = 0$.

Our second guiding principle is that everyone who uses a given cost element equally should be charged equally for it, i.e., equal usage gives rise to equal charge. Our second property, termed *Symmetry*, captures this idea and states that if two individuals enter symmetrically into the cost function, then they are charged equally. A formal statement of this property is as follows:

Property #2 (Symmetry) *Let* i, j *be any two individuals. If for any coalition* $S \subseteq N \setminus \{i, j\}$ *the opportunity costs satisfy* $c(S \cup \{i\}) = c(S \cup \{j\})$*, then for any coalition* T *such that* $i, j \in T$ *the cost sharing rule satisfies*

$$CS(i, T) = CS(j, T). \tag{2}$$

Once again, consider the opportunity costs $c_{AB}(\cdot)$ associated with power line (A, B) as defined above. Note that any two towns in the set $\{B, C, D\}$ satisfy the symmetry property above. For example, consider towns B and C, and any set $S \subseteq \{A, D\}$. By definition, we have that

$$c(S \cup \{B\}) = c(S \cup \{C\}) = 300.$$

Hence, if our cost sharing rule satisfies Symmetry, then for any coalition T such that $B, C \in T$, we have that

$$CS(B, T) = CS(C, T).$$

Our third guiding principle is that the resulting allocation should be additive over the allocations associated with each cost element. Our third and final property, termed *Additivity*, states that our cost sharing rule should be linear. A formal statement of this property is as follows:

Property #3 (Additivity) *If the opportunity cost function $c(\cdot)$ decomposes into the sum of two function $c'(\cdot)$ and $c''(\cdot)$, i.e., $c(S) = c'(S) + c''(S)$ for every coalition $S \subseteq N$, then the cost allocation derived for c is precisely the sum of the cost allocations derived for c' and c''. That is, for any coalition $S \subseteq N$ and individual $i \in S$ we have*

$$CS(i, S; c) = CS(i, S; c') + CS(i, S; c'') \tag{3}$$

where we write the notation $CS(i, S; c)$ to denote the cost shares derived for the cost sharing problem with opportunity costs $c(\cdot)$.

Once again, consider the power transmission problem described above. Recall that the opportunity costs are decomposable, i.e., for any coalition $S \subseteq N$ we have that

$$c(S) = c_{OA}(S) + c_{AB}(S) + c_{BC}(S) + c_{BD}(S)$$

where $c_x(\cdot)$ is the opportunity costs associated with power line x. Accordingly, if the cost sharing rule satisfies the additivity property then for any coalition $S \subseteq N$ and any individual $i \in S$ we have that

$$CS(i, S; c) = CS(i, S; c_{OA}) + CS(i, S; c_{AB}) + CS(i, S; c_{BC}) + CS(i, S; c_{BD}).$$

4 Cost Sharing Mechanisms

In this section we explore several specific cost sharing mechanisms to identify whether they satisfy Properties #1-3. We will then focus on whether or not these mechanisms provide allocations in the core. In the following, we will assume that N and $c : 2^N \to \mathbb{R}$ are given and will focus directly on the derivation of the cost shares $CS(\cdot)$.

4.1 Equal Share

The first cost sharing mechanism that we consider is equal share, where for each coalition $S \subseteq N$ and individual $i \in S$ we have

$$CS^{\mathrm{ES}}(i, S) = \frac{c(S)}{|S|}. \tag{4}$$

The equal share mechanism is clearly a valid cost sharing rule. Furthermore, it is straightforward to show that this mechanism satisfies both our Symmetry and Additivity Properties. However, it unfortunately fails our Dummy Property; this is easy to see on the power line example in the previous section, which (for example) would charge A for part of the cost of every line section.

