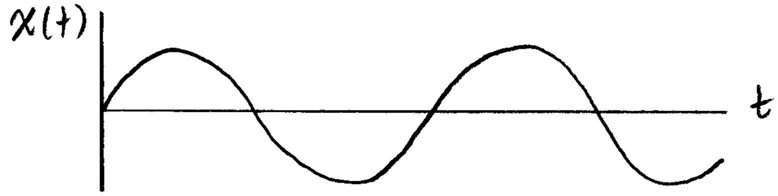
Periodic sampling as a frequency conversion method

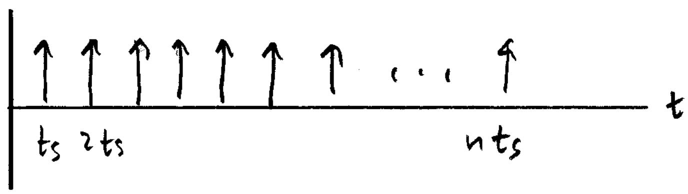
Reference: R. Lyons, Understanding DSP, Second Ed., Prentice-Hall, 2004.
(on RBS website)

Objective:

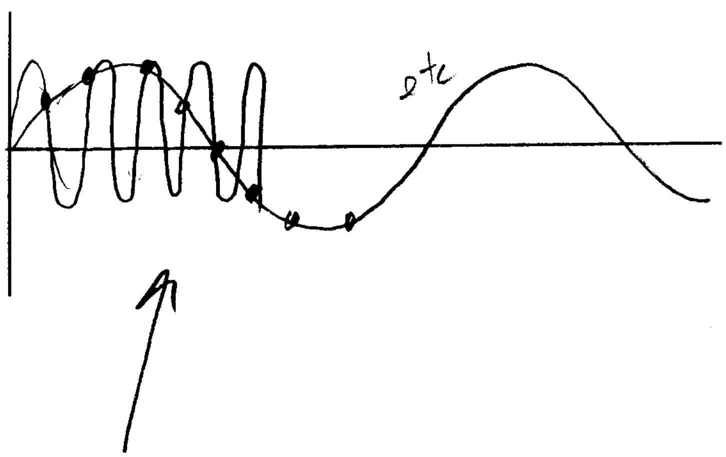
Preserve information content
shift frequency up or down

Consider a single sinusoidal signal $x(t) = \sin(2\pi f_0 t)$:





$$y(t) = x(t)s(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t - kt_s) = y(kt_s)$$

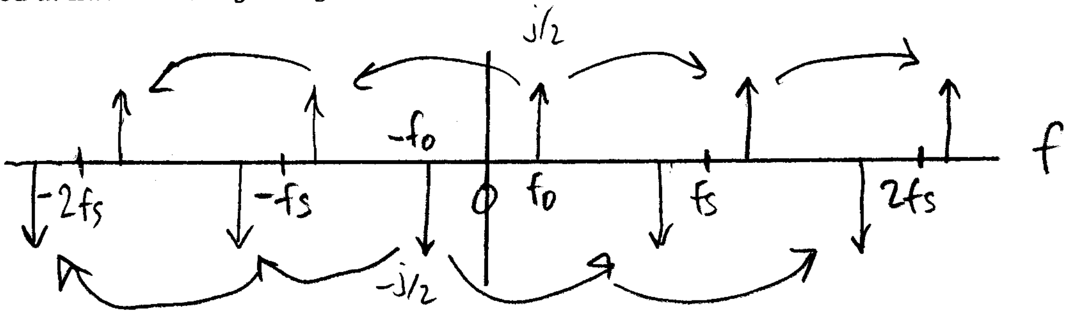


BUT: There are an infinite number of frequencies that could give the same sample set.

$$x(n) = \sin(2\pi f_0 n t_s) = \sin(2\pi (f_0 + k f_s) n t_s)$$

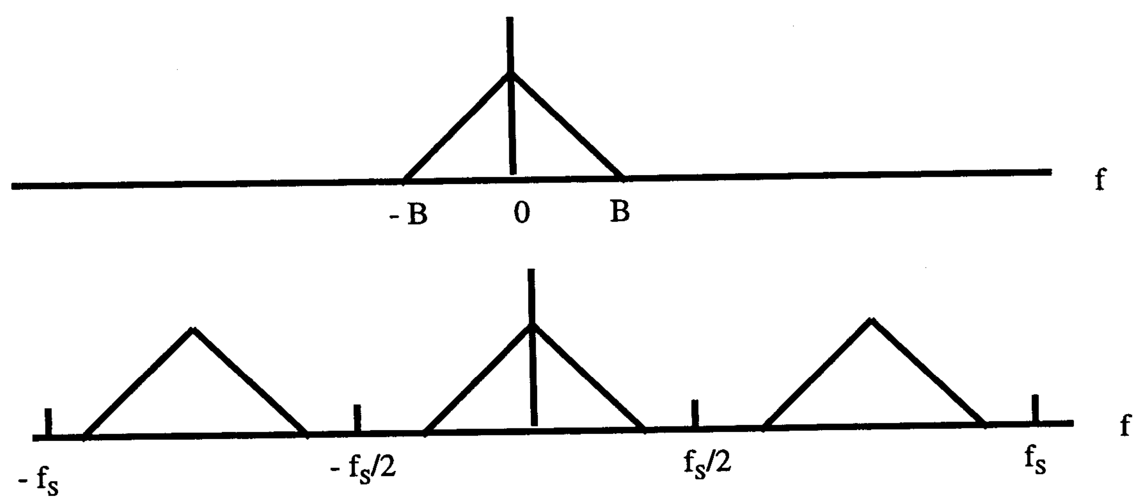
for any positive or negative integer k.

Therefore, the frequency spectrum consists of replicas of the frequency spectrum of x(t) spaced at intervals of $f_S = 1/t_S$.



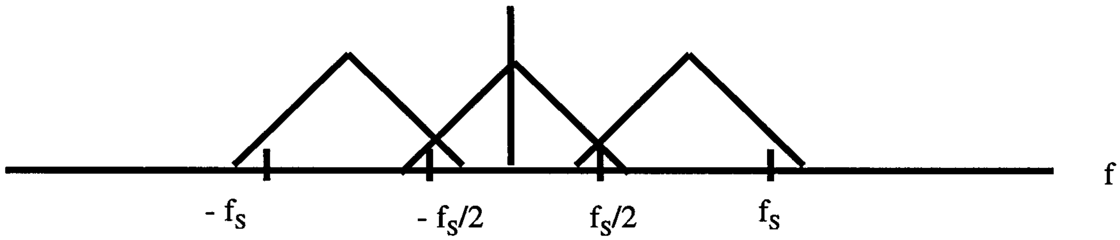
$$Y(f) = X(f) * \frac{1}{t_s} \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{t_s})$$

Example: Low Pass Sampling. Take a baseband continuous spectrum, bandlimited between –B and B, and sample it at frequency f_S.

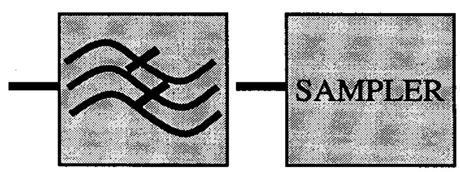


Note that the sampled signals fold around + or – $nf_S/2$. So, $f_s \geq 2B$ is required to avoid aliasing. This is the Nyquist criterion.

If we reduce sampling rate below Nyquist, we get spectral overlap that corrupts the information – cannot be undone.

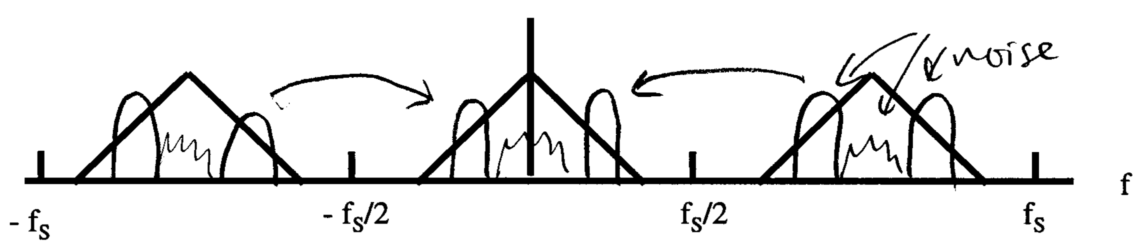


Thus, an analog anti-aliasing filter will be needed at the input of the sampler to strictly band-limit signals to $-\frac{f_s}{2} \leq f \leq \frac{f_s}{2}$.