4.2 Marginal Contribution

The second cost sharing mechanism that we consider is marginal contribution, where for each coalition $S \subseteq N$ and individual $i \in S$ we have

$$CS^{\mathrm{MC}}(i, S) = c(S) - c(S \setminus \{i\}). \tag{5}$$

The marginal cost mechanism assigns each individual a cost that is equal to their marginal contribution ot the coalition S, i.e., the additional cost required for individual i to join the coalition $S \setminus \{i\}$. Recall the three town example in the previous lecture; we restate it here with new opportunity costs

$$\begin{array}{ccc} c(\{A\}) = 11, & c(\{B\}) = 7, & c(\{C\}) = 8, \\ c(\{A,B\}) = 15, & c(\{A,C\}) = 13, & c(\{B,C\}) = 10, \\ & c(\{A,B,C\}) = 20. & \end{array}$$

Here, the marginal contribution cost mechanism results in the following cost shares

$$\begin{aligned} CS^{\mathrm{MC}}(A, \{A\}) &= c(\{A\}) - c(\emptyset) = 11 - 0 = 11, \\ CS^{\mathrm{MC}}(A, \{A,B\}) &= c(\{A,B\}) - c(\{B\}) = 15 - 7 = 8, \\ CS^{\mathrm{MC}}(A, \{A,C\}) &= c(\{A,C\}) - c(\{C\}) = 15 - 8 = 7, \\ CS^{\mathrm{MC}}(A, \{A,B,C\}) &= c(\{A,B,C\}) - c(\{B,C\}) = 20 - 10 = 10, \\ CS^{\mathrm{MC}}(B, \{A,B,C\}) &= c(\{A,B,C\}) - c(\{A,C\}) = 20 - 13 = 7, \\ CS^{\mathrm{MC}}(C, \{A,B,C\}) &= c(\{A,B,C\}) - c(\{A,B\}) = 20 - 15 = 5, \end{aligned}$$

and the others can be derived in a similar fashion. It is straightforward to show that the marginal contribution mechanism satisfies Properties #1-3 (and this will be covered in the homework). However, note that it is not necessarily a valid cost sharing mechanism as $c(S)$ may not equal $\sum_{i \in S} CS^{\mathrm{MC}}(i, S)$ for all coalitions $S \subseteq N$. As an example, note that

$$CS^{\mathrm{MC}}(A, \{A,B,C\}) + CS^{\mathrm{MC}}(B, \{A,B,C\}) + CS^{\mathrm{MC}}(C, \{A,B,C\}) = 10 + 7 + 5 = 22$$

while $c(\{A,B,C\}) = 20$.

4.3 Shapley Value

While the equal share mechanism was straightforward and simple, it unfortunately did not satisfy Property #1: Dummy. In turn, we proposed the marginal cost sharing mechanism to remedy this but unfortunately this mechanism was no longer a valid cost sharing rule. In this section we propose the Shapley value cost sharing mechanism, where for each coalition $S \subseteq N$ and individual $i \in S$ we have

$$CS^{\mathrm{SV}}(i,S) = \sum_{T \subseteq S \setminus \{i\}} \frac{|T|!(|S|-|T|-1)!}{|S|!} \left(c(T \cup \{i\}) - c(T)\right). \tag{6}$$

While the Shapley value computation looks complicated, the general idea surrounding the structure of (6) is relatively simple. The marginal contribution rule assigns each individual a cost share that is equal to the marginal contribution to the coalition of interest. The Shapley value, on the other hand, assigns each individual a cost share that is equal to the that individual's *average* marginal contribution to all sub-coalitions of the coalition of interest.

Consider the three town example discussed previously. The Shapley value computation considered in (6) takes on the form

$$\begin{aligned} CS^{\mathrm{SV}}(A,\{A,B,C\}) = {} & w_{BC}\left(c(\{A,B,C\})) - c(\{B,C\})\right) \\ & + w_B\left(c(\{A,B\}) - c(\{B\})\right) \\ & + w_C\left(c(\{A,C\}) - c(\{C\})\right) \\ & + w_\emptyset\left(c(\{A\}) - c(\{\emptyset\})\right) \end{aligned}$$

where $w_{BC}, w_B, w_C, w_\emptyset$ are the appropriate weights for each sub-coalition. For example, the weight on sub-coalition B from (6) would be

$$w_B = \frac{|\{B\}|!(|\{A,B,C\}| - |\{B\}| - 1)!}{|\{A,B,C\}|!} = \frac{1! \cdot 1!}{3!} = \frac{1}{6}.$$

These weights could have been derived from the standpoint of the different ordered lists associated with the towns. For our three town example, there are $3! = 6$ orders of the towns, i.e.,

$$\begin{aligned} & A \leftarrow B \leftarrow C \\ & A \leftarrow C \leftarrow B \\ & B \leftarrow A \leftarrow C \\ & C \leftarrow A \leftarrow B \\ & B \leftarrow C \leftarrow A \\ & C \leftarrow B \leftarrow A \end{aligned}$$