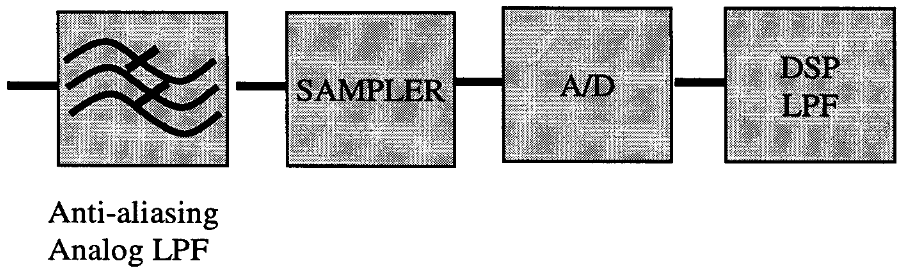


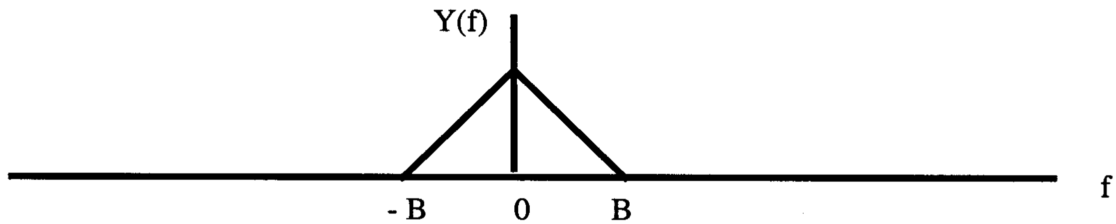
Anti-aliasing
Analog LPF

This is also needed to prevent out-of-band spectral content (noise, for example) from being aliased into the desired $-B \leq f \leq B$ frequency range. Note that sampling reflects signal about $\pm\frac{nf_s}{2}$.



To extract just one baseband spectrum, the output of the sampler must also be low-pass filtered. This is often done after A/D conversion with a digital filter.

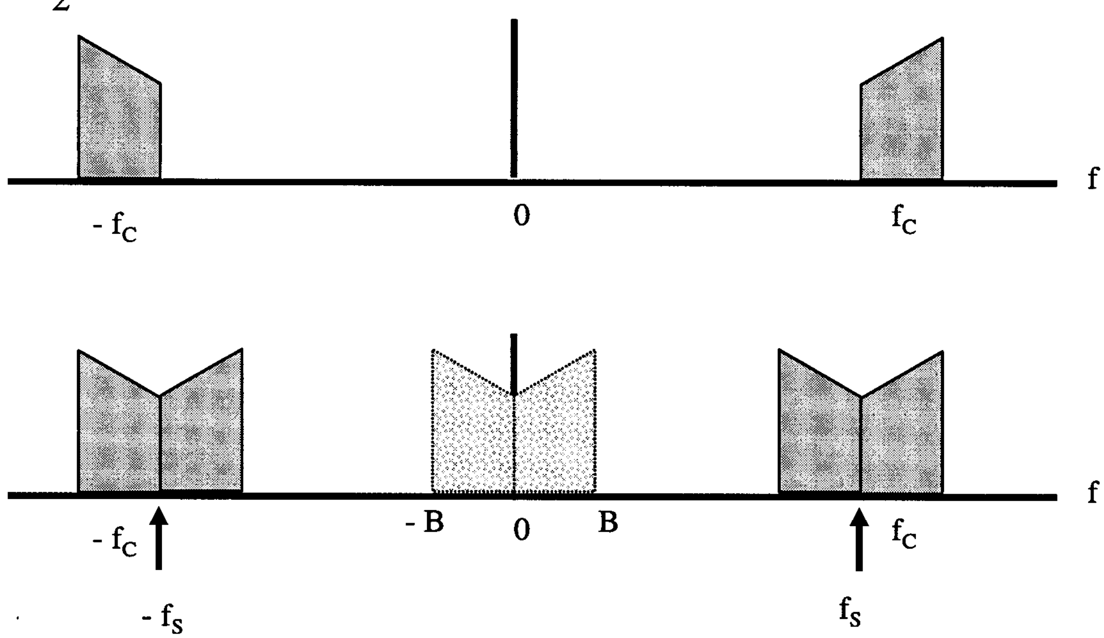




Anti-aliased and low-pass filtered sampled output spectrum.

Bandpass Sampling:

Just as sampling a baseband lowpass continuous time input generates replicas about $\pm\frac{nf_s}{2}$, we can sample a signal at $\pm nf_s$ and obtain a baseband replica.



Note that we are "under-sampling". That is, $f_s < 2f_c$. We still can avoid aliasing if we are careful in our choice of f_S. We are using the spectral replication for frequency translation. If we choose f_S incorrectly, we can get folding and corrupted signals.

In the example above, the positive and negative replications of f_C just adjoined at f = 0. In general, for this to occur with m replications,

$$mf = 2f_c - B$$

$$or$$

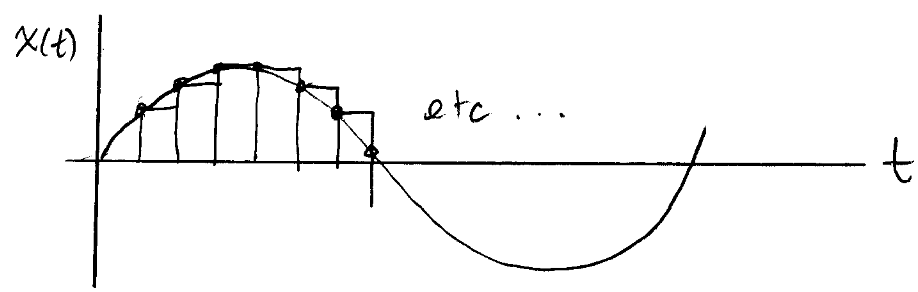
$$f_s = \frac{2f_c - B}{m}$$

In our example, there were two replications, so m = 2. There are minimum and maximum f_S values for a given m, f_C and B that will avoid overlap and aliasing

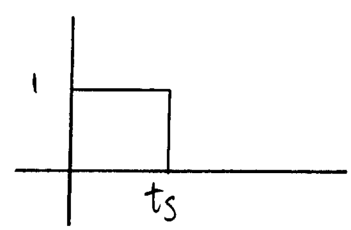
Practical issues: How can we generate an impulse train at time interval tS?

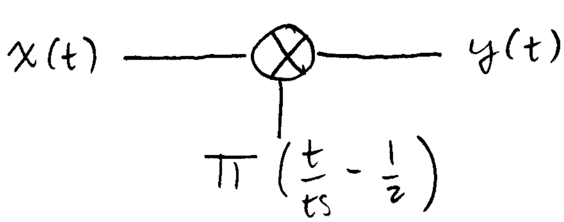
Answer: We don't! Instead use zero order hold, or sample-hold, or track-hold functions.

Hold Function: Capture the sampled value – generally with a capacitor, right after the sampling switch opens. Hold this value until the next sample.



We have a rectangle function $\Pi\left(\frac{t}{t_s} - \frac{1}{2}\right) = u(t) - u(t - t_s)$





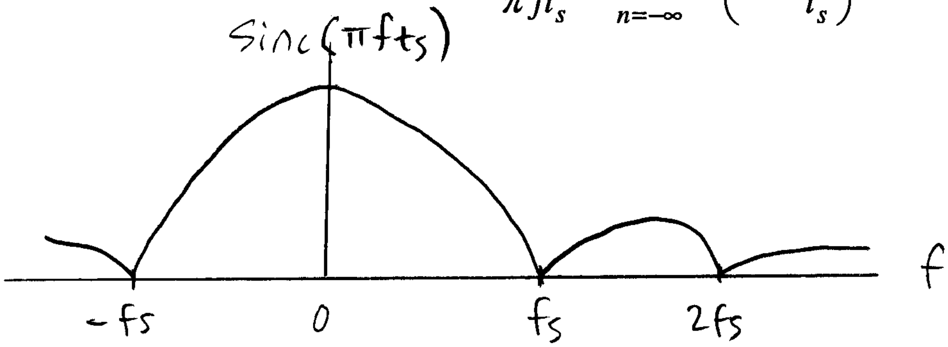
This has the effect of taking the periodic train of impulses and convolving them with the rectangle:

$$y(t) = \left[x(t) \sum_{k=-\infty}^{\infty} \delta(t - kt_s) \right] \otimes \Pi\left(\frac{t}{t_s} - \frac{1}{2} \right)$$

So, we multiply frequency spectra to get the spectrum when the signal is held:

$$Y_{held}(f) = Y(f) F\left\{ \Pi\left(\frac{t}{t_s} - \frac{1}{2} \right) \right\}$$