Further, one can define the incremental marginal cost of each town relative to a given order as the marginal contribution to the coalition of individuals that came before in the given order. Focusing on town A, the incremental marginal cost relative to each of these six orders is given by:

$$\begin{aligned}
A \leftarrow B \leftarrow C &\Rightarrow c(\{A\}) - c(\emptyset) = 11 \\
A \leftarrow C \leftarrow B &\Rightarrow c(\{A\}) - c(\emptyset) = 11 \\
B \leftarrow A \leftarrow C &\Rightarrow c(\{A,B\}) - c(\{B\}) = 15 - 7 = 8 \\
C \leftarrow A \leftarrow B &\Rightarrow c(\{A,C\}) - c(\{C\}) = 13 - 8 = 5 \\
B \leftarrow C \leftarrow A &\Rightarrow c(\{A,B,C\}) - c(\{B,C\}) = 20 - 10 = 10 \\
C \leftarrow B \leftarrow A &\Rightarrow c(\{A,B,C\}) - c(\{B,C\}) = 20 - 10 = 10
\end{aligned}$$

The weights defined above in (6) correspond to the proportion of orderings that give rise to an incremental marginal cost that equals that marginal cost of the sub-coalition. Accordingly, we have that $w_{BC} = 2/6, w_B = 1/6, w_C = 1/6, w_\emptyset = 2/6$. Hence, the Shapley value computation in (6) can be viewed as the average incremental marginal contribution over all potential orderings, i.e.,

$$CS^{\text{SV}}(A, \{A,B,C\}) = \frac{11 + 11 + 8 + 5 + 10 + 10}{6} = \frac{55}{6}.$$

Computing the Shapley value using the definition in (6) or using incremental marginal costs will provide exactly the same answer.[1]

The Shapley value is clearly more complicated than either the equal share or marginal contribution cost sharing mechanism, and hence its analysis is also more complicated. Note though the Shapley value is in fact a valid cost sharing mechanism, i.e., for any coalition $S \subseteq N$ we have

$$\sum_{i \in S} CS^{\text{SV}}(i, S) = c(S).$$

To see this, consider the three town example once again where the the incremental costs of each town to a given order is of the form:

$$A \leftarrow B \leftarrow C \;\Rightarrow\; \begin{cases} CS(A, \{A,B,C\}) = c(\{A\}) - c(\emptyset) \\ CS(B, \{A,B,C\}) = c(\{A,B\}) - c(\{A\}) \\ CS(C, \{A,B,C\}) = c(\{A,B,C\}) - c(\{A,B\}) \end{cases}$$

which satisfies

$$CS(A, \{A,B,C\}) + CS(B, \{A,B,C\}) + CS(C, \{A,B,C\}) = c(\{A,B,C\})$$

as all the intermediate terms, e.g., $c(\{A\})$ and $c(\{A,B\})$ cancel out. Since the Shapley value is defined as the average incremental marginal costs of all potential orderings this completes the result.

Recall that our goal is to find a cost sharing rule that satisfies our three properties: Dummy, Symmetry, and Additivity. The following Theorem demonstrates that one needs to look no further than the Shapley value.

[1] When implementing this, beware the algorithmic considerations. It may be tempting to simply generate all orderings (permutations) for a given coalition and compute the Shapley value as the average of the resulting incremental marginal costs, but this will result in an algorithm with a factorial runtime. Using (6) directly will result in "only" exponential runtime.

Theorem 4.1 (Shapley, 1953) *The Shapley value is the unique cost sharing rule that satisfies the Dummy, Symmetry, and Additivity Properties.*

The above theorem demonstrates that the Shapley value is the only cost sharing mechanism that satisfies our desired three desired properties, which is quite surprising. Accordingly, if our goal is to find a cost sharing mechanism that satisfies our three properties and is in the core, we only need to check whether or not the Shapley value gives us an allocation in the core. Thankfully, there are several classes of cost sharing problems, i.e., structures of opportunity costs, for which there are such positive results. One such example is as follows:

Proposition 4.1 *If the cost function decomposes into distinct cost elements (like the example on electricity distribution networks), then the Shapley value produces an allocation in the core.*

Interestingly, a similar conclusion does not hold true for minimum spanning tree games as the Shapley value does not necessarily provide an allocation in the core, even though the core is non-empty which we showed last lecture. This implies that the proposed cost sharing mechanism for minimum spanning tree games must fail one of our three properties. Which one?