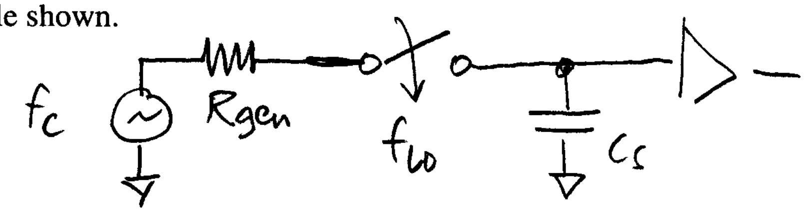
$$= \frac{\sin(\pi f t_s)}{\pi f t_s} \sum_{n=-\infty}^{\infty} X\left(f - \frac{n}{t_s} \right)$$



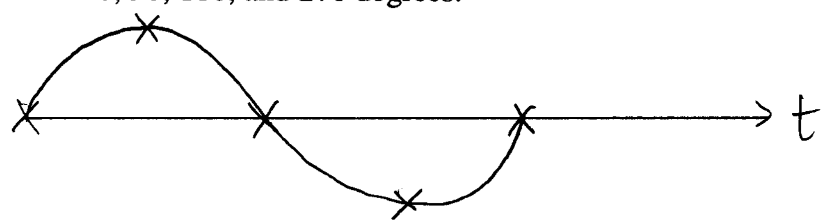
The hold function produces a sinc amplitude distortion. This can be equalized if necessary with an inverse sinc filter. In many cases, the distortion is acceptable. The amplitude is reduced by about 4 dB at $f_s/2$.

An interesting application: Tayloe mixer.

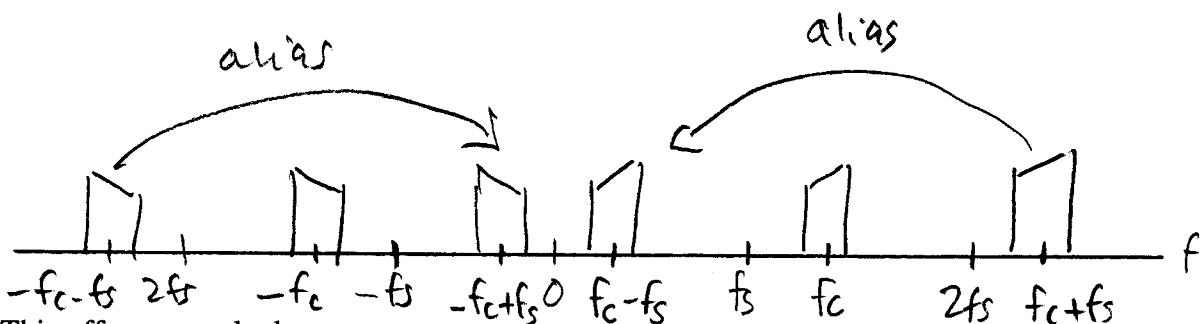
Refer to the block diagram from the patent. The input continuous time signal is sampled 4 times per period. The sampling frequency is the same as the carrier frequency in the example shown.



Sampling occurs at 0, 90, 180, and 270 degrees.



The capacitor performs the hold function and low pass filters the out put at the same time. So, only the downconverted (baseband) replica remains.



This offers several advantages:

- Power is lost in the upconverted sideband in a normal switching mixer. This leads to conversion loss. In the case of this sampling mixer, the upconverted signal is aliased to exactly the downconverted signal frequency.
  $f_c + f_s - 2f_s = f_c - f_s$
- No spectral inversion. Thus, the conversion loss is reduced – on the order of 1 dB rather than 4 dB for a DBM.

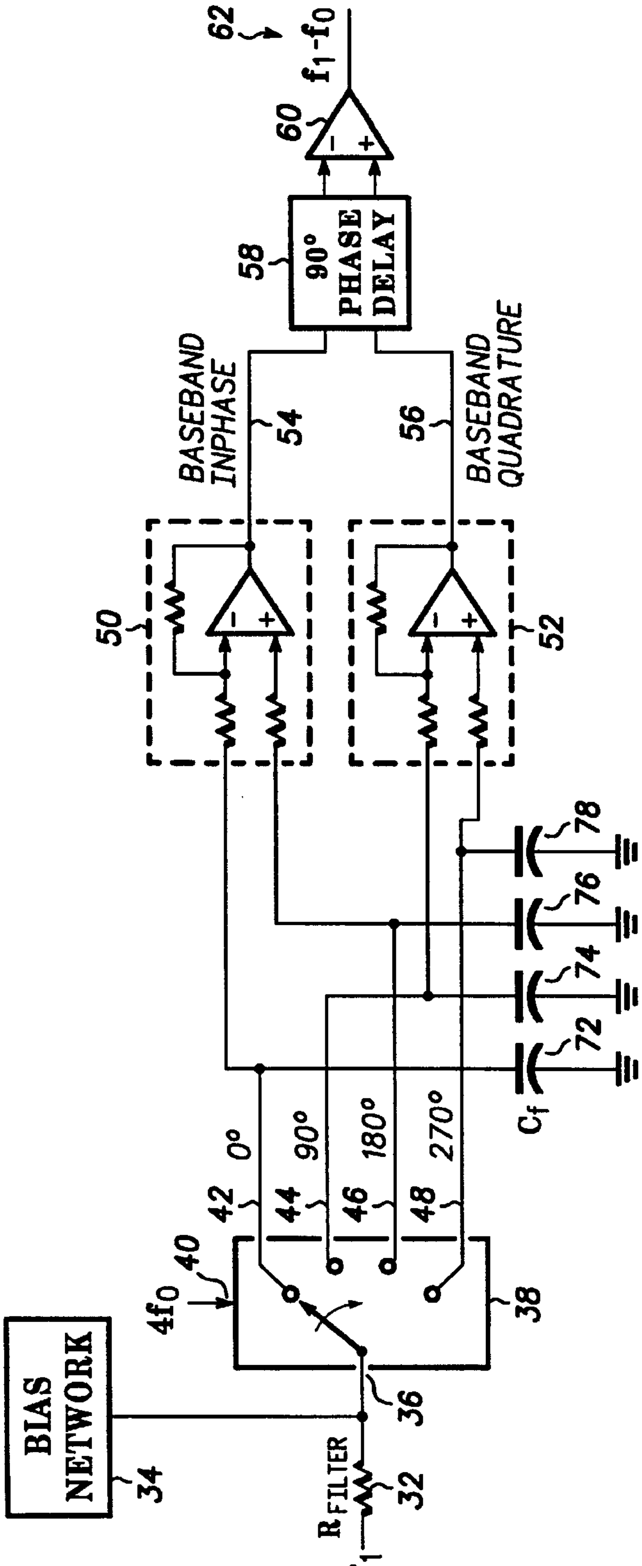

62
f_1-f_0
60
58
90° PHASE DELAY
BASEBAND INPHASE
54
BASEBAND QUADRATURE
56
50
52
78
76
74
72
C_f
0°
90°
180°
270°
42
44
46
48
40
$4f_0$
38
36
BIAS NETWORK
34
R_{FILTER}
32
f_1

FIG. 3

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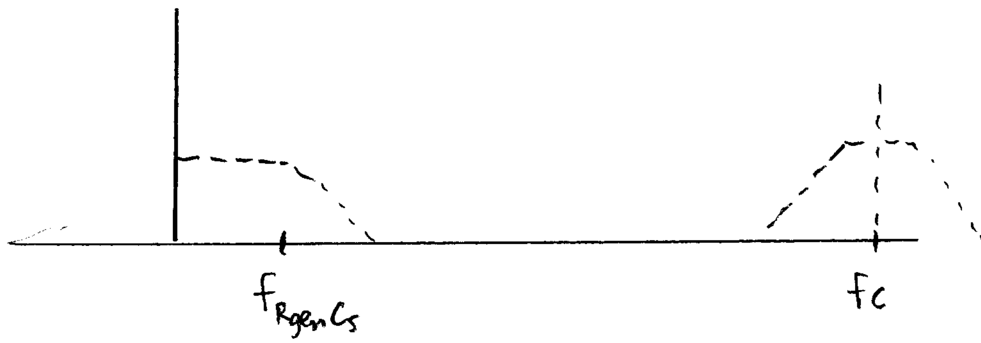
Secondly, the lowpass filtering performed by the R_{gen} C_S network filters the output. This is equivalent to filtering the bandpass input with a very narrow filter at f_C.



Because the capacitor is charged for only ¼ of the period, the effective source resistance is 4 Rgen. In the equation below, n = # samples/period.

$$B = 2\frac{1}{2\pi n C_s R_{gen}} = \frac{1}{4\pi C_s R_{gen}}$$

$$Q_{eff} = \frac{f_c}{B}$$

The effective Q can therefore be very high, controlled by the lowpass filtering of the sampler.