5 Conclusion

The past two lectures focused on the important problem of cost sharing. That is, how should one divide the costs associated with a joint venture to incentivize participation in the grand coalition? Last lecture focused on the solution concept of the core, and whether or not the core was non-empty. In this lecture we focused more directly on deriving cost sharing mechanisms that yield desirable outcomes. Rather than focus on the core as a desirable outcome, we instead identified three properties that any reasonable cost sharing mechanism should satisfy: Dummy, Symmetry, and Additivity. We then showed that the Shapley value is the unique cost sharing mechanism that satisfies all three properties. Further, we identified some important classes of cost sharing problems where the Shapley value mechanism provides an allocation in the core (in addition to satisfying our three reasonable properties).

The results covered in these lectures can be thought of in both a negative and positive light. First, it is worth reiterating that the Shapley value is the only cost sharing mechanism that satisfies our three desired properties, which is a very tidy result from a characterization standpoint. On the negative side, computing the Shapley value in general is computationally prohibitive which implies that meeting these desired conditions is hard and requires substantial (and potentially prohibitive) computational effort.

6 Exercises

1. Consider the following cost sharing problem from the previous set of exercises:

- Player set: $N = \{1, 2, 3\}$
- Opportunity costs: $c : 2^N \to R$

$$\begin{array}{rrr} c(\{1\}) = 9, & c(\{2\}) = 8, & c(\{3\}) = 9 \\ c(\{1,2\}) = 14, & c(\{1,3\}) = 15, & c(\{2,3\}) = 13 \\ c(\{1,2,3\}) = 21 & & c(\emptyset) = 0 \end{array}$$

(a) Compute the marginal contribution for each player to the grand coalition $\{1, 2, 3\}$.

(b) Compute the Shapley value for each player to the grand coalition $\{1, 2, 3\}$ using equation (6).

(c) Compute the Shapley value for each player to the grand coalition $\{1, 2, 3\}$ using the ordering approach.

(d) Verify approaches in (b) and (c) result in same answer.

2. Recall Problem 3 from the previous lecture where a scientist has been invited for consultation at three distant cities. Building upon your results from the previous exercises, compute the Shapley value for each host. How much is the trip going to cost and how much should each host contribute to the travel expenses?

3. Show that the marginal contribution mechanism satisfies Property #1: Dummy, Property #2: Symmetry, and Property #3: Additivity.

4. The Shapley value can also be used to gauge a player's index of power in a weighted majority game. We define a weighted majority game by the following:

   - Set of players: $N = \{1, 2, \dots, n\}$
   - Positive weight for each player $i \in N$: $w_i > 0$
   - Quota: $q > 0$

   Here, we assume that q is greater than half the sum of the weights and less than or equal to the sum of the weights. A coalition is called a winning coalition in such a game if the sum of the weights of its members is $\geq q$. Otherwise, the coalition is called a losing coalition. Accordingly, the valuation function satisfies

$$v(S) = \begin{cases} 1 & \text{if } \sum_{i \in S} w_i \geq q \\ 0 & \text{otherwise} \end{cases}$$

   The electoral college is effectively a weighted majority game.

   (a) Consider a four player majority game where $w_1 = 3$, $w_2 = w_3 = w_4 = 1$, and $q = 3.1$. What is the Shapley value for each player to the coalition N? This quantity is known as the Shapley-Shubik power index. Does this power index agree with our intuition that the power index of an individual is aligned with the individual's fraction of weight?

(b) Consider a three player majority game where $w_1 = 7$, $w_2 = 1$, $w_3 = 7$, and $q = 8$. What is the Shapley-Shubik power index for the three players? Are these results surprising?

5. In the minimum spanning tree games presented in the previous lecture, the Shapley value does not necessarily provide an allocation in the core, even though the core is always non-empty. This implies that the proposed cost sharing mechanism for minimum spanning tree games must fail at least one of our three properties.

   (a) Provide an example graph for which the Shapley value does not provide an allocation in the core. (hint: try to do this using as few nodes as possible!)

   (b) Identify at least 1 of the 3 properties that the proposed cost sharing mechanism fails